Test 2
Phys 111, Fall 2010, Section 1
Name: ______________________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

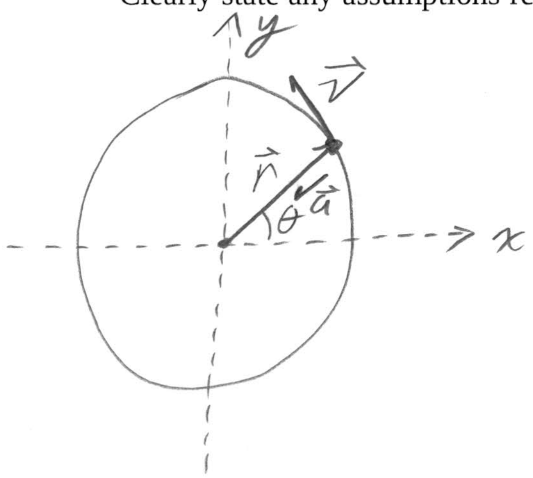
SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

1) Starting with the unit vector expression for the position, r, of a particle constrained to move in a circle, derive an expression for the magnitude of the velocity vector and the magnitude of the acceleration vector assuming that the particle is moving in **uniform circular motion.**

Include a picture with the position vector, r, the velocity vector, v, the acceleration vector, a, and the position angle, θ, clearly marked.

Clearly state any assumptions required by the proof.

assume: r and $\frac{d\theta}{dt}$ are const.



$$\vec{r} = r_x \hat{i} + r_y \hat{j}$$

$$\vec{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = r\frac{d}{dt}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$= r\left(-\frac{d\theta}{dt}\sin\theta\,\hat{i} + \frac{d\theta}{dt}\cos\theta\,\hat{j}\right)$$

$$\vec{v} = r\frac{d\theta}{dt}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$\boxed{v = r\omega} \quad \omega \equiv \frac{d\theta}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = r\omega\frac{d}{dt}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$= r\omega\frac{d\theta}{dt}(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

$$\boxed{a = r\omega^2}$$

SAMPLE TEST 2
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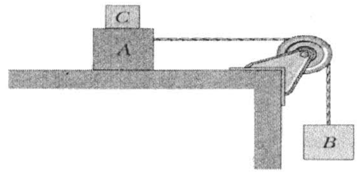
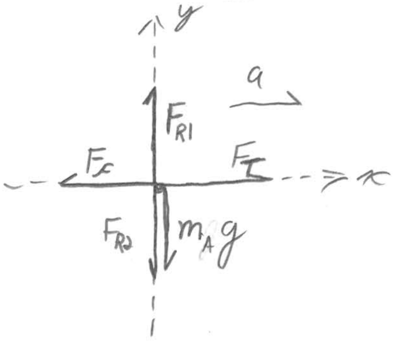
2) A and B are blocks with ~~weights~~ masses of 44 ~~N~~ kg and 22 ~~N~~ kg respectively. The coefficient of friction between the table and the block is 0.20.

a) Determine the minimum weight of block C so that block A does not slide.

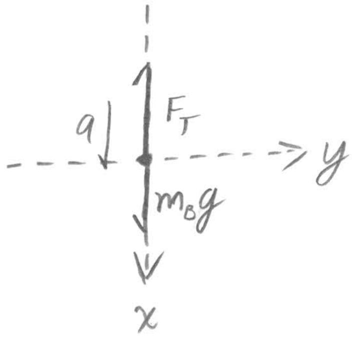
b) If block C is suddenly lifted off of A, what is the acceleration of A?



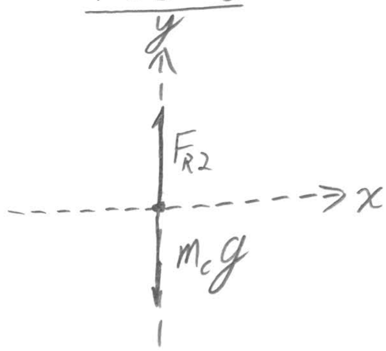
FBD A



FBD B



FBD C



NSL A

$x: \sum F_x = ma_x$

$F_T - F_f = 0$

$$① \underbrace{F_T - \mu_s F_{R1} = 0}_{\text{slipping condition}}$$

$y: \sum F_y = ma_y$

$② F_{R1} - F_{R2} - m_A g = 0$

NSL B

$x: \sum F_x = ma_x$

$③ m_B g - F_T = 0$ static

$y: \sum F_y = ma_y$

$0 = 0$

NSL C

$x: \sum F_x = ma_x$

$0 = 0$

$y: \sum F_y = ma_y$

$④ F_{R2} - m_c g = 0$

* Find m_c in terms of μ_s, m_A, and m_B. Need to eliminate all reaction forces

Add ② + ④ : $F_{R1} - \cancel{F_{R2}} - m_A g + \cancel{F_{R2}} - m_c g = 0 + 0$

$⑤ F_{R1} - (m_A + m_c) g = 0$

Add ① + ③ : $\cancel{F_T} - \mu_s F_{R1} + m_B g - \cancel{F_T} = 0 + 0$

$⑥ m_B g - \mu_s F_{R1} = 0$

continued ↓

continued (ST2, P2)

Solve 5 for F_{R1} and subst into 6

$$F_{R1} = (m_A + m_C)g$$

$$⑦\ m_B g - \mu_s (m_A + m_C) g = 0$$

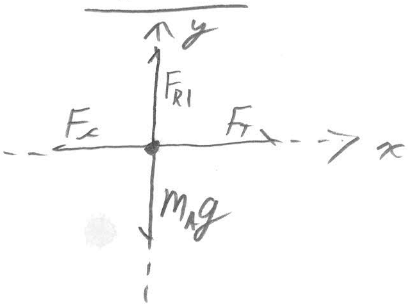
Solve 7 for m_C

$$m_B \cancel{g} = \mu_s m_A \cancel{g} + \mu_s m_C \cancel{g}$$

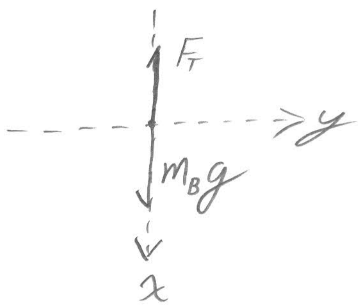
$$\boxed{m_C = \frac{1}{\mu_s}\left[m_B - \mu_s m_A\right]}$$

$$m_C = \frac{1}{0.2}\left[22 - (0.2)(44)\right] = \boxed{66\ kg}$$

b) FBD A



FBD B



NSL A

$$x: \Sigma F_x = ma_x$$

$$F_T - F_c = m_A a$$

$$①\ F_T - \mu_k F_{R1} = m_A a$$

$$y: \Sigma F_y = ma_y$$

$$②\ F_{R1} - m_A g = 0$$

NSL B

$$x: \Sigma F_x = ma_x$$

$$③\ m_B g - F_T = m_B a$$

$$y: \Sigma F_y = ma_y$$

$$0 = 0$$

continued ↓

continued (ST2, P2)

Solve 2 for F_{R1} and subst. into 1

4 $F_T - \mu_K m_A g = m_A a$

add 4 + 3

5 $\cancel{F_T} - \mu_K m_A g + m_B g - \cancel{F_T} = m_A a + m_B a$

Solve 5 for a

$$\boxed{a = \frac{m_B - \mu_K m_A}{m_A + m_B} g} = \frac{22 - (0.2)(44)}{44 + 22}(9.8) = \boxed{1.96\ m/s^2}$$

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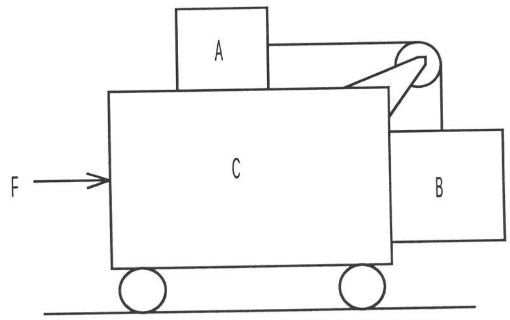
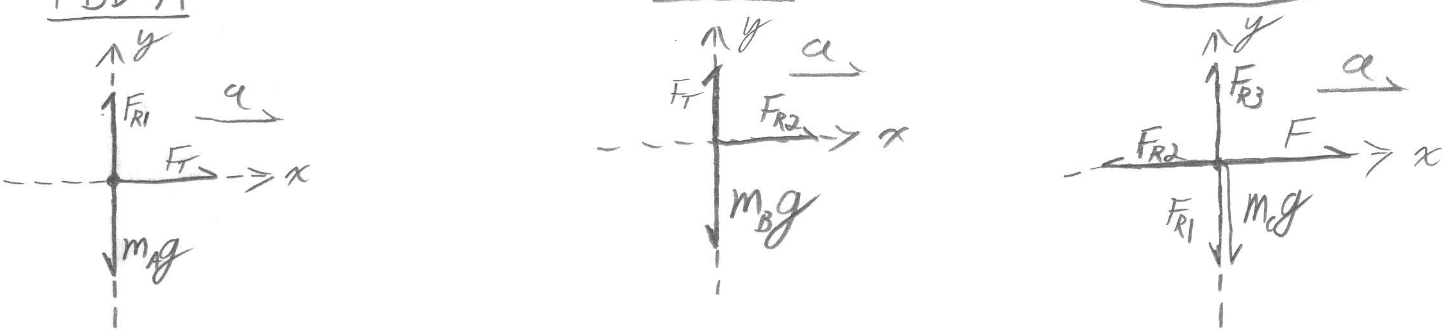
3) Find an expression for the horizontal force, F, that be must be applied to the cart so that the blocks remain stationary relative to the cart. Assume all surfaces are frictionless and that all wheels and pulleys are massless and frictionless.



FBD A   FBD B   FBD C



NSL A

$x: \sum F_x = ma_x$

① $F_T = m_A a$

$y: \sum F_y = ma_y$

② $F_{R1} - m_A g = 0$

NSL B

$x: \sum F_x = ma_x$

③ $F_{R2} = m_B a$

$y: \sum F_y = ma_y$

④ $F_T - m_B g = 0$

NSL C

$x: \sum F_x = ma_x$

⑤ $F - F_{R2} = m_C a$

$y: \sum F_y = ma_y$

⑥ $F_{R3} - F_{R1} - m_C g = 0$

I want F, so I'll start with ⑤ and eliminate F_{R2} by adding ⑤ + ③

$$F - \cancel{F_{R2}} + \cancel{F_{R2}} = m_C a + m_B a \Rightarrow \boxed{⑦\; F = (m_B + m_C) a}$$

Now I need to eliminate a From ⑦ using ①. But First I'll eliminate F_T From ① by substituing From ④.

continued ↓

Continued (ST2, P3)

From ④: $F_T = m_B g$

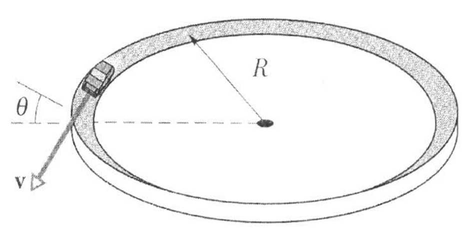
Subst into ①: $m_B g = m_A a \Rightarrow \boxed{a = \frac{m_B}{m_A} g}$

Subst a into ⑦:

$$\boxed{F = (m_B + m_C)\frac{m_B}{m_A} g}$$

Sample Test 2
Phys 111 Spring 2010

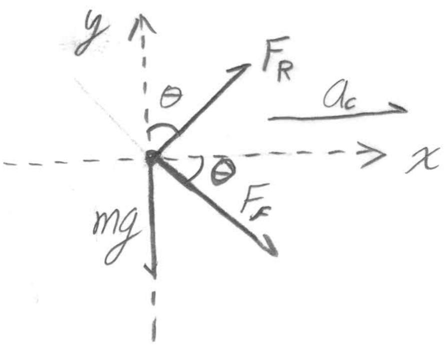
4) Curves on roadways are often banked so that the reaction force from the road provides part of the centripetal acceleration required to keep the car moving around the circle.



Assume the road is a circle of radius R, the car is going some velocity v, and the tires have some coefficient of static friction μ_s.

Find an expression for the minimum angle required to keep the car on the road in terms of R, v, and μ_s.

FBD - side view



NSL

$x: \sum F_x = m a_x$

① $F_R \sin\theta + \mu_s F_R \cos\theta = m \frac{v^2}{R}$

$y: \sum F_y$

$F_R \cos\theta - \mu_s F_R \sin\theta - mg = 0$

② $\Rightarrow F_R \cos\theta - \mu_s F_R \sin\theta = mg$

Divide $\frac{①}{②}$:

$$\frac{\cancel{F_R}(\sin\theta + \mu_s \cos\theta)}{\cancel{F_R}(\cos\theta - \mu_s \sin\theta)} = \frac{\cancel{m} v^2}{\cancel{m} g R}$$

$gR\sin\theta + \mu_s gR\cos\theta = v^2\cos\theta - \mu_s v^2 \sin\theta$

$\Rightarrow (gR + \mu_s v^2)\sin\theta = (v^2 - \mu_s g R)\cos\theta$

$$\Rightarrow \boxed{\tan\theta = \frac{v^2 - \mu_s g R}{gR + \mu_s v^2}}$$

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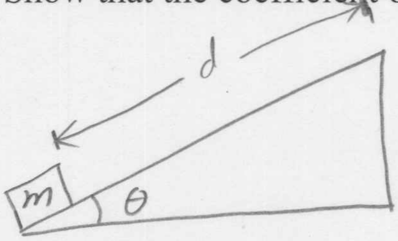
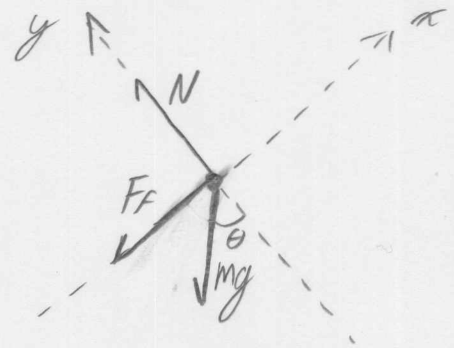
A block is projected up an incline plane making an angle θ with the horizontal. It returns to its initial position with half of its initial speed. Show that the coefficient of kinetic friction is: $\mu_k = 3/5\tan(\theta)$.



So: the block has an initial velocity V_0, it goes up the ramp a distance d, stops, and comes back down. Its final velocity is:

$$V_f = \tfrac{1}{2} V_0$$

On the way up – Friction opposes motion



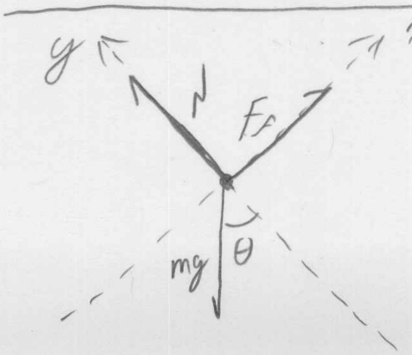
$$x: -\mu_k N - mg\sin\theta = ma_{up}$$

$$y: N - mg\cos\theta = 0$$

$$\Rightarrow \mu_k \cancel{m}g\cos\theta + \cancel{m}g\sin\theta = -\cancel{m}a_{up}$$

$$\Rightarrow \boxed{a_{up} = -g(\sin\theta + \mu_k\cos\theta)}$$

On the way down – Friction opposes motion



$$x: \mu_k N - mg\sin\theta = ma_{dwn}$$

$$y: N - mg\cos\theta = 0$$

$$\boxed{a_{dwn} = -g(\sin\theta - \mu_k\cos\theta)}$$

so the acceleration up is different than the acceleration down. So we'll take the kinematiccs in two parts, up and then down.

up - a distance d and stops

$x_f = x_0 + v_0 t + \frac{1}{2} a_{up} t^2$ $\qquad$ $v_f = v_0 + a_{up} t$

$d = 0 + v_0 t + \frac{1}{2} a_{up} t^2$ $\qquad$ $0 = v_0 + a_{up} t$

Eliminate t $\qquad$ $t = -\frac{v_0}{a_{up}}$

$$d = -\frac{v_0^2}{a_{up}} + \frac{1}{2}\frac{v_0^2}{a_{up}}$$

$$\boxed{d = -\frac{1}{2}\frac{v_0^2}{a_{up}}}$$

down - goes a distance d from rest

$x_f = x_0 + v_0 t + \frac{1}{2} a_{dwn} t^2$ $\qquad$ $v_f = v_0 + a_{dwn} t$

$0 = d + 0 + \frac{1}{2} a_{dwn} t^2$ $\qquad$ $v_f = a_{dwn} t$

$0 = d + \frac{1}{2}\frac{v_f^2}{a_{dwn}}$ $\qquad$ $t = \frac{v_f}{a_{dwn}}$

Plug in d

$\Rightarrow 0 = -\cancel{\frac{1}{2}}\frac{v_0^2}{a_{up}} + \cancel{\frac{1}{2}}\frac{v_f^2}{a_{dwn}}$ and use $v_f = \frac{1}{2} v_0$

$$\frac{\cancel{v_0^2}}{a_{up}} = \frac{1}{4}\frac{\cancel{v_0^2}}{a_{dwn}} \Rightarrow 4 a_{dwn} = a_{up}$$

continued ↓

$$+4\cancel{g}(\sin\theta - \mu_k\cos\theta) = \cancel{g}(\sin\theta + \mu_k\cos\theta)$$

$$4\sin\theta - 4\mu_k\cos\theta = \sin\theta + \mu_k\cos\theta$$

$$3\sin\theta = 5\mu_k\cos\theta$$

$$\boxed{\mu_k = \tfrac{3}{5}\tan\theta}$$