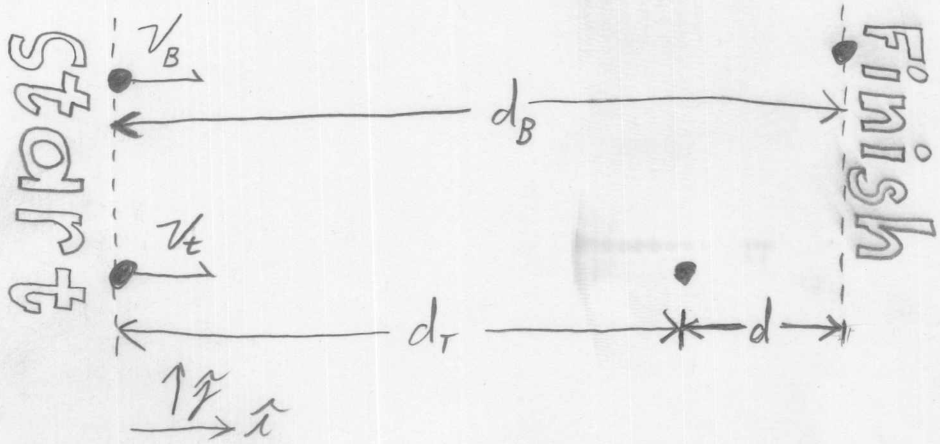
Kinematics

Bill and Ted are going to race. Bill can run 15 mi/hr and Ted can only run 11 mi/hr. How far from the finish line is Ted when Bill finishes a quarter mile race?



$v_B = 15 \,{}^{mi}/_{hr}$

$v_T = 11 \,{}^{mi}/_{hr}$

$d_B = 0.25 \, mi$

$d_T = ?$

$d = d_B - d_T$

What's the physics principle?

① need to relate distance and velocity.

② velocity is constant.

③ There are two players.

$$v_x = \frac{dx}{dt} \Rightarrow \int_{t_0}^{t} v_x \, dt = \int_{x_0}^{x} dx \quad \text{relates } d \text{ and } v$$

Relate <u>position</u> to <u>velocity</u> for each player

Bill: $$\int_0^t v_B \, dt = \int_0^{d_B} dx$$ (d_B ← Final Position; 0 ← initial position) what t? When Bill finishes

Ted: $$\int_0^t v_T \, dt = \int_0^{d_T} dx$$ (d_T ← Final position; 0 ← initial position (same as Bill's))

continued ↓

Bill and Ted continued

① $v_B t = d_B$, ② $v_T t = d_T$ We have equations!

$\quad ? \qquad\qquad ? \quad ?$

I want d_T ... Well, really, I want d so lets put that in

$d = d_B - d_T \Rightarrow d_T = d_B - d$

and ② becomes: $v_T t = d_B - d$

$\quad ? \qquad ?$

so I have 2 equations and 2 unknowns.

I know: v_B, v_T, and d_B

I do not know: t and d

I want d so I'll eliminate t

by: Solving ① for t : $t = \frac{d_B}{v_B}$

and putting it back into ②:

$$v_t \frac{d_B}{v_B} = d_B - d \quad \text{and solve for } d$$

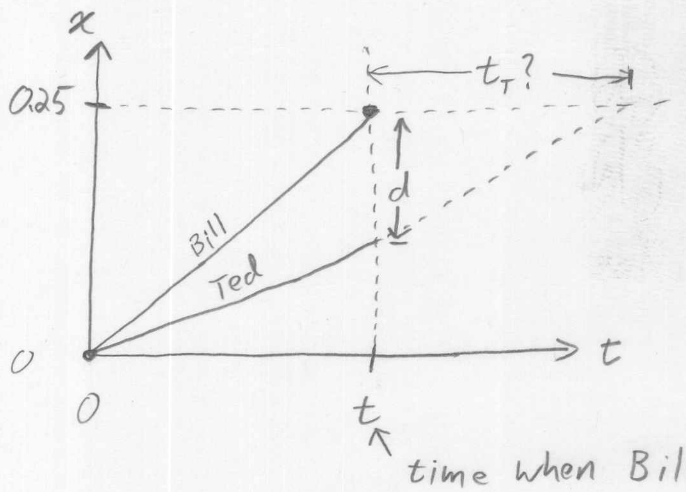
$$d = d_B - d_B \frac{v_t}{v_B} = d_B\left(1 - \frac{v_t}{v_B}\right) = d_B \frac{v_B - v_t}{v_B}$$

$$\boxed{d = d_B \frac{v_B - v_T}{v_B}}$$

Oh yes, numbers... $d = 0.25\,mi \; \frac{15\,mi/h - 11\,mi/h}{15\,mi/h} = 0.07\,mi$

continued ↓

Graph it:



Corrallary question? How much Looonger does it take for Ted to finish?

Let's use this new tool:

$$V_x = \frac{dx}{dt} \Rightarrow \int_0^t V_x\,dt = \int_{x_0}^{x} dx \Rightarrow V_x t = x - x_0$$

$$\underset{\text{Final}}{x(t)} = \underset{\text{initial}}{x_0} + V_x t$$ constant Velocity

Bill

$$d_B = 0 + V_B t_B$$

Ted

$$d_B = 0 + V_T t_T$$

We want $\Delta t = t_T - t_B$

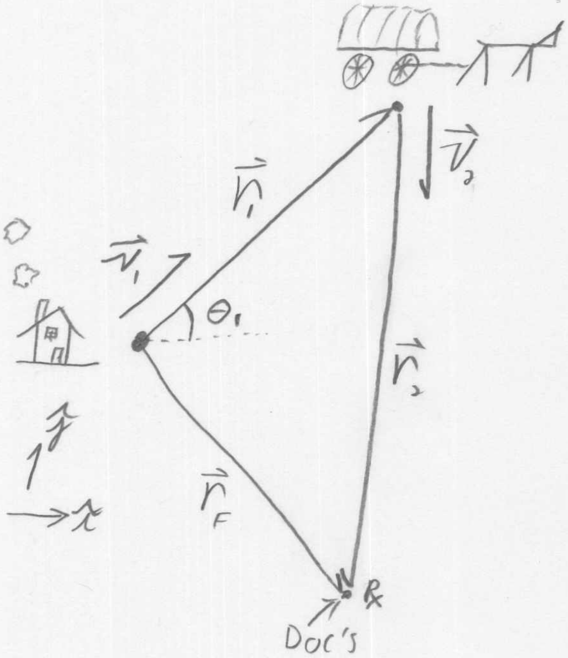
$$\Delta t = \frac{d_B}{V_T} - \frac{d_B}{V_B} = d_B\left(\frac{1}{V_T} - \frac{1}{V_B}\right) = d_B\left(\frac{V_B - V_T}{V_B V_T}\right)$$

$$\boxed{\Delta t = d_B\left(\frac{V_B - V_T}{V_B V_T}\right)}$$

Kinematics

Ma and Pa Ingalls are going to have another baby! Ma says "It's time!" so Pa gets the team hitched up and they take off for Doc Weber's place. Doc's place is 15 miles south and 10 miles east of the homestead. But In his excitement, Pa drives northeast at 18 miles per hour for 1.5 hours. The baby will arrive 4 hours after Ma said "It's time!"

How fast and in what direction does Pa have to drive to make it to Doc's before the baby arrives? Find the magnitude and the direction.



$\vec{r}_F = (10\hat{\imath} - 15\hat{j})$ mi

$|\vec{v}_1| = 18$ mi/hr

$\theta_1 = 45°$

$t_1 = 1.5$ hr

$t_T = 4$ hr

This is a vector displacement problem with constant velocities.

Vector sum: $\vec{r}_1 + \vec{r}_2 = \vec{r}_F$ ①

And constant velocity:

$$\vec{r} = \vec{v}t + \vec{r}_0$$

Two legs of the trip: $\vec{r}_1 = \vec{v}_1 t_1$ ②

$\vec{r}_2 = \vec{v}_2 t_2$ ③

continued ↓

Ma and Pa continued

combine equations ①, ②, and ③

$$\vec{V}_1 t_1 + \vec{V}_2 t_2 = \vec{r}_F$$

But... we don't know t_2. We know $t_1 + t_2 = t_T$

$$\Rightarrow \underline{t_2 = t_T - t_1}$$

$$\vec{V}_1 t_1 + \vec{V}_2 (t_T - t_1) = \vec{r}_F$$

Now solve for $\vec{V}_2$

$$\boxed{\vec{V}_2 = \frac{\vec{r}_F - \vec{V}_1 t_1}{(t_T - t_1)}}$$

Now we can solve for the components of $\vec{V}_2$

$$x: \; V_{2x} = \frac{r_{Fx} - V_{1x} t_1}{t_T - t_1}, \quad V_{1x} = |\vec{V}_1| \cos(\theta_1)$$

$$V_{2x} = \frac{r_{Fx} - |\vec{V}_1| \cos(\theta_1) t_1}{t_T - t_1} = \frac{10 - (18)\cos(45)(1.5)}{4 - 1.5} = \boxed{-3.6 \text{ mi/hr}}$$

$$y: \; V_{2y} = \frac{r_{Fy} - V_{1y} t_1}{t_T - t_1}, \quad V_{1y} = |\vec{V}_1| \sin(\theta_1)$$

$$V_{2y} = \frac{r_{Fy} - |\vec{V}_1| \sin(\theta_1) t_1}{t_T - t_1} = \frac{-15 - (18)\sin(\theta_1)(1.5)}{4 - 1.5} = \boxed{-14 \text{ mi/hr}}$$

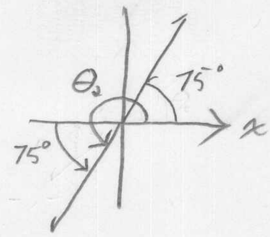
continued ↓

Ma and Pa continued

Finally, calculate magnitude and direction

$$|\vec{v}_2| = (-3.6^2 + -14^2)^{1/2} = \boxed{14.5 \text{ mi/hr}}$$

$$\theta_2 = \tan^{-1}\left(\frac{-14}{-3.6}\right)$$



My calculator gives: 75°

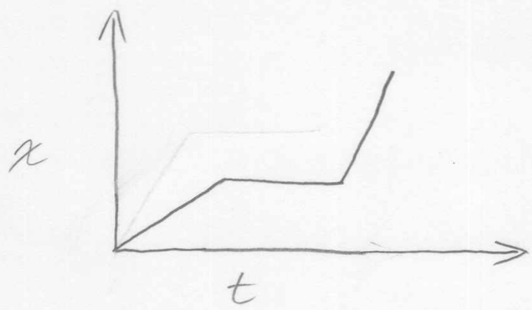
The real angle is in Quadrant 4: $\theta_2 = 180 + 75 = \boxed{255°}$

Just a touch west of south

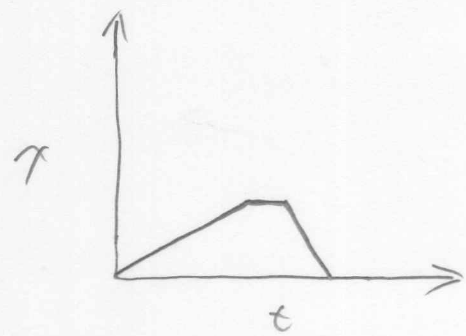
1D Kinematics

1. Sketch position vs. time graphs for the following situations. You should label your axes, but you don't need to include numbers.

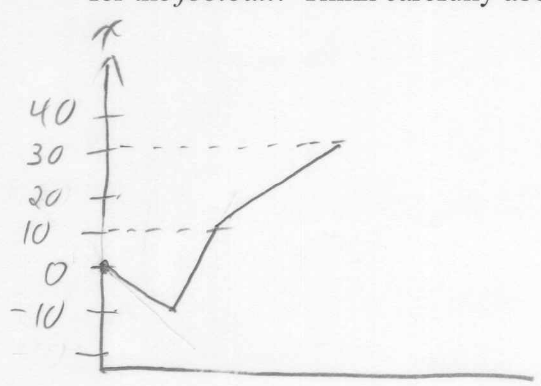
(a) A student walks to the bus stop, waits for the bus, then rides to campus. Assume that all the motion is along a straight street.



(b) A student walks slowly from home to the bus stop, realizes he forgot his paper that is due, and *quickly* walks home to get it.



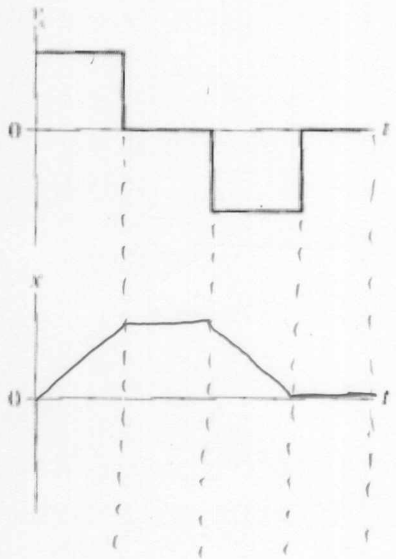
(c) The quarterback drops back 10 yards from the line of scrimmage, then throws a pass 20 yards to the receiver, who catches it and sprints 20 yards to the goal. Draw your graph for the *football*. Think carefully about what the slopes of the lines should be.



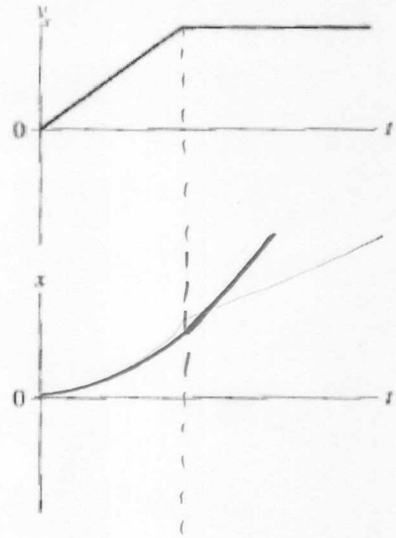
1D Kinematics

2. Below are four velocity vs. time graphs. For each, draw the corresponding position vs. time graph.

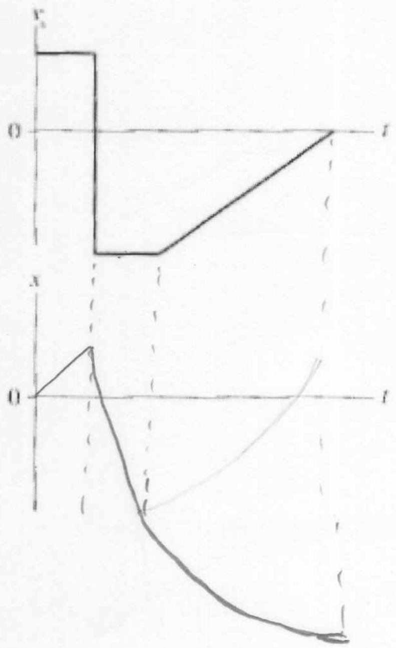
a.



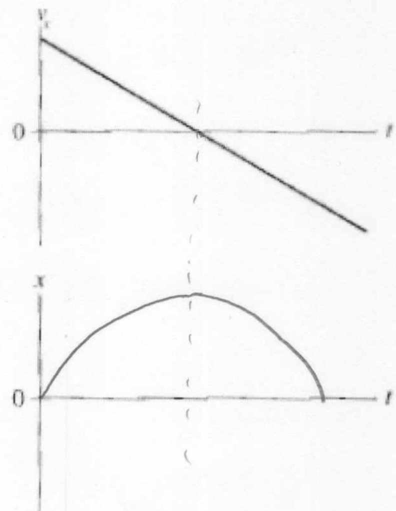
b.



c.



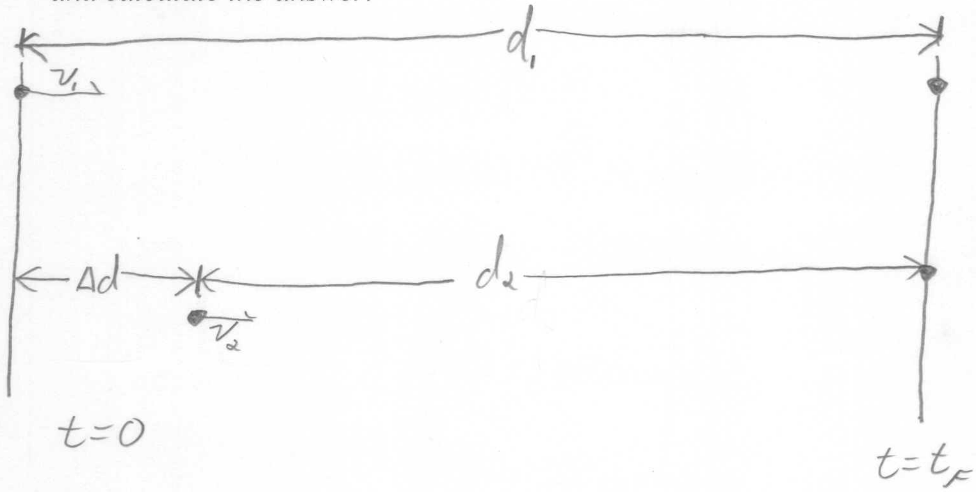
d.



1D Kinematics

4. I can run at 90 m/s, 20% faster than my little brother (we're super heroes). We are running a 100m dash. How much of a head start (distance) should I give him so that we cross the finish line at the same time?

Make a sketch of the situation. Define a coordinate system and all of the relevant variables. Derive the ***analytical*** *solution (no numbers!). Finally, plug in the numbers and calculate the answer.*



Given:

$v_1 = 90\,m/s$

$P = 0.2$

$d_1 = 100\,m$

Want: $\underline{\Delta d}$

$\Delta d = d_1 - \underline{d_2} \leftarrow$ Find d_2

Displacement: $x = \frac{1}{2}at^2 + v_{0x}t + x_0$

$d_2 = v_2 \underline{t_F}$

$d_1 = v_1 \underline{t_F}$

$\Longrightarrow \dfrac{d_2}{d_1} = \dfrac{v_2}{v_1} \Rightarrow d_2 = d_1 \dfrac{v_2}{v_1}$

$$\boxed{v_1 = v_2(1+P)}$$

$\Delta d = d_1 - \dfrac{v_2}{v_1} d_1$

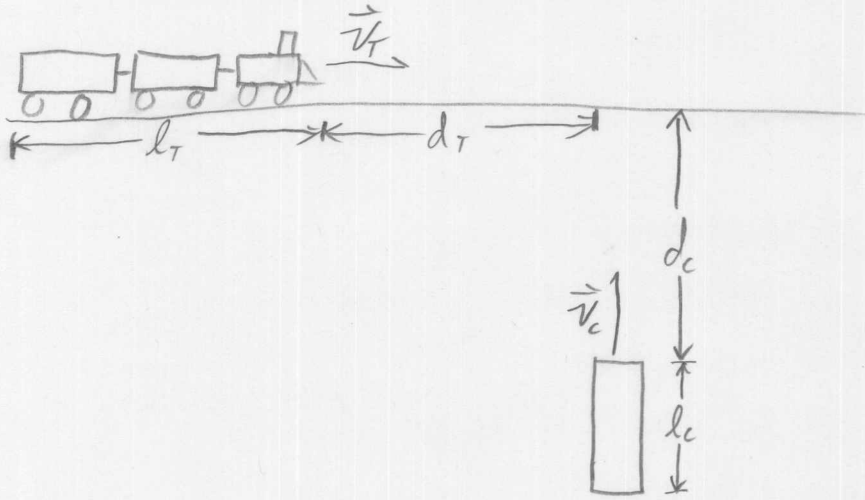
$\Delta d = d_1\left(1 - \dfrac{v_2}{v_1}\right) = d_1\left(1 - \dfrac{\cancel{v_2}}{\cancel{v_2}(1+P)}\right) = d_1\left(\dfrac{P}{1+P}\right)$

Kinematics

You are traveling north in your car approaching a rail-road crossing. A 200 ft passenger train traveling east at 50 mi/hr is 40 ft away from the intersection. The front of your 10 ft long car is 30 ft away from the intersection.

a) What minimum speed does your car need so that the back end of your car clears the tracks before the train gets to the intersection?

b) What is the maximum speed that your car can be traveling so that it clears the back end of the train?



$l_T = 200\,ft$

$V_T = 50\,mi/hr \cdot \frac{5280\,ft/mi}{3600\,s/hr} = 73\,ft/s$

$d_T = 40\,ft$

$l_c = 10\,ft$

$d_c = 30\,ft$

a) car — train

$y = y_0 + V_0 t$ — $x = x_0 + V_0 t$

$d_c + l_c = 0 + V_c t$ — $d_T = V_t t \Rightarrow t = \frac{d_T}{V_t}$

$$d_c + l_c = V_c \frac{d_T}{V_T}$$

$$\boxed{V_c = V_T \frac{d_c + l_c}{d_T}}$$

$$\boxed{V_c = 73\,ft/s \, \frac{30\,ft + 10\,ft}{40\,ft} = 73\,ft/s}$$

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Continued ↓

Train vs car continued

b) car

$y = y_0 + v_0 t$

$d_c = v_c t$

$t = \frac{d_c}{v_c}$

train

$x = x_0 + v_0 t$

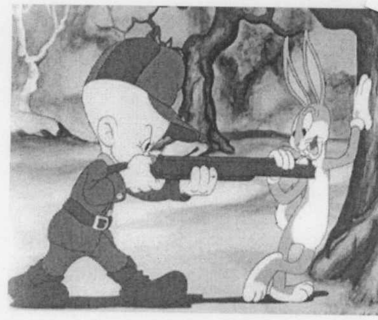
$l_T + d_T = v_T t$

$l_T + d_T = v_T \frac{d_c}{v_c}$

$$\boxed{v_c = v_T \frac{d_c}{l_T + d_T}}$$

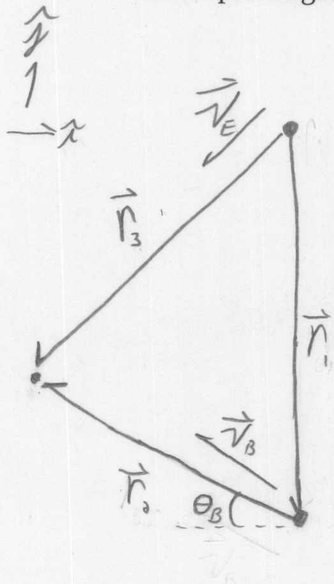
$$\boxed{v_c = 73 \cdot \frac{30}{200 + 40} = 9.1 \text{ mi/hr}}$$

Kinematics



Elmer Fudd is hunting wabbits. He spots Bugs Bunny 25 feet due south of him. Bugs takes off running at 25 mi/hr 15^{o} north of west. Bullets from Elmer's .38 special travel 600 ft/s.

What direction should Elmer fire so that his bullet intercepts Bugs.



$$\vec{r}_1 = (0\hat{i} - 25\hat{j})\ ft$$

$$|\vec{v}_B| = 25\ mi/hr \cdot 5280\ \frac{ft}{mi} \cdot \frac{1}{3600}\ \frac{hr}{s} = 37\ ft/s$$

$$\theta_B = 15^{o}$$

$$|\vec{v}_E| = 600\ ft/s$$

① $\vec{r}_1 + \vec{r}_2 = \vec{r}_3$

② $\vec{r}_2 = \vec{v}_B t$

③ $\vec{r}_3 = \vec{v}_E t$

combine ①, ②, and ③

$$\vec{r}_1 + \vec{v}_B t = \vec{v}_E t$$

solve for $\vec{v}_E$

$$\vec{v}_E = \frac{1}{t}(\vec{r}_1 + \vec{v}_B t)$$

$$\boxed{\vec{v}_E = \frac{\vec{r}_1}{t} + \vec{v}_B}$$

continued ↓

Bugs continued

separate into x and y

$$v_{Ex} = \frac{r_{ix}}{t} + v_{Bx} \qquad v_{Ey} = \frac{r_{iy}}{t} + v_{By}$$

$$|\vec{v}_E| \cos\theta_E = \frac{\cancel{0}}{\cancel{t}} + -|\vec{v}_B| \cos(\theta_B) \; , \; |v_E| \sin\theta_E = \frac{r_{iy}}{t} + |\vec{v}_B| \sin(\theta_B)$$

Oh! The x-equation has only 1 unknown!

$$\cos(\theta_E) = -\frac{|\vec{v}_B|}{|\vec{v}_E|} \cos(\theta_B)$$

$$\boxed{\theta_E = \cos^{-1}\left[-\frac{|\vec{v}_B|}{|\vec{v}_E|} \cos(\theta_B)\right]}$$

$$\theta_E = \cos^{-1}\left[-\frac{37}{600} \cos(15)\right] = \pm 93^\circ$$

$\boxed{\text{or } 3^\circ \text{ West of South}}$