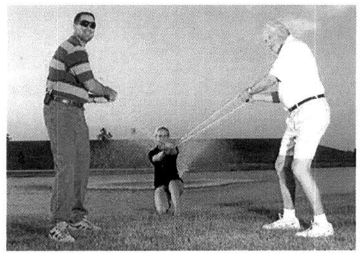
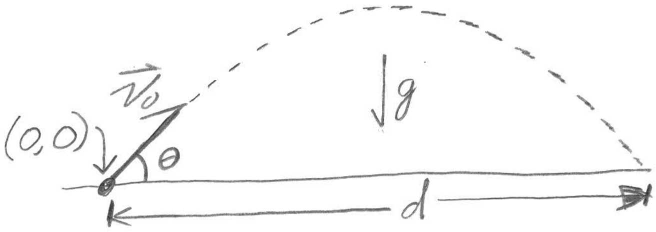
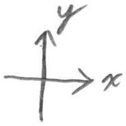
Kinematics

You and your buddies have created a water balloon slingshot. Using a radar gun, you measure that the velocity of the water balloon as it leaves the slingshot. Because you're taking physics, your friends want you to work out how far the balloon will go for a given launch angle.



Find an equation for the horizontal distance the balloon will travel in terms of the magnitude of it's initial velocity and the launch angle with respect to the horizontal.





Given	Want
v_0	d
θ	

Kinematics equations:

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \qquad \vec{v} = \vec{v}_0 + \vec{a}t$$

Treat x and y axis independently:

x: $$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad v_x = v_{0x} + a_x t$$

① $$d = 0 + |\vec{v}_0|\cos\theta\, t + 0$$ ② $$\boxed{v_x = |\vec{v}_0|\cos\theta} + 0$$

$\boxed{\text{need } t}$

Velocity in x is constant — no accel.

y: $$y = y_0 + v_{0y}t + \tfrac{1}{2}at^2 \qquad v_y = v_{0y} + a_y t$$

③ $$0 = 0 + |\vec{v}_0|\sin\theta\, t - \tfrac{1}{2}gt^2$$ ④ $$v_y = |\vec{v}_0|\sin\theta - gt$$

continued ↓

Water Balloon continued

3 equations, (1, 3, 4) and 3 unknowns.

in Equation 3, only $\underline{t}$ is unknown, so we can use it to eliminate $\underline{t}$ from 1 and 4

From 3: $|\vec{v}_0| \sin\theta \cancel{t} = \frac{1}{2} g t^{\cancel{2}} \Rightarrow \underline{t = \frac{2|\vec{v}_0|\sin\theta}{g}}$ (5)

Plug into 1:

$$d = |\vec{v}_0| \cos\theta \cdot \frac{2|\vec{v}_0|\sin\theta}{g}$$

$$\boxed{\text{math aside: } \sin(2\theta) = 2\sin\theta\cos\theta}$$

$$\boxed{d = \frac{|\vec{v}_0|^2}{g} 2\sin\theta\cos\theta} \Rightarrow \boxed{d = \frac{|\vec{v}_0|^2}{g}\sin(2\theta)} \quad (6)$$

Although the problem statement doesn't request it, let's solve for v_y

$$v_y = |\vec{v}_0|\sin\theta - gt$$

$$= |\vec{v}_0|\sin\theta - \cancel{g} \cdot \frac{2|\vec{v}_0|\sin\theta}{\cancel{g}} = -|\vec{v}_0|\sin\theta$$

$$\boxed{\underline{v_y} = -|\vec{v}_0|\sin\theta = -\underline{v_{0y}} \quad \text{nifty!}}$$

continued ↓

Water Balloon continued.

What is the balloon's maximum height?

Maximize a function by taking the derivative and setting it to zero.

But that's the velocity!

So... max height when $\underline{v_y = 0}$

From eq. 4

$$v_y = |\vec{v}_0| \sin\theta - gt, \quad \text{let } v_y = 0$$

$$0 = |\vec{v}_0| \sin\theta - g t_{apex}$$

$$\Rightarrow t_{apex} = \frac{|\vec{v}_0| \sin\theta}{g} \Rightarrow \text{Time at max height}$$

Compare to eq 5: $\underline{t_{apex} = \frac{1}{2} t}$

So: $y_{max} = \cancel{y_0}^{0} + v_{0y} t - \frac{1}{2} g t^2$

$$y_{max} = |\vec{v}_0| \sin\theta \frac{|\vec{v}_0| \sin\theta}{g} - \frac{1}{2} \cancel{g} \frac{(|\vec{v}_0| \sin\theta)^2}{g^{\cancel{2}}}$$

$$\boxed{y_{max} = \frac{1}{2} \frac{|\vec{v}_0|^2 \sin^2\theta}{g}}$$

continued ↓

One last bit: <u>What's x at the apex?</u>

$$x_{apex} = \cancel{x_0}^{\,0} + v_{0x} t_{apex} + \cancel{\tfrac{1}{2} a_x t^2}^{\,0}$$

$$x_{apex} = |\vec{v}_0| \cos\theta \frac{|\vec{v}_0| \sin\theta}{g} = \frac{|\vec{v}_0|^2}{g} \sin\theta\cos\theta$$

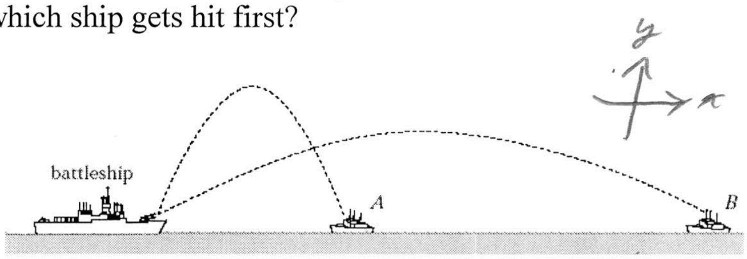
$$x_{apex} = \frac{|\vec{v}_0|^2}{g} \frac{1}{2} \sin(2\theta)$$

compare to eq. 6 we see that

$$\boxed{x_{apex} = \tfrac{1}{2} d \quad \text{groovy!}}$$

A battleship simultaneously fires two shells at enemy ships. If the shells follow the parabolic trajectories shown, which ship gets hit first?

1. *A*
2. both at the same time
3. *B*
4. need more information



Explain.

Total time of flight is determined by the initial y-velocity, which is a function of launch angle.

6. At what point in its trajectory does a batted baseball have its minimum speed? Why?

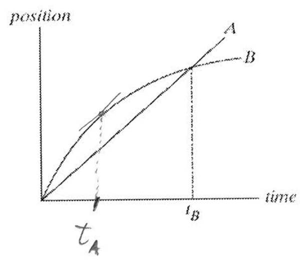
At its apex. x-Velocity is constant throughout the flight. y-Velocity is zero at it's apex.

7. A friend claims that bullets fired by some high-powered rifles travel for many meters in a straight-line path before they start to fall. Another friend disputes this claim and states that all bullets from any rifle drop beneath a straight-line path a vertical distance given by $\frac{1}{2}gt^2$ and that the curved path is apparent for low velocities and less apparent for high velocities. Now it's your turn: Will all bullets drop the same vertical distance in equal times? Explain.

Yes. (unless they have lift)
Time to the ground is independent of x-velocity.

Group Problems Set 4 – Kinematics

The graph shows position as a function of time for two trains running on parallel tracks. Which statement is true?



1. At time t_B, both trains have the same velocity.
2. Both trains speed up all the time.
3. Both trains have the same velocity at some time before t_B.
4. Somewhere on the graph, both trains have the same acceleration.

Explain.

(1) No. At t_B, A is going faster than B

(2) No. Train A has constant velocity and B is slowing down

(3) Yes! at t_A, they have the same velocity

(4) No, Train A has zero acceleration and train B has some non-zero acceleration.

Suppose you roll a ball off a table top. Will the time to hit the floor depend on the speed of the ball? Why?

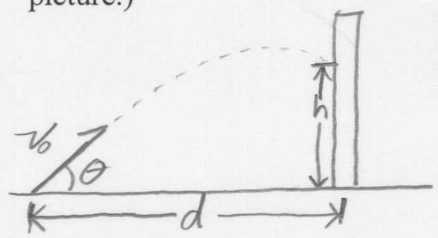
No. The time from the table top to the floor depends only on the y axis where the acceleration is. Time to floor is independent of x-axis velocity.

x and y axis are independent.

1D Kinematics, Part 2

2. You throw a ball toward a wall with a speed of 25.0 m/s and at an angle of 40.0º above the horizontal. The wall is 22.0 m from the release point of the ball.

   (a) How far above the release point does the ball hit the wall? (Be sure to draw a picture.)



Given

$V_0 = 25\ m/s$

$\theta = 40°$

$d = 22\ m$

Want

h

Part a

x

$x_f = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$

Solve for t $\rightarrow d = 0 + V_0 \cos\theta\, t + 0$

$t = \dfrac{d}{V_0 \cos\theta}$ $\xrightarrow{\text{Plug into } y \text{ eq}}$ $h = \cancel{V_0} \sin\theta \dfrac{d}{\cancel{V_0}\cos\theta} - \frac{1}{2} g \dfrac{d^2}{V_0^2 \cos^2\theta}$

y

$y_f = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$

$h = 0 + V_0 \sin\theta\, t - \frac{1}{2} g t^2$

(b) What are the horizontal and vertical components of its velocity as it hits the wall?

$\boxed{h = d\tan\theta - \frac{1}{2}\dfrac{gd^2}{V_0^2\cos^2\theta}}$ *

$h = 22 \cdot \tan(40) - \frac{1}{2}\dfrac{(9.8)(22)^2}{(25)^2\cos^2(40)}$

$\boxed{h = 12.0\ m}$

Part B - Find $\vec{V_f}$, Final velocity

x

$V_{fx} = V_{0x} + a_x t$

y

$V_{fy} = V_{0y} + a_y t$

(c) When it hits, has it passed the highest point on its trajectory? How do you know?

$\boxed{V_{fx} = V_0 \cos\theta}$

$V_{fx} = (25)\cos(40)$

$= \boxed{19.2}\ m/s$

get from part a

$V_{fy} = V_0 \sin\theta - gt$

$\boxed{V_{fy} = V_0 \sin\theta - \dfrac{gd}{V_0 \cos\theta}}$

$V_{fy} = 25 \sin 40 - \dfrac{(9.8)(22)}{(25)\cos(40)}$

$\boxed{V_{fy} = 4.8\ m/s}$

Part C Not past apex because V_{yF} is positive

SAMPLE TEST 1

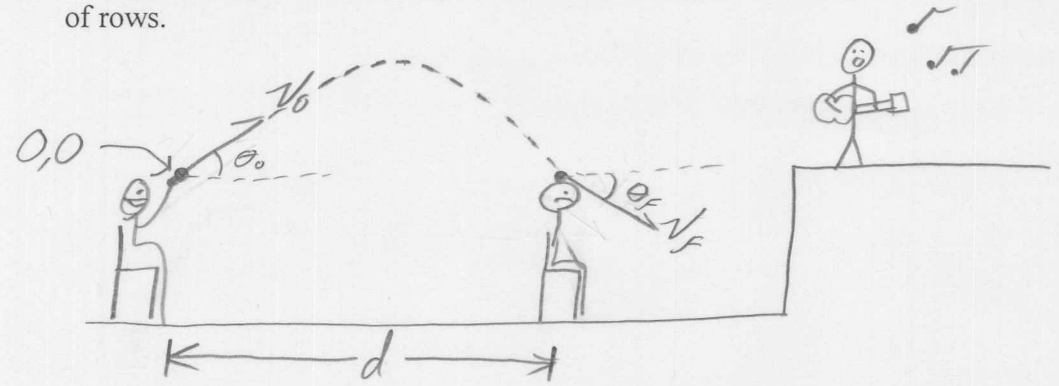
PHYS 111 FALL 2008

4 20pts) You are sitting in the middle of a field listening to a concert when a water balloon hits you from behind at a speed of 14 m/s coming in at an angle of -60° as measured off of the x axis. If the rows are separated by 1.5 m, how many rows behind you are the vandals sitting?

a. (5pts) Draw a sketch of the situation showing ALL relevant variables and define the coordinate system.

b. (13pts) Using the kinematics equations, derive an expression for the number of rows in terms of GIVEN variables defined in part a.

c. (2pts) Using your expression from part b and numbers given in the problem statement, calculate the number of rows.



GIVEN

$|\vec{V}_f| = 14\ m/s$

$\theta = -60°$

$r = 1.5\ m/row$

want

d in rows

$x_f = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$

$d = 0 + V_{0x}t + 0$

$y_f = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$

$0 = 0 + V_{0y}t - \frac{1}{2}gt^2$

hmm... use velocity eqs

$V_{fx} = V_{0x} + at$

$|\vec{V}_f| \cos\theta_f = V_{0x} + 0$

$d = |\vec{V}_f| \cos\theta_f\, t$

$V_{fy} = V_{0y} + at$

$|\vec{V}_f| \sin\theta = V_{0y} - gt$

$V_{0y} = |\vec{V}_f| \sin\theta + gt$

$0 = |\vec{V}_f| \sin\theta + gt - \frac{1}{2}gt$

$0 = |\vec{V}_f| \sin\theta + \frac{1}{2}gt$

Continued ↓

Solve for t

$$t = -\frac{2|\vec{V}_i|\sin\theta}{g}$$

$$d = -\frac{2|\vec{V}_i|^2\sin\theta\cos\theta}{g}$$

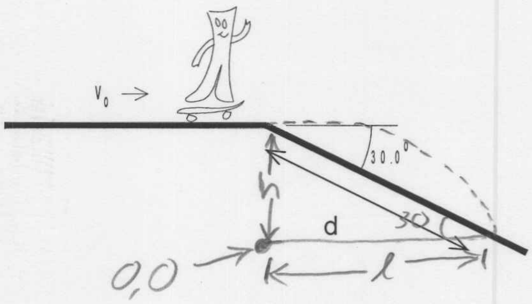
don't freak out!
Remember $\theta = -60$ so $\sin\theta$ will be negative and it will cancel the negative

$$d = -\frac{2(14)^2\sin(-60)\cos(-60)}{9.8}$$

$$= 17.3\,m \cdot \frac{1}{1.5}\,\text{rows}/m = \boxed{11.5\ \text{rows}}$$

1D Kinematics, Part 2

3. Gumby has just purchased a new skateboard; but, unfortunately, he does not know how to stop. Traveling at 8.0 m/s, he reaches the top of a hill sloping down at 30.0°.



How far down the hill does Gumby crash (i.e., find the distance *d*)?

Given

$V_{0x} = 8.0\ m/s$

$V_{0y} = 0$

$\theta = 30^\circ$

want

d

x

$x_f = x_0 + V_{0x}t + \frac{1}{2}at^2$

$l = 0 + V_{0x}t + 0$

y

$y_f = y_0 + V_{0y}t + \frac{1}{2}at^2$

$0 = h + 0 - \frac{1}{2}gt^2$

Look at picture:
write l and h in terms of d and θ

$l = d\cos\theta, \quad h = d\sin\theta$

$d\cos\theta = V_{0x}t$

$t = \dfrac{d\cos\theta}{V_{0x}}$

$d\sin\theta = \frac{1}{2}gt^2$

$\cancel{d}\sin\theta = \frac{1}{2}g\dfrac{d^{\cancel{2}}\cos^2\theta}{V_{0x}^2}$

Solve for d

$\boxed{d = \dfrac{2V_{0x}^2\sin\theta}{g\cos^2\theta}} \Rightarrow d = \dfrac{2(8)^2\sin(30)}{(9.8)\cos^2(30)} =$

$\boxed{d = 8.7\ m}$