SAMPLE TEST 6

PHYS 111, FALL 2010, SECTION 1

Name: ______________________________

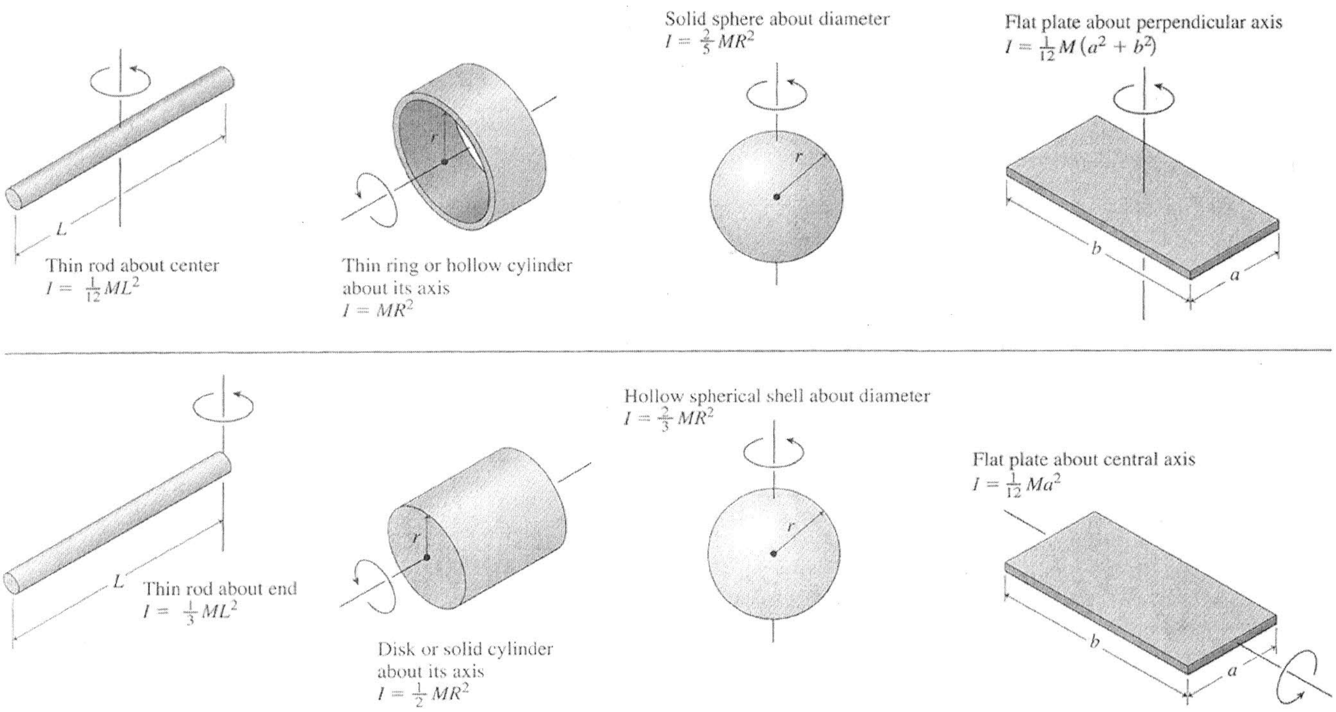
By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

A few potentially useful equations

Moment of Inertia, discrete definition	$I=\sum m_i r_i^2$
Moment of Inertia, integral definition	$I=\int r^2\,dm$
Parallel Axis Theorem	$I=I_{cm}+Md^2$
Superposition	$I_{Total}=\sum I_i$

TABLE 10.2 Rotational Inertias



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1) Derivations

a) (10pts) Given a differential equation of the form $\frac{d^2x(t)}{dt^2} = -\omega^2 x(t)$, write the general solution for $x(t)$, $v(t)$, and $a(t)$ in terms of the angular frequency ω, the amplitude A, and the phase angle φ.

assume: $x(t) = A\cos(\omega t + \phi)$ ①

then: $v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$ ②

$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$ ③

b) (10pts) Given the boundary conditions $x(t_0) = x_0$ and $v(t_0) = v_0$, derive an expression for the phase angle φ and the amplitude A.

From eq ①: $x_0 = A\cos(\omega t_0 + \phi)$

From eq ②: $v_0 = -\omega A \sin(\omega t_0 + \phi)$

Divide $\frac{②}{①}$: $\frac{v_0}{x_0} = \frac{-\omega \sin(\omega t + \phi)}{\cos(\omega t + \phi)} \Rightarrow \tan(\omega t + \phi) = -\frac{v_0}{x_0 \omega}$

$\Rightarrow \omega t + \phi = \tan^{-1}\left(-\frac{v_0}{x_0\omega}\right) \Rightarrow \boxed{\phi = \tan^{-1}\left(-\frac{v_0}{x_0\omega}\right) - \omega t}$

If we divide v_0 by ω, we have

$v_0/\omega = -A\sin(\omega t + \phi)$

then we can write: $\left(x_0^2 + \left(\frac{v_0}{\omega}\right)^2\right)^{1/2} = \left(A^2\cos^2(\omega t + \phi) + A^2\sin^2(\omega t + \phi)\right)^{1/2}$

$= A\left(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right)^{1/2}$

$\Rightarrow \boxed{A = \left(x_0^2 + \left(\frac{v_0}{\omega}\right)^2\right)^{1/2}}$

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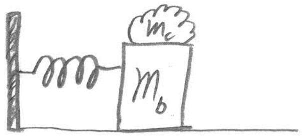
A block of mass $m_b = 5.0$ kg is attached to a spring of spring constant $k = 4$ N/m where it is allowed to oscillate horizontally on a frictionless surface. The spring is compressed a distance $d = 0.5$ m from equilibrium and released. As the block passes equilibrium, a wad of clay, $m_c = 10.0$ kg, falls from directly above and sticks to the block.

a) What is the angular frequency of the block/clay system?
b) What is the velocity of the block/clay system just after the collision?
c) What is the amplitude of the block/clay system?

a)



$\sum F = ma$

$-kx = (m_b + m_c)a \Rightarrow a = -\boxed{\frac{k}{m_b + m_c}}x$

$\boxed{\omega = \left(\frac{k}{m_b + m_c}\right)^{1/2}} = \left[\frac{4}{15}\right]^{1/2} = \underline{0.52 \text{ rad/s}}$

b) To know the velocity after the collision, I need to know the velocity before the collision.

at $t=0$, $x(0) = d$, $v(0) = 0$

$x \Rightarrow$ so: $d = A\cos(\phi)$ ①

$v \Rightarrow$ and: $0 = -\omega \sin\phi$ ② $\Rightarrow \sin(\phi) = 0 \Rightarrow \boxed{\phi = 0}$

then: $d = A\cos(0) \Rightarrow \boxed{d = A}$

Before collision

At equilibrium, $v = \boxed{v_{max} = \omega A}$, where $\boxed{\omega = \sqrt{\frac{k}{m_b}}}$

collide: $P_I = P_F \Rightarrow m_b v_{max} = (m_b + m_c) v_F$

continued ↓

Sample Test 6 P_3 continued

$$m_b \sqrt{\frac{k}{m_b}}\, d = (m_b + m_c)\, v_F$$

$$\Rightarrow \boxed{v_F = \frac{d\sqrt{m_b k}}{m_b + m_c}} = \frac{(0.5)(5.0 \cdot 4)^{1/2}}{15} = 0.15\ m/s$$

c) Let $t=0$, $v(0) = 0.15\ m/s$

$x(0) = 0$

$$x \Rightarrow 0 = A\cos(\phi) \Rightarrow \phi = \pi/2,\ 3\pi/2$$

$$v \Rightarrow 0.15 = -\omega A\, \cancel{\sin(3\pi/2)}^{-1}$$

$$0.15 = \omega A \Rightarrow A = \frac{0.15}{0.52} = \underline{0.29\ m}$$

SAMPLE TEST 6

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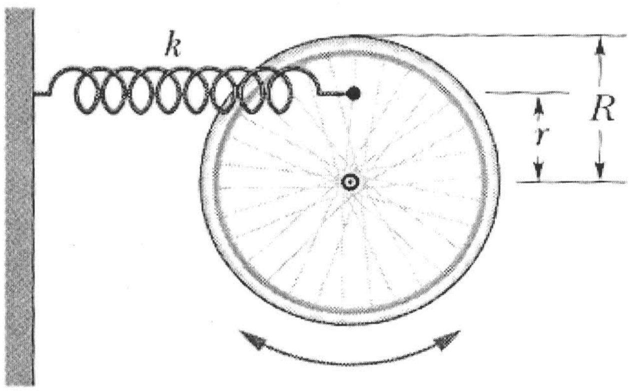
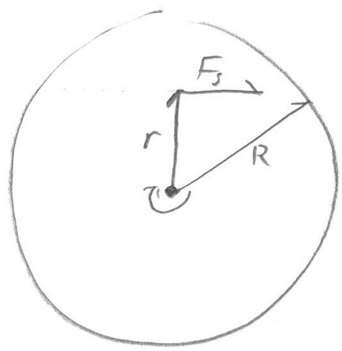
A wheel is free to rotate about a fixed axle. A spring with a spring constant k is attached to one of its spokes at a distacne r from the axle, as shown in the picture. Assume that the wheel is a hoop of mass m and radius R (the spokes have negligible mass).

a) Using **Newton's Second Law**, find the angular frequency of small oscillations in terms of m, R, r and the spring constant k.
b) Using **Energy techniques**, find the angular frequency of small oscillations in terms of m, R, r and the spring constant k.
b) What is the angular frequency if r = R.
c) What is the angular frequency if r = 0.





a) $\sum \tau = I\alpha$

$F_s r = I\alpha$

$-kxr = I\alpha$, $x = r\theta$

$-kr\theta r = I\alpha$

$\alpha = -\boxed{\frac{kr^2}{I}}\theta \Rightarrow \omega = \left[\frac{kr^2}{I}\right]^{1/2}$, $I = mR^2$

$\Rightarrow \boxed{\omega = \left[\frac{kr^2}{mR^2}\right]^{1/2}}$

continued

Sample Test 6, P4 continued

b) $E_T = \frac{1}{2}kx^2 + \frac{1}{2}I\omega_v^2$ angular velocity!

$$\frac{dE_T}{dt} = \cancel{\frac{1}{2}}\, k\, \cancel{2}\, x \frac{dx}{dt} + \cancel{\frac{1}{2}}\, I\, \cancel{2}\, \omega_v \frac{d\omega_v}{dt} = 0$$

$$\Rightarrow kxv + I\omega_v\alpha = 0 \quad , \quad v = r\omega_v \Rightarrow \omega_v = \frac{v}{r}$$

$$\Rightarrow kx\cancel{v} + mR^2\frac{\cancel{v}}{r}\cdot\frac{a}{r} = 0 \quad a = r\alpha \Rightarrow \alpha = \frac{a}{r}$$

$$\Rightarrow kx + m\frac{R^2}{r^2}a = 0$$

$$\Rightarrow a = -\boxed{\frac{k}{m}\frac{r^2}{R^2}}\,x \Rightarrow \boxed{\omega = \left[\frac{k}{m}\frac{r^2}{R^2}\right]^{1/2}}$$

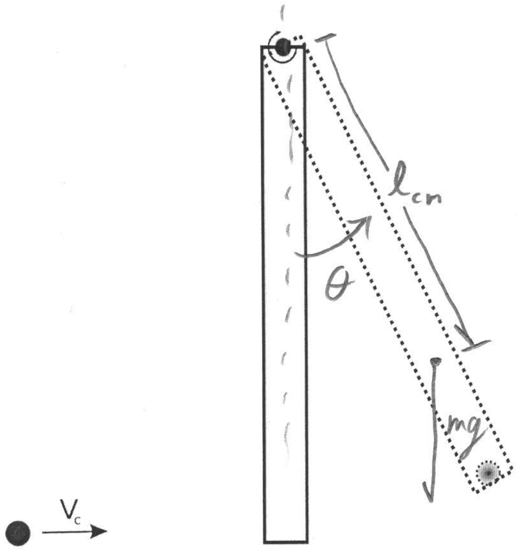
c) if $R = r$, $\omega = \sqrt{\frac{k}{m}}$

d) if $r = 0$, $\underline{\omega = 0}$

Sample Test 6
Phys 111, Fall 2010, Section 1

A 1 kg meter stick is hung from its and allowed to pivot. A small wad of clay with a mass of 0.25 kg with a velocity V_c = 2 m/s impacts the bottom of the meter stick. Assuming that the resulting oscillations are small:

a) (10 pts) Find the angular frequency of the resulting pendulum.
~~b) (10 pts) Find the amplitude of the resulting pendulum in terms of θ.~~



$$T = -mg\, l_{cm} \sin\theta$$

a) $\Sigma T = I\alpha$

$$-mg\, l_{cm} \sin\theta = I\alpha \Rightarrow \alpha = \frac{-mg\, l_{cm}}{I} \sin\theta$$

And for small θ, $\sin\theta \cong \theta$

$$\Rightarrow \alpha = \frac{-m g\, l_{cm}}{I}\theta \Rightarrow \omega = \left[\frac{m_T g\, l_{cm}}{I}\right]^{1/2}$$ where $m_T = m + M$

Need to find I and l_{cm}

$$I = I_s + I_c, \quad I_s = I_{cm} + Md^2$$

$$I_s = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \boxed{\frac{1}{3}ML^2}$$

$$I_c = mL^2$$

$$\boxed{I = \left(\tfrac{1}{3}M + m\right)L^2}$$

continued ↓

Sample Test 6, P5 continued

$$l_{cm} = \frac{\sum m_i l_i}{\sum m_i} = \frac{M\frac{L}{2} + mL}{M + m} = \boxed{\frac{\frac{1}{2}M + m}{M + m} L}$$

$$\omega = \left[\underbrace{\cancel{(m+M)}}_{m_T} g \underbrace{\frac{\frac{1}{2}M + m}{\cancel{M+m}} \cancel{L}}_{l_{cm}} \underbrace{\frac{1}{(\frac{1}{3}M + m)L^2}}_{\frac{1}{I}} \right]^{1/2}$$

$$\boxed{\omega = \left[\frac{\frac{1}{2}M + m}{\frac{1}{3}M + m} \frac{g}{L} \right]^{1/2}}$$

$m = \frac{1}{4}M, \quad L = 1$

$$\omega = \left[\frac{\frac{1}{2}\cancel{M} + \frac{1}{4}\cancel{M}}{\frac{1}{3}\cancel{M} + \frac{1}{4}\cancel{M}} \frac{g}{1} \right]^{1/2} \Rightarrow \boxed{\omega = \left[\frac{9}{7} g \right]^{1/2}}$$

b) ~~conserve angular momentum to~~ find