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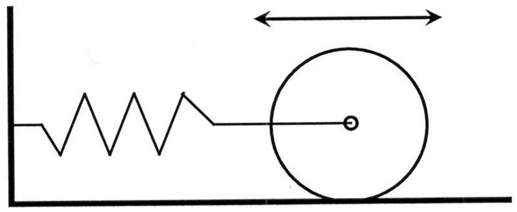
Oscillation – Set ~~2~~ 3

Use **ENERGY** techniques to answer the following question.

A solid cylinder of mass M=2 kg and radius R=1.0 m is attached to a horizontal spring with spring constant k=100 N/m. The cylinder can roll without slipping along the horizontal plane. When the system is displaced from the equilibrium position, it executes simple harmonic motion.

a) Derive an expression for the period of the oscillations in terms of M, k, ~~and I~~.

b) If the cylinder has a translational velocity of v_0=5.0 m/s as it passes through equilibrium, find the phase constant, the amplitude, and the maximum acceleration of the system.



$I = \frac{1}{2} m R^2$ $\quad \omega = \frac{v}{R}$ $\quad \alpha = \frac{a}{R}$

a) $E_T = \frac{1}{2} k x^2 + \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$

$$\frac{dE_T}{dt} = \frac{1}{2} k\, 2x \frac{dx}{dt} + \frac{1}{2} I\, 2\omega \frac{d\omega}{dt} + \frac{1}{2} m\, 2v \frac{dv}{dt} = 0$$

$$\Rightarrow kxv + I\omega\alpha + mva = 0$$

$$\Rightarrow kx\cancel{v} + \frac{1}{2} m\cancel{R^2} \frac{\cancel{v}}{\cancel{R}} \frac{a}{\cancel{R}} + m\cancel{v}a = 0$$

$$\Rightarrow kx + \frac{1}{2} ma + ma = 0$$

$$\Rightarrow \frac{3}{2} ma = -kx$$

$$\Rightarrow a = -\frac{2}{3}\frac{k}{m} x \Rightarrow \frac{d^2x}{dt^2} = -\left[\frac{2}{3}\frac{k}{m}\right] x$$

$$\boxed{\omega = \left[\frac{2}{3}\frac{k}{m}\right]^{1/2}}$$

Oscillation set 3, P1 continued

b) $V_0 = 5.0$ m/s. Let's let $t=0$ then.

$\boxed{v(0) = 5.0 \text{ m/s}}$

At equilibrium, $x = 0$ so:

$\boxed{x(0) = 0}$

General Solution

$x(t) = A\cos(\omega t + \phi)$

$v(t) = -\omega A \sin(\omega t + \phi)$

$x(0) = A\cos(\phi) \Rightarrow 0 = A\cos(\phi) \Rightarrow \boxed{\phi = 0, \pi}$

$v(0) = -\omega A \sin\phi \Rightarrow 5.0 = -\omega A \sin\phi$, let $\phi = \pi$

$\Rightarrow A = \frac{5.0}{\omega} = \sqrt{\frac{3}{2}\frac{m}{k}}\, 5.0$

$\boxed{A = \left(\frac{3}{\cancel{2}}\frac{\cancel{2}}{100}\right)^{1/2} \cdot 5.0 = 0.87 \text{ m}}$

Oscillation – Set ~~2~~ 3

An oscillating block-spring system has a mechanical energy of 1.00 J, an amplitude of of 10.0 cm, and a maximum speed of 1.20 m/s.

a) What is the spring constant?
b) What is the mass of the block?
c) What is the frequency of oscillation?

$$E_T = U + K \Rightarrow E_T = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2$$

a) Use the amplitude. When the spring is compressed, $K = 0$ and $x = A$

So: $E_T = \tfrac{1}{2}kA^2 \Rightarrow \boxed{k = \frac{2E_T}{A^2}} \Rightarrow \boxed{k = \frac{(2)(1)\,J}{(0.1\,m)^2} = 200\ N/m}$

b) Use the max speed.
Max speed is at equilibrium when $U = 0$

$E_T = \tfrac{1}{2}mV_{max}^2 \Rightarrow \boxed{m = \frac{2E_T}{V_{max}^2}} \Rightarrow \boxed{m = \frac{2(1J)}{(1.2\,m/s)^2} = 1.4\ kg}$

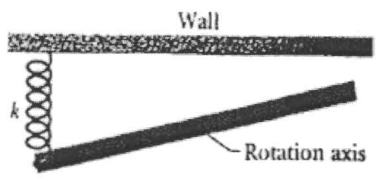
c) We can use V_{max} and A to find ω.

$v(t) = -\omega A\cos(\omega t + \phi)$

$\Rightarrow V_{max} = \omega A \Rightarrow \boxed{\omega = \frac{V_{max}}{A}} \Rightarrow \boxed{\omega = \frac{1.2\,m/s}{0.1\,m} = 12\ rad/s}$

Oscillation – Set 3

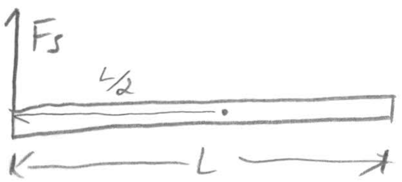
A long uniform rod of length L and mass m is free to rotate in a *horizontal* plane about a vertical axis through its center (the picture shows a *top* view). A spring with force constant k is connected horizontally between one end of the rod and a fixed wall. When the rod is in equilibrium, it is parallel to the wall.



What is the period of the *small* oscillations that result when the rod is rotated slightly and then released? $I_{cm} = \frac{1}{12}ML^2$ for the rod.

a) Use **Newton's Second Law** to find the oscillator frequency.
b) Use **Energy Techniques** to find the oscillator frequency.

a)



$$\Sigma T = I\alpha \Rightarrow \alpha = \frac{\Sigma T}{I}$$

$$\Rightarrow \alpha = \frac{\frac{L}{2}F_s}{\frac{1}{12}ML^2} \Rightarrow \alpha = \frac{-\frac{L}{2}kx}{\frac{1}{12}ML^2},\quad x = \theta \frac{L}{2}$$

$$\Rightarrow \alpha = \frac{-\cancel{6}^{3}\,\theta \frac{\cancel{L}}{2}k}{M\cancel{L}} \Rightarrow \boxed{\alpha = -3\frac{k}{M}\theta}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\left[\frac{3k}{M}\right]\theta \Rightarrow \boxed{\omega = \left[\frac{3k}{M}\right]^{1/2}}$$

b) $E_T = \frac{1}{2}kx^2 + \frac{1}{2}I\omega^2$

$$\frac{dE_T}{dt} = \frac{1}{2}k2x\frac{dx}{dt} + \frac{1}{2}I2\omega\frac{d\omega}{dt} = 0 \Rightarrow kxv + I\omega\alpha = 0$$

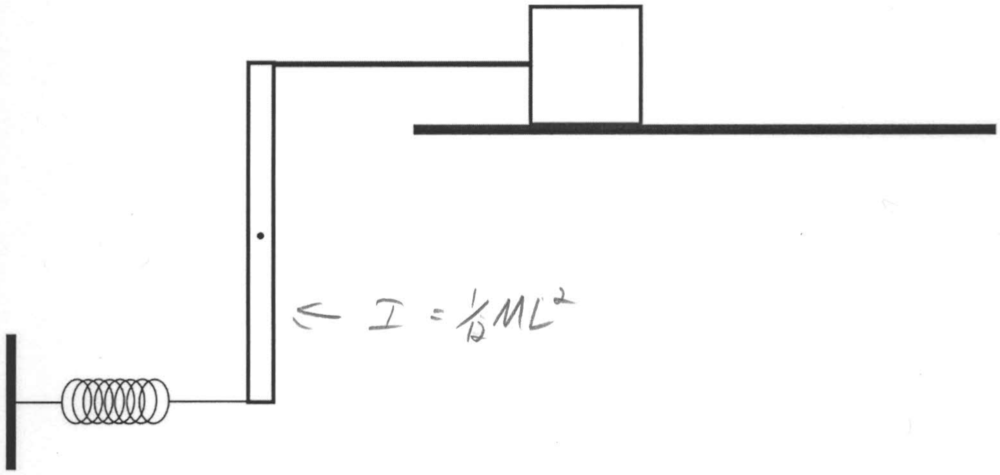
$$x = \frac{L}{2}\theta,\quad v = \frac{L}{2}\omega$$

$$k\frac{L}{2}\theta\frac{L}{2}\omega + \frac{1}{12}ML^2\omega\alpha = 0 \Rightarrow \frac{1}{12}ML^2\alpha = -\frac{1}{4}kL^2 \Rightarrow \boxed{\alpha = -3\frac{k}{M}\theta}$$

$$\boxed{\omega = \left[\frac{3k}{M}\right]^{1/2}}$$

Use **ENERGY** techniques to answer the following question.

A block of mass M resting on a frictionless surface is attached to a stiff rod of negligible mass. The other end of the rod is attached to the top of a thin bar of length l mass M that is allowed to rotate about its center. The bottom of a bar is attached to a light spring of spring constant k. The spring is relaxed when the bar is vertical. Find the frequency of small oscillations.



$$E_T = \frac{1}{2}kx^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$\frac{dE_T}{dt} = \cancel{\frac{1}{2}}k\cancel{2}x\frac{dx}{dt} + \cancel{\frac{1}{2}}I\cancel{2}\omega\frac{d\omega}{dt} + \cancel{\frac{1}{2}}m\cancel{2}v\frac{dv}{dt} = 0$$

$$\Rightarrow kxv + I\omega\alpha + mva = 0, \quad \omega = \frac{v}{L/2}, \quad \alpha = \frac{a}{L/2}$$

$$\Rightarrow kx\cancel{v} + \frac{1}{12}m\cancel{L^2}\frac{\cancel{v}}{\cancel{L}/2}\cdot\frac{a}{\cancel{L}/2} + m\cancel{v}a = 0$$

$$\Rightarrow kx + \frac{1\cancel{4}}{3\cancel{12}}ma + ma = 0$$

$$\Rightarrow \frac{4}{3}ma = -kx \Rightarrow a = -\frac{3}{4}\frac{k}{m}x \Rightarrow \boxed{\frac{d^2x}{dt^2} = -\left[\frac{3}{4}\frac{k}{m}\right]x}$$

$$\boxed{\omega = \left[\frac{3}{4}\frac{k}{m}\right]^{1/2}}$$

Two particles are in simple harmonic motion in a straight line. They have the same amplitude and a period of 1.5 s but differ in phase by π/6 radians.

a) How far apart are they from one another (in terms of A) when the lagging particle is at its maximum position?

b) Are they moving in the same direction or opposite directions?

c) How far apart are they 0.5 seconds later?

d) Are they moving in the same or opposite directions then?

$A_1 = A_2 = A, \quad T_1 = T_2 = T, \quad \phi_1 = 0, \quad \phi_2 = \pi/6,$

a) $x_1(t) = A_1 \cos(\omega_1 t + \phi_1)$

$x_2(t) = A_2 \cos(\omega_2 t + \phi_2)$

$\boxed{\omega = \frac{2\pi}{T}}$

what's t_0?

$x_1(t_0) = A\cos(\omega t_0)$ At max, $\underline{x = A}$

$A = A\cos(\omega t_0) \Rightarrow \cos(\omega t) = 1$

$\Rightarrow \omega t = 0 \Rightarrow \underline{\underline{t = 0}}$

$x_2(0) = A\cos(\phi_2) \Rightarrow x_2(0) = A\cos(\pi/6) = \frac{\sqrt{3}}{2} A$

$\boxed{x_2 - x_1 = \left(\frac{\sqrt{3}}{2} - 1\right) A}$

b) $v_1(0) = -\omega A \sin(0) = 0$

hmm... neither...

trick question :)

c) at $t = 0.5$

$x_1(0.5) = A\cos(\omega \cdot 0.5)$, $T = \frac{3}{2} \Rightarrow \frac{2\pi}{3/2} = \frac{4}{3}\pi = \omega$

$x_1(0.5) = A\cos\left(\frac{4}{3}\pi \frac{1}{2}\right)$ $\omega = \frac{4}{3}\pi$

$x_1(0.5) = A\cos\left(\frac{2}{3}\pi\right)$

$x_1(0.5) = -\frac{1}{2}A$

$x_2(0.5) = A\cos\left(\frac{4}{3}\pi \cdot \frac{1}{2} + \frac{\pi}{6}\right)$

$= A\cos\left(\frac{5}{6}\pi\right)$

$= -\frac{\sqrt{3}}{2}A$

$$\boxed{x_2 - x_1 = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)A}$$

d) $v_1(0.5) = -\omega A \sin\left(\frac{2}{3}\pi\right) = -\omega A \frac{\sqrt{3}}{2}$

$v_2(0.5) = -\omega A \sin\left(\frac{5}{6}\pi\right) = -\omega A \frac{1}{2}$

same sign, same direction