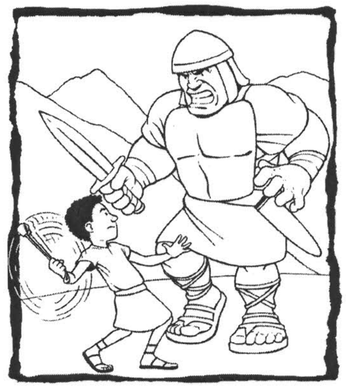
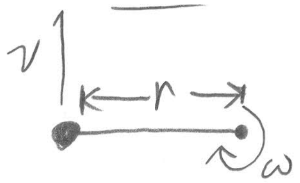
Force Problems – Set 3

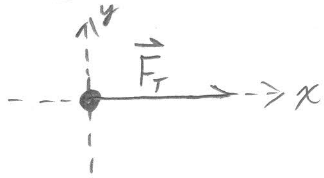
David is fighting Goliath. To defeat him, David's rock must leave his 0.5 m sling at 20 m/s. Unfortunately, his sling was damaged and the maximum tension the rope can withstand is 500 N. What is the mass of largest rock that David can sling? Ignore gravity.



Picture



FBD



NSL

$$\sum \vec{F} = m\vec{a}$$

x: $F_T = m r\omega^2$ ← Uniform circular motion, $a = r\omega^2$ or $a = \frac{v^2}{r}$

① $F_T = m\frac{v^2}{r}$

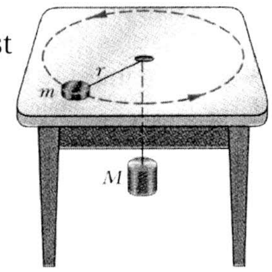
y: $0 = 0$

so, solve ① for m

From ①: $\boxed{m = F_T \frac{r}{v^2}}$

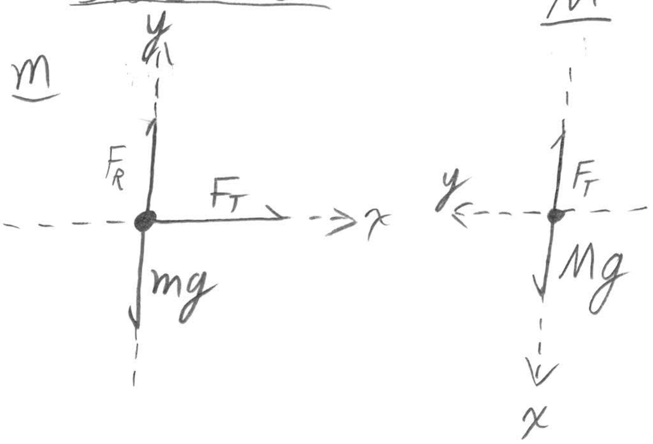
$$m = 500\,N \cdot \frac{0.5\,m}{(20\,m/s)^2} = \boxed{0.625\,kg}$$

A mass m on a frictionless table is attached to a hanging mass M by a cord through a hole in the table. Find the speed with which m must move in order for M to stay at rest.



FBD

Side view



NSL

m

x: $\sum F_x = ma_x$

① $F_T = m\frac{v^2}{r} \Rightarrow$ UCM

y: $\sum F_y = ma_y$

② $F_R - mg = 0 \Rightarrow$ No motion in y

NSL

M

x: $\sum F_x = ma_x$

③ $Mg - F_T = 0$

y: $0 = 0$

So, from ③: $F_T = Mg$

then into ①: $Mg = m\frac{v^2}{r}$

$$\Rightarrow \boxed{v = \left[\frac{M}{m}gr\right]^{1/2}}$$

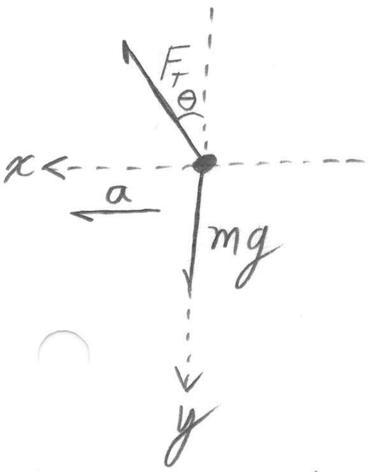
Force Problems – Set 3

Napoleon lands a flippin' sweet blow to the tether ball, which is attached to a rope of whose length is $L = 0.60\text{m}$ sending it whipping around the pole with a period of of 0.75s.

a) Napoleon is jealous of your awesome physics skills and needs your help finding an expression for the angle, θ, that the rope makes with the pole.

b) Calculate a numeric value for θ using your expression from part a and use it to calculate the tension in the rope.

FBD



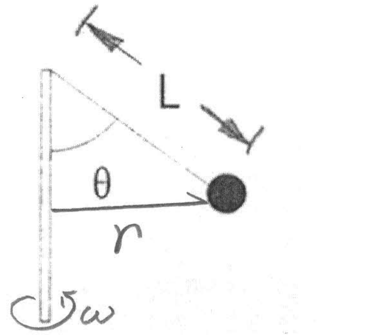
NSL

$x: \sum F_x = ma_x$

① $F_T \sin\theta = m\frac{v^2}{r}$

$y: \sum F_y = ma_y$

② $F_T \cos\theta - mg = 0$



$r = L\sin\theta$

Let's eliminate F_T Between ① and ② First and then deal with the period.

From ②: $F_T \cos\theta = mg$ ③

Divide $\frac{①}{③}$: $\frac{F_T \sin\theta}{F_T \cos\theta} = \frac{m\frac{v^2}{r}}{mg} \Rightarrow$ $\boxed{\tan\theta = \frac{v^2}{rg}}$ ④

Now we can relate v and r to the period.

$v = \frac{d}{t} \Rightarrow \boxed{v = \frac{2\pi r}{P}}$ Plug into ④

continued ↓

Napoleon - continued

From (4): $\tan\theta = \frac{4\pi^2 r}{P^2} \frac{1}{g}$

$$\tan\theta = \frac{4\pi^2 r}{g P^2} \quad (5)$$

But $r = L\sin\theta$

So: $\tan\theta = \frac{4\pi^2}{g P^2} L\sin\theta$

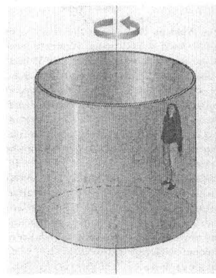
$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{4\pi^2 L}{g P^2}\sin\theta$$

$$\Rightarrow \boxed{\cos\theta = \frac{g P^2}{4\pi^2 L}}$$

b) $\theta = \cos^{-1}\left(\frac{gP^2}{4\pi^2 L}\right) = \cos^{-1}\left[\frac{(9.8)(0.75)^2}{(4)\pi^2(0.6)}\right]$

$$\boxed{\theta = 76^\circ}$$

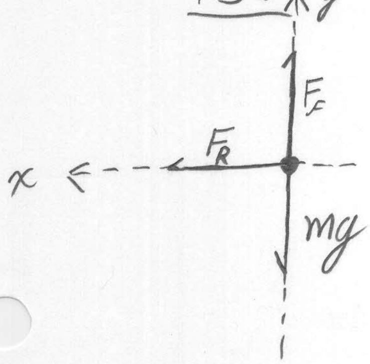
The Cyclone is an exciting state fair ride consisting of a large vertical cylinder that spins about its axis. Riders stand inside the cylinder with their backs against the wall. When the cylinder is spinning fast enough, the floor drops away and the rider "sticks" to the wall.



If the The coefficient of static friction between a wall and person is μ_s, and the radius of the cylinder is R, show that the maximum period of revolution is given by:

$$P=\left(\frac{4\pi^2 R\mu_s}{g}\right)^{\frac{1}{2}}$$

FBD



NSL

$x: \sum F_x = ma_x$

① $F_R = mr\omega^2$

$y: \sum F_y = ma_y$

$F_f - mg = 0$

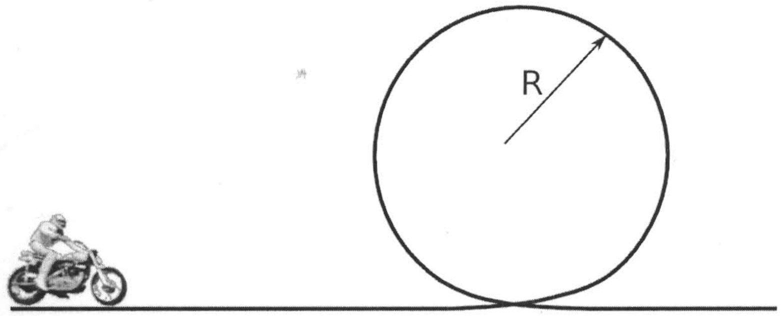
② $\mu_s F_R - mg = 0$

Subst ① → ② : $\mu_s mr\omega^2 = mg$ $\qquad \omega = \frac{2\pi}{P}$

$\mu_s r \frac{4\pi^2}{P^2} = g$

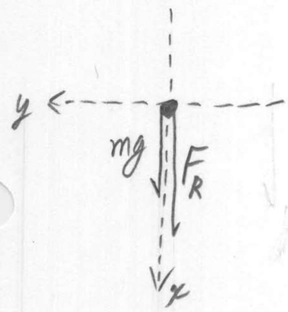
$$\boxed{P = \left(\frac{4\pi^2 \mu_s r}{g}\right)^{1/2}}$$

Evel Knievel is going to do the loop the loop as a warm up to jumping 50 busses, a tank of piranha, and a tank of sharks. The loop is a circle with radius R. What is the minimum speed that Evel can be going **at the top** of the loop without falling?



At the top: have to be going Fast

FBD



NSL

$$F_R + mg = m\frac{v^2}{r} \Rightarrow$$ Must be true to stay in UCM

When the motorcycle just starts to fall,

$F_R = 0$.

When the motorcycle goes really fast,

$F_R > 0$

So, the critical speed is when $F_R = 0$

$$0 + mg = m v^2/r \Rightarrow \boxed{v = \sqrt{gr}}$$