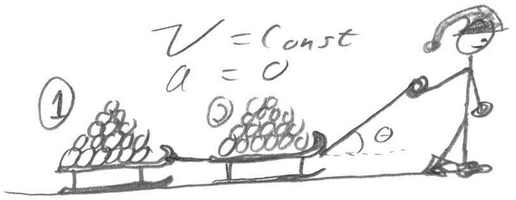
Force Problems – Set 2

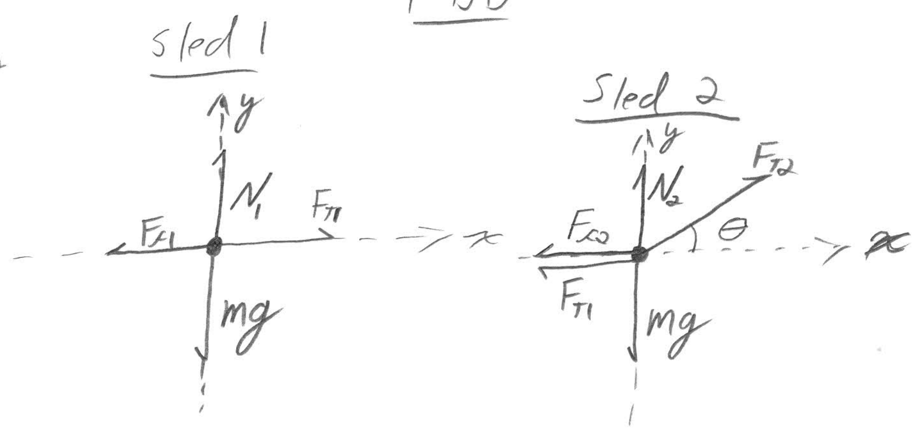
Little Laura Ingalls has to haul firewood to the house on her double sled (two sleds attached with a rope). Each loaded sled is 45kg. The rope connecting the two sleds is horizontal. The rope that Laura is pulling makes an angle of 30° with the horizontal. The coefficient of friction between the sled and the ground is $\mu_k=0.30$.

a) Find the tension in the rope between the two sleds that will keep the sled moving at a constant velocity.

b) Find the tension in the rope that Laura is pulling that will keep both sleds moving at a constant velocity.



FBD



NSL: Sled 1

① x: $F_{T1} - F_{F1} = 0$

② y: $N_1 - mg = 0$

$F_{F1} = N_1 \mu_K$

From ①: $F_{T1} = N_1 \mu_K$

From ②: $N_1 = mg$

Combine: $\boxed{F_{T1} = mg\mu_K}$ ⑤

Sled 2

③ x: $F_{T2}\cos\theta - F_{F2} - F_{T1} = 0$

④ y: $N_2 + F_{T2}\sin\theta - mg = 0$

$F_{F2} = N_2 \mu_K$

From ⑤

From ③: $F_{T2}\cos\theta = N_2\mu_K + mg\mu_K$

From ④: $N_2 = mg - F_{T2}\sin\theta$

continued ↓

Laura's Sled continued

combine:

$$F_{T2}\cos\theta = (mg - F_{T2}\sin\theta)\mu_k + mg\mu_k$$

$$F_{T2}\cos\theta = 2mg\mu_k - \mu_k F_{T2}\sin\theta$$

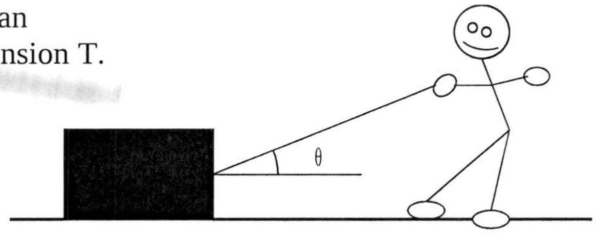
$$F_{T2}(\cos\theta + \mu_k\sin\theta) = 2mg\mu_k$$

$$\boxed{F_{T2} = \frac{2mg\mu_k}{\cos\theta + \mu_k\sin\theta}}$$

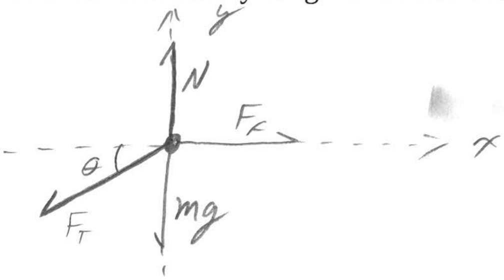
$$F_{T1} = (45\,kg)(9.8\,m/s^2)(0.3) = \underline{132\,N}$$

$$F_{T2} = \frac{(2)(45)(9.8)(0.3)}{\cos(30) + (0.3)\sin(30)} = \underline{260\,N}$$

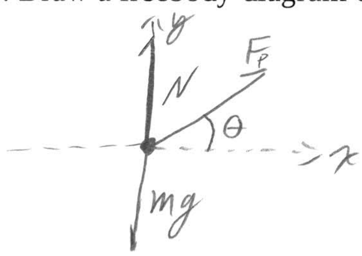
While standing on a rough surface, Stickman is pulling an ice block to the right with a tension T.



a. Draw a freebody diagram of Stickman. (He does not slide)



b. Draw a freebody diagram of the ice block. (It's frictionless)



c. Assume that the Box is frictionless and calculate the velocity of the box after it has traveled a distance d starting from rest. Your velocity should be in terms of m,T, θ, and d.

NSL

x: $F_P \cos\theta = ma \Rightarrow \boxed{a = \frac{F_P}{m}\cos\theta}$

y: $N + F_P \sin\theta - mg = 0$ Okay... Boring.

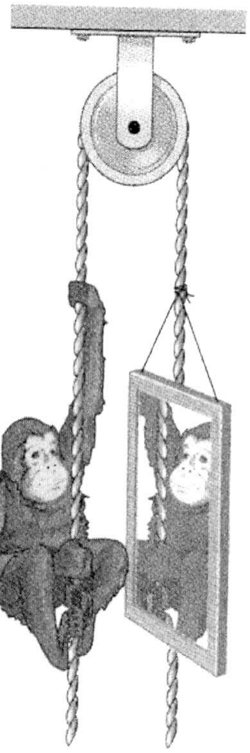
Kinematics

$x = x_0 + v_0 t + \frac{1}{2}at^2$ $\quad v = v_0 + at$

$d = 0 + 0 + \frac{1}{2}at^2$ $\quad v = 0 + at$

$t = \left(\frac{2d}{a}\right)^{1/2}$ $\quad v = a\left(\frac{2d}{a}\right)^{1/2} \Rightarrow \boxed{v = (2d\frac{F_P}{m}\cos\theta)}$

Force Problems – Set 2 3



A massless rope is strung over a massless frictionless pulley. A monkey holds onto the rope and a mirror having the same mass as the monkey is attached to the other side of the rope at the monkey's level. Can the monkey get away from its image by climbing up, climbing down, or letting go of the rope?

Consider the Free-body diagrams:

Monkey

Mirror

NSL: Monkey

$$F_T - m_{monkey}g = m_{monkey}a_{monkey}$$

NSL: Mirror

$$F_T - m_{mirror}g = m_{mirror}a_{mirror}$$

Solve both for F_T and set equal.

$$m_{monkey}g + m_{monkey}a_{monkey} = m_{mirror}g + m_{mirror}a_{mirror}$$

If $m_{monkey} = m_{mirror} = m$,

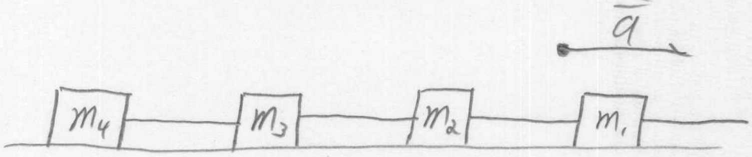
$$\cancel{mg} + \cancel{m}a_{monkey} = \cancel{mg} + \cancel{m}a_{mirror}$$

$$a_{monkey} = a_{mirror} \quad \text{for all } a$$

The monkey can't escape!

Force Homework Packet

Penguins



$m_1 = 20\text{ kg}$ $\quad T_1 = 222\text{ N}$

$m_2 = 15\text{ kg}$ $\quad T_2 = ?$

$\boxed{m_3 = ?}$ $\quad T_3 = 111\text{ N}$

$m_4 = 12\text{ kg}$ $\quad T_4 = ?$

Surface is frictionless so I'll ignore N and mg.

FBD

NSL

1 m_4: $T_4 = m_4 a$

2 m_3: $T_3 - T_4 = m_3 a$

3 m_2: $T_2 - T_3 = m_2 a$

4 m_1: $T_1 - T_2 = m_1 a$

The system is linked by ropes so a is the same for all penguins.

We want m_3, so we'll eliminate T_2, T_4, and a

use eq 3 and 4 to eliminate T_2:

$T_2 = T_3 + m_2 a \qquad T_2 = T_1 - m_1 a$

5 $T_3 + m_2 a = T_1 - m_1 a$

continued ↓

Penguins continued

Use eq 1 and 2 to eliminate T_4

$$T_4 = m_4 a \qquad T_4 = T_3 - m_3 a$$

$$m_4 a = T_3 - m_3 a \quad (6)$$

Re-arrange 5 and 6 to get a on one side

$$T_1 - T_3 = m_2 a + m_1 a \qquad T_3 = m_4 a + m_3 a$$

$$T_1 - T_3 = a(m_2 + m_1) \qquad T_3 = a(m_3 + m_4)$$

divide

$$\frac{T_1 - T_3}{T_3} = \frac{\cancel{a}(m_2 + m_1)}{\cancel{a}(m_3 + m_4)}$$

Isolate m_3

$$(T_1 - T_3)(m_3 + m_4) = T_3(m_2 + m_1)$$

$$T_1 m_3 + T_1 m_4 - T_3 m_3 - T_3 m_4 = T_3 m_2 + T_3 m_1$$

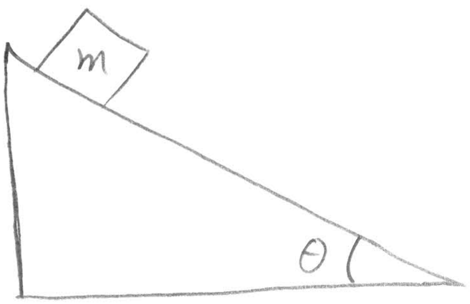
$$m_3(T_1 - T_3) = T_3 m_2 + T_3 m_1 + T_3 m_4 - T_1 m_4$$

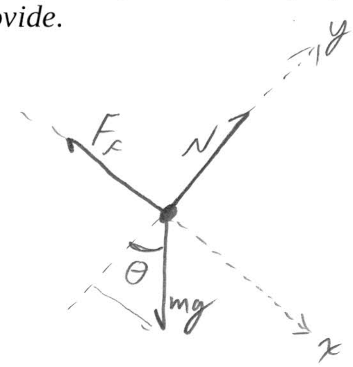
$$\boxed{m_3 = \frac{T_3(m_1 + m_2 + m_4) - T_1 m_4}{T_1 - T_3}} \qquad m_3 = \frac{111(20 + 15 + 12) - 222 \cdot 12}{222 - 111}$$

$$\boxed{= 23\,\text{kg}}$$

A block rests on an incline plane. The coefficient of friction between the block and the plane is μ_s. Find an expression for the maximum angle of the incline before the block slips.

HINT: The equation for the force of static friction ($F = \mu_s N$) represents the ***maximum*** *force that friction can provide.*





NSL

① x: $mg\sin\theta - F_f = 0$ ⇒ When this is true, the block will be on the verge of slipping.

y: $N - mg\cos\theta = 0$

② ⇒ $N = mg\cos\theta$

From ①: $mg\sin\theta = \mu_s N$

Subst N from ②: $\cancel{mg}\sin\theta = \mu_s \cancel{mg}\cos\theta$

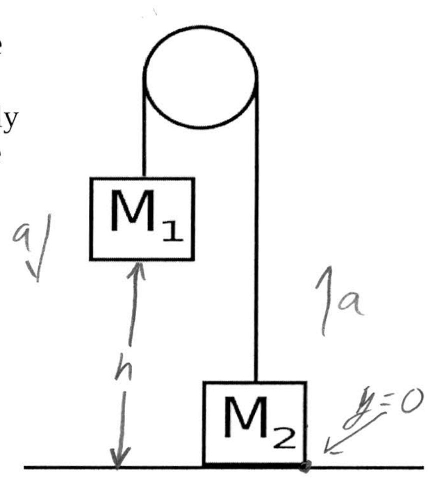
$\tan\theta = \mu_s$

$$\boxed{\theta = \tan^{-1}(\mu_s)}$$

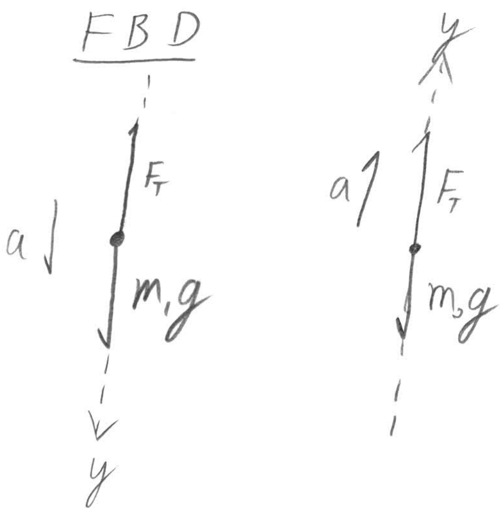
Force Problems – Set 2

One end of a rope is connected to a mass M_1=10kg. The rope passes over a massless frictionless pulley and the other end is connected to a mass M_2 = 5kg. M_2 is initially resting on the ground and M_1 is suspended 3m above the ground. The system is initially at rest.

If M_1 is released and allowed to hit the ground, what is the maximum height that M_2 will reach?



*HINT: When M_1 hits the ground, M_2 will still have an upward velocity. The rope will go slack and M_2 will **continue** upward until its velocity is zero.*



NSL

① $m_1 g - F_T = m_1 a$

② $F_T - m_2 g = m_2 a$

From ②: $F_T = m_2 a + m_2 g$

Into ①: $m_1 g - m_2 a - m_2 g = m_1 a$

$$(m_1 - m_2) g = (m_1 + m_2) a$$

③ $$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2} g}$$

continued

Atwood — continued

We have acceleration, now we'll do kinematics.

M_2 will accelerate through a distance h with a from the force analysis. Then, it will essentially be in "free fall" with some initial upward velocity $\underline{V_1}$.

Find V_1

$$y = y_0 + V_0 t + \tfrac{1}{2} a t^2$$

$$h = 0 + 0 + \tfrac{1}{2} a t^2$$

$$t = \left(\frac{2h}{a}\right)^{1/2}$$

$$V = V_0 + at$$

$$V_1 = 0 + at$$

$$V_1 = a\left(\frac{2h}{a}\right)^{1/2} = (2ha)^{1/2}$$

$$\boxed{V_1 = (2ha)^{1/2}} \quad 4$$

Now find h_{max}

$$y = y_0 + V_0 t + \tfrac{1}{2} a t^2$$

$$V = V_0 + at$$

$$0 = V_1 - gt$$

$$t = \frac{V_1}{g}$$

$$h_{max} = h + V_1 t - \tfrac{1}{2} g t^2$$

$$h_{max} = h + \frac{V_1^2}{g} - \frac{1}{2}\frac{V_1^2}{g}$$

$$\boxed{h_{max} = h + \tfrac{1}{2}\frac{V_1^2}{g}} \quad 5$$

continued

↓

Atwood continued.

combine 3, 4, and 5

$$h_{max} = h + \frac{\cancel{2}ha}{\cancel{2}g}$$

$$h_{max} = h + \frac{h}{\cancel{g}} \frac{m_1 - m_2}{m_1 + m_2} \cancel{g}$$

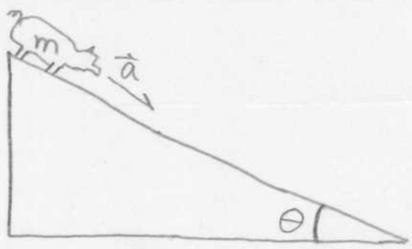
$$h_{max} = h\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right) = h \frac{m_1 + \cancel{m_2} + m_1 - \cancel{m_2}}{m_1 + m_2}$$

$$\boxed{h_{max} = \frac{2m_1}{m_1 + m_2} h}$$

So... If $m_2 < m_1$, $h_{max} > h$.

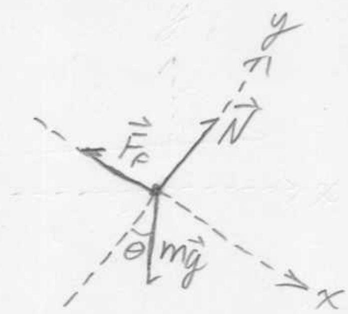
$$h_{max} = \frac{(2)(10)}{10 + 5}(3) = \frac{20}{15} 3 = 4_m$$

Force Homework, #3



$\theta = 35^\circ$

$t_F = 2t_{NF}$



In general (Newton's 2^{nd} law)

$F_x = mg\sin\theta - F_F = ma$ $\qquad$ $F_y = N - mg\cos\theta = 0$

$mg\sin\theta - \mu_s N = ma$ $\qquad$ $N = mg\cos\theta$

$$\cancel{m}g\sin\theta - \mu_s \cancel{m}g\cos\theta = \cancel{m}a$$

$$\boxed{a = g(\sin\theta - \mu_s\cos\theta)}$$

Kinematics

$x(t) = \cancel{x_0}^{0} + \cancel{v_0}^{0}t + \frac{1}{2}at^2$

$d = \frac{1}{2}at^2 \Rightarrow \boxed{d = \frac{1}{2}g(\sin\theta - \mu_s\cos\theta)t^2}$

Frictionless

$\mu_s = 0$

$d = \frac{1}{2}g\sin\theta\, t_{NF}^2$

Friction

$d = \frac{1}{2}g(\sin\theta - \mu_s\cos\theta)t_F^2$

$d = \frac{1}{2}g(\sin\theta - \mu_s\cos\theta)4t_{NF}^2$

divide

$$\frac{\cancel{d}}{\cancel{d}} = \frac{\cancel{\frac{1}{2}g}\sin\theta\,\cancel{t_{NF}^2}}{\cancel{\frac{1}{2}g}(\sin\theta - \mu_s\cos\theta)4\cancel{t_{NF}^2}} \Rightarrow \boxed{\mu_s = \frac{3}{4}\tan\theta}$$

$\mu_s = 0.53$