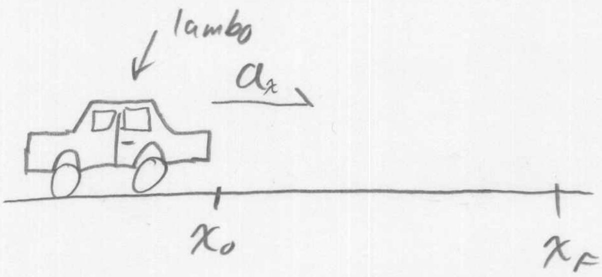
Group Problems, Set 3 – Kinematics

The salesman where I bought my car told me that my new Lamborghini can go from 0 to 60 miles per hour in 5 seconds. I tried bragging to my physicist friends, but they said "No car is worth owning that doesn't have an acceleration of at least 16 ft/s^2. And if you're going to own a car like that, you have to know how FAR it's gone after 5 seconds."

a) Is my car cool enough for physics?
b) Am I cool enough for physics?



$V_{x0} = 0$

$V_{xF} = 60 \text{ mi/hr} \cdot \dfrac{5280 \text{ ft/mi}}{3600 \text{ s/hr}} = 88 \text{ ft/s}$

$t = 5 \text{ s}$

let: $x_0 = 0$

a) $a_x = \dfrac{dV_x}{dt} \Rightarrow \displaystyle\int_0^t a_x dt = \int_{V_{x0}}^{V_{xF}} dV_x$

$$\Rightarrow \boxed{a_x t = V_{xF} - V_{0x}}$$ * constant acceleration only!!!

$$\boxed{a_x = \frac{V_{Fx} - V_{0x}}{t}} = \frac{88 \text{ ft/s} - 0}{5 \text{ s}} = 17.1 \text{ ft/s}^2$$

My car is totally cool enough

continued ↓

Lambo continued

b) Position: $v_x = \frac{dx}{dt} \Rightarrow \int_0^t v_x dt = \int_{x_0}^{x_F} dx$

v_x <u>changes</u> with time.

From part a: $a_x t = v_{xF} - v_{x0}$

$$\Rightarrow v_{xF} = v_{x0} + a_x t$$

in general: $v_x(t) = v_{x0} + a_x t$

So: $\int_0^t (v_{x0} + a_x t)\, dt = \int_{x_0}^{x_F} dx$

$$\Rightarrow v_{x0} t + \frac{1}{2} a_x t^2 = x_F - x_0$$

$$\boxed{x_F = x_0 + v_{x0} t + \frac{1}{2} a_x t^2} \ *$$ Const. accel <u>only</u>

$x_0 = 0, \ v_0 = 0$

So: $x_F = \frac{1}{2} a_x t^2 = \frac{1}{2}(17.1)(5)^2 = \boxed{214 \text{ Ft}}$

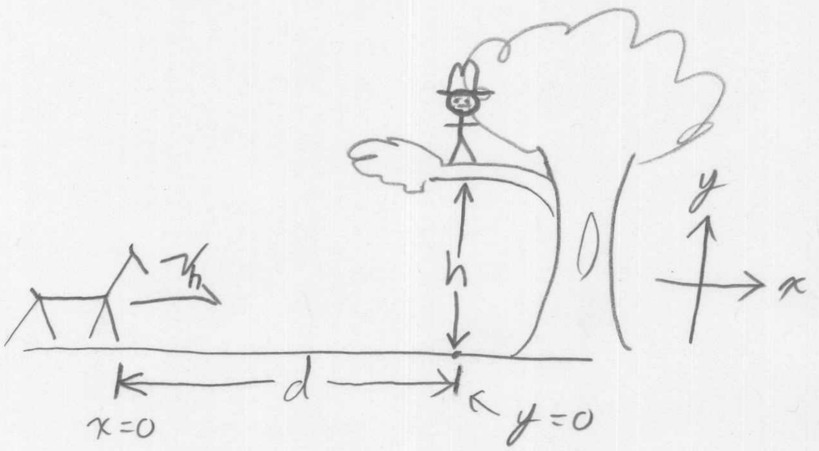
I am <u>so</u>! cool enough for physics.

(1)

A daring ranch hand sitting on a tree limb wishes to drop vertically onto a horse galloping under the tree. The constant speed of the horse is 10.0 m/s, and the distance from the limb to the level of the saddle is 3.00 m.

What must be the horizontal distance between the saddle and the limb when the ranch hand makes his move?

Make a sketch of the situation. Define a coordinate system and all of the relevant variables. Derive the ***analytical*** *solution (no numbers!). Finally, plug in the numbers and calculate the answer.*



Given

$V_h = 10\ m/s$

$h = 3.0\ m$

$g = +9.8\ m/s^2$

$d = ?$

2 players, 2 sets of equations: horse, hand

Want to be at same <u>position</u> after t.

<u>Horse</u> – x only

Constant Velocity

$$x_F = x_0 + V_0 t + \tfrac{1}{2}at^2$$

$$d = 0 + V_h t + 0$$

(1) $d = V_h t$

<u>Ranch hand</u> – y only

const acceleration

$$y_F = y_0 + V_0 t + \tfrac{1}{2}a_y t^2$$

$$0 = h + 0 - \tfrac{1}{2}gt^2$$

(2) $0 = h - \tfrac{1}{2}gt^2$

Solve (2) for t and plug into (1)

continued ↓

Ranch Hand continued

From 2: $h = \frac{1}{2} g t^2 \Rightarrow t = \left(\frac{2h}{g}\right)^{1/2}$

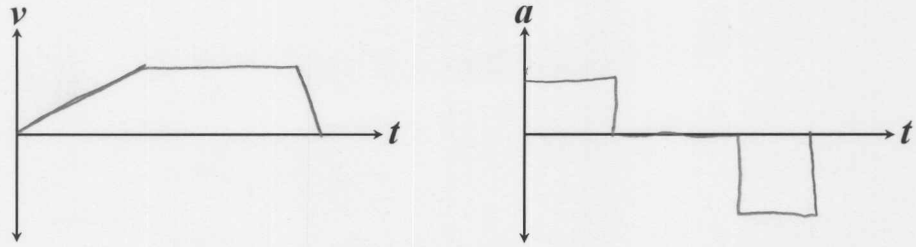
Plug into 1: $\boxed{d = v_h \left(\frac{2h}{g}\right)^{1/2}}$

$$d = 10\,\text{m/s}\left(\frac{(2)(3.0\,\text{m})}{9.8\,\text{m/s}^2}\right)^{1/2} = \boxed{7.8\,\text{m}}$$

1D Kinematics, Part 2

1. A subway train starts from rest at a station and accelerates at a rate of 1.60 m/s^2 for 14.0s. It runs at a constant speed for 70.0 s and slows down at a rate of 3.50 m/s^2 until it stops at the next station.

   Accurately sketch the velocity and acceleration vs. time graphs for this situation.



During the interval of constant speed, how fast is the train going?

Given

$v_0 = 0$

$a = 1.6\ m/s^2$

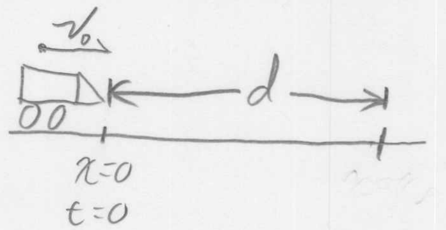
$t_f = 14.0s$

want: v_f

$v(t) = v_0 + at$

$v_f = 0 + at_f \Rightarrow \boxed{v_f = at_f} = (1.6\ m/s^2)(14.0s) = 22.4\ m/s$

How far does the train travel as it decelerates to a stop?



Given

$v_0 = 22.4\ m/s$

$a = +3.50\ m/s$

$v_f = 0$

want

d

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$

$d = 0 + v_0 t + \frac{1}{2}at^2$

$v(t) = v_0 + at$

$0 = v_0 + at$

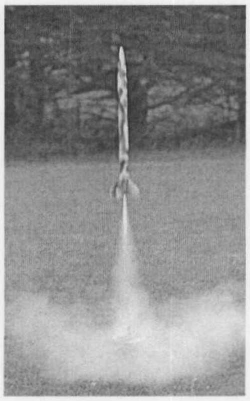
$\Rightarrow t = -\frac{v_0}{a}$

$d = \frac{v_0^2}{a} - \frac{1}{2}\frac{v_0^2}{a}$

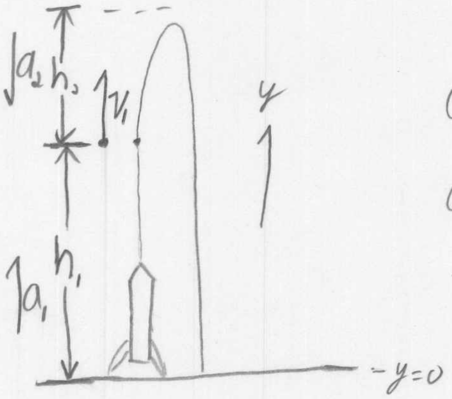
$\boxed{d = \frac{1}{2}\frac{v_0^2}{a}} = \frac{1}{2}\frac{(22.4\ m/s)^2}{3.50\ m/s^2} = \boxed{71.7\ m}$

(1)

A model rocket, initially at rest, is fired vertically with an acceleration of 4.00 m/s2. After 6.00s, it's fuel is exhausted and it continues with the acceleration due to gravity.



a) Find the rocket's height and velocity at the moment that its the fuel runs out.
b) Find the rocket's maximum height.
c) Find the total time that the rocket is in the air.



$h_0 = 0$

$a_1 = 4.00\ m/s^2$

$a_2 = -9.8\ m/s^2$

$t_1 = 6.0s$

2 accelerations so we have to treat them separately.

a) $y = y_0 + V_0 t + \frac{1}{2} a_1 t_1^2$ ⇒ Basic equation

$$\boxed{h_1 = 0 + 0 + \tfrac{1}{2} a_1 t_1^2}$$

$$h_1 = \tfrac{1}{2}(4.0)(6.0s) = \boxed{12m}$$

b) $y = y_0 + V_0 t + \frac{1}{2} a t^2$ ⇒ Basic equation

(1) $h_2 = h_1 + V_1 t_2 + \frac{1}{2} a_2 t_2^2$ ⇒ Don't know V_1 or t_2

we can get V_1 by solving the velocity eq. for stage 1.

continued ↓

Rocket continued

Stage 1: $v = v_0 + a_1 t_1$

$\boxed{v_1 = 0 + a_1 t_1} \Rightarrow v_1 = (4.00)(6.00) = \boxed{24.0\ m/s}$

Stage 2: Final stage 1 velocity becomes initial stage 2 velocity.

Find time by solving velocity eq. for stage 2.

$v = v_0 + at$

or: $v_2 = v_1 + a_2 t_2 \leftarrow$ time to apex

At the top of the trajectory, $v_2 = 0$

so: $0 = v_1 + a_2 t_2$

$\Rightarrow \boxed{t_2 = -\frac{v_1}{a_2}}$

Plug back into equation for h_2:

$$h_2 = h_1 + v_1 \frac{-v_1}{a_2} + \frac{1}{2} a_2 \left(\frac{-v_1}{a_2}\right)^2$$

continued

↓

Rocket continued

$$h_2 = h_1 - \frac{v_1^2}{a_2} + \frac{1}{2}\cancel{a_2}\frac{v_1^2}{a_2^{\cancel{2}}}$$

$$= h_1 - \frac{v_1^2}{a_2} + \frac{1}{2}\frac{v_1^2}{a_2}$$

$$\boxed{h_2 = h_1 - \frac{1}{2}\frac{v_1^2}{a_2}}$$

$$h_2 = 12m - \frac{1}{2}\frac{(24.0\,m/s)^2}{-9.8\,m/s^2}$$

$$\boxed{h_2 = 41m}$$

c) Although it's tempting to say $t_t = t_1 + t_2$, <u>resist!</u>

t_2 is the time to the apex. We want time from $y = h_1$ all the way up, then back down to $y = 0$:

$$y = y_0 + v_0 t + \tfrac{1}{2} a_2 t^2$$

$$0 = h_1 + v_1 t_3 + \tfrac{1}{2} a_2 t_3^2 \Rightarrow \text{Quadratic in } t_3$$

$$t_3 = \frac{-v_1 \pm \sqrt{v_1^2 - 2a_2 h_1}}{a_2} \Rightarrow \text{Quadratic Equation}$$

continued ↓

Rocket continued

which sign?

$$t_3 = \frac{-24.0 \pm \sqrt{(24)^2 - (2)(-9.8)(12)}}{-9.8}$$

choose + sign. − gives negative time.

$t_3 = 5.3\,s$

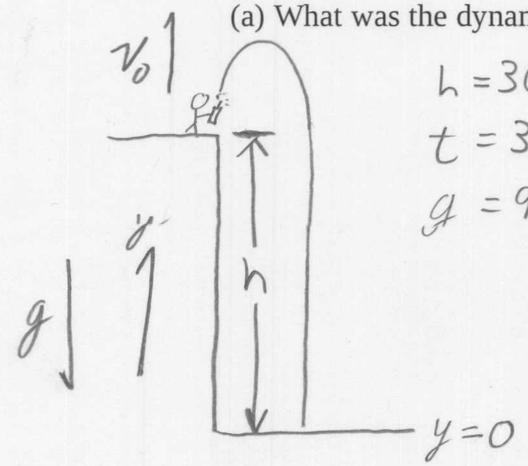
and $t_T = t_1 + t_3$

$t_T = 6.0\,s + 5.3\,s$

$\boxed{t_T = 11.3\,s}$

Wile E. Coyote is standing on the edge of a cliff throwing dynamite. The cliff is 30.0 m high, and Wile E. throws the dynamite straight up in the air. The dynamite falls past the edge of the cliff and hits the ground 3.50 s after it was thrown.

(a) What was the dynamite's initial velocity?



$h = 30.0\,m$

$t = 3.5\,s$

$g = 9.8\,m/s^2$

$$y = y_0 + V_0 t + \tfrac{1}{2}at^2 \Rightarrow \text{Basic Eq.}$$

$$0 = h + V_0 t - \tfrac{1}{2}gt^2 \Rightarrow \text{Plug in variables}$$

$$\Rightarrow \boxed{V_0 = \tfrac{1}{2}gt - \frac{h}{t}} \qquad V_0 = \tfrac{1}{2}(9.8)(3.5) - \frac{(30)}{(3.5)}$$

$$\boxed{V_0 = 8.6\,m/s}$$

(b) Emboldened by his first throw, Wile E. decides to light the next stick and repeat his throw (same trajectory), this time trying to hit Roadrunner. The dynamite explodes 1.60 seconds after The Coyote throws it. Is he toast?

What is y after 1.6 seconds?

$y_0 = h = 30.0\,m$

$V_0 = 8.6\,m/s$

$t = 1.6\,s$

$a = g = 9.8\,m/s^2$

$$y = y_0 + V_0 t + \tfrac{1}{2}at^2$$

$$\boxed{y = h + V_0 t - \tfrac{1}{2}gt^2}$$

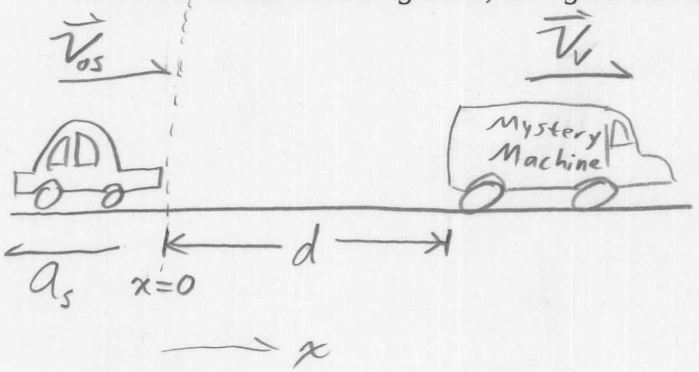
$$y = 30.0\,m + (8.6\,m/s)(1.6\,s) - \tfrac{1}{2}(9.8\,m/s^2)(1.6\,s)^2$$

$y = 31.2\,m$ or $1.2\,m$ above the cliff edge

Wile E. Blew his little face off...

Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. She applies her brakes but can accelerate only at –2.00 m/s² because the road is wet. Will there be a collision? If so, determine how far into the tunnel and at what time the collision occurs. If not, determine the distance of closest approach between Sue's car and the van.

Make a sketch of the situation. In the sketch define your coordinate system and appropriate variables. In your solution give equations in symbols, next equation with numerical values including units, then give answer.



$V_{os} = 30.0\ \mathrm{m/s}$

$d = 155\ \mathrm{m}$

$V_v = 5.0\ \mathrm{m/s}$

$a_s = 2.0\ \mathrm{m/s}$

Tracking two players, each gets kinematics equations.

Sue

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$x_s = 0 + V_{os} t - \tfrac{1}{2} a_s t^2$$

Van

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$x_v = d + V_v t + 0$$

There's a collision when: $x_s = x_v$

$$V_{os} t - \tfrac{1}{2} a_s t^2 = d + V_v t$$

$$\Rightarrow \tfrac{1}{2} a_s t^2 + (V_v - V_{os}) t + d = 0 \quad \text{Quadratic in } t$$

If t has real roots, there is a collision.

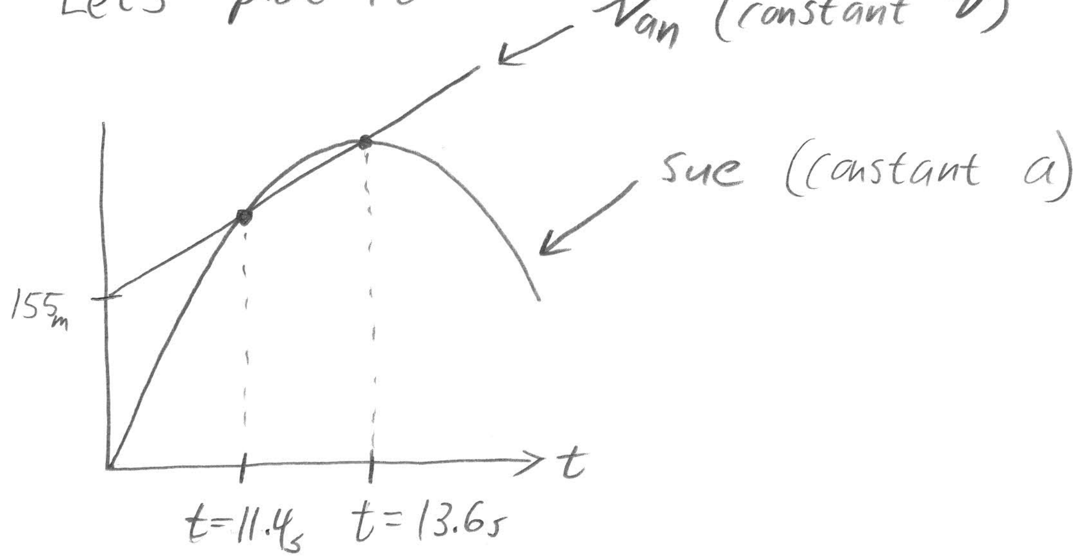
Speedy Sue continued

$$t = \frac{-(v_v - v_{0s}) \pm \sqrt{(v_v - v_{0s})^2 - 2a_s d}}{a_s}$$

$$t = \frac{-(5.0 - 30.0) \pm \sqrt{(5.0-30.0)^2 - (2)(2.0)(155)}}{2.0}$$

$t = 11.4\,s$ and $13.6\,s$ which one?

Let's plot it



First intersection point is the "real" collision.

$\boxed{t = 11.4}$

Where is the van (and sue) after 22.5 s

$\boxed{x_v = d + v_v t} \Rightarrow x_v = 155\,m + (5.0\,m/s)(11.4)$

$\boxed{x_v = x_s = 212\,m}$