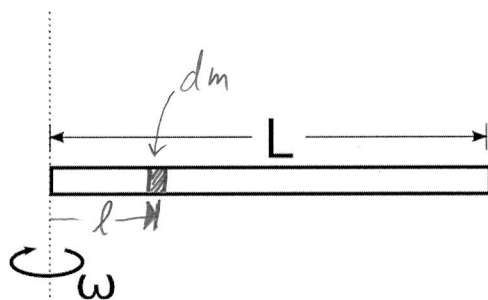


Rotation – Set 2

1

Calculate the moment of inertia of a uniform bar of length L and mass M about the axis of rotation shown.



$$\lambda = \frac{M}{L}$$

$$I = \int r^2 dm, \quad dm = \lambda dl$$
$$dm = \frac{M}{L} dl, \quad \underline{r = l}$$

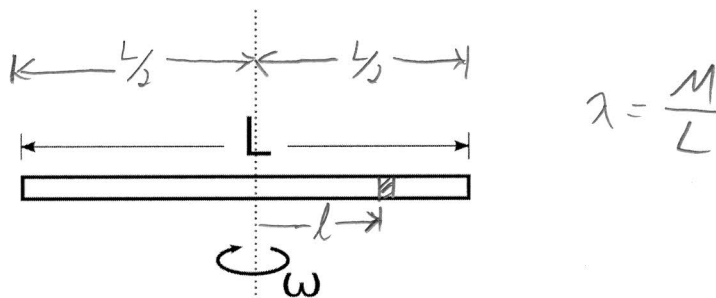
$$I = \int_0^L l \frac{M}{L} dl = \frac{M}{L} \int_0^L l dl$$
$$= \frac{M}{L} \left(\frac{1}{3} l^3 \right) \Big|_0^L$$
$$= \frac{M}{L} \frac{1}{3} L^3$$

$$I = \frac{1}{3} ML^2$$

Rotation – Set 2

2

Calculate the moment of inertia of a uniform bar of length L and mass M about the axis of rotation shown.



$$I = \int r^2 dm, \quad dm = \lambda dl$$

$$dm = \frac{M}{L} dl, \quad \underline{r = l}$$

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} l^2 \frac{M}{L} dl = \frac{M}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} l^2 dl = \frac{M}{L} \left(\frac{1}{3} l^3 \right) \Big|_{-\frac{L}{2}}^{\frac{L}{2}}$$

$$= \frac{M}{L} \frac{1}{3} \left(\frac{L^3}{8} + \frac{L^3}{8} \right) = \frac{M}{L} \frac{1}{3} \frac{L^3}{4}$$

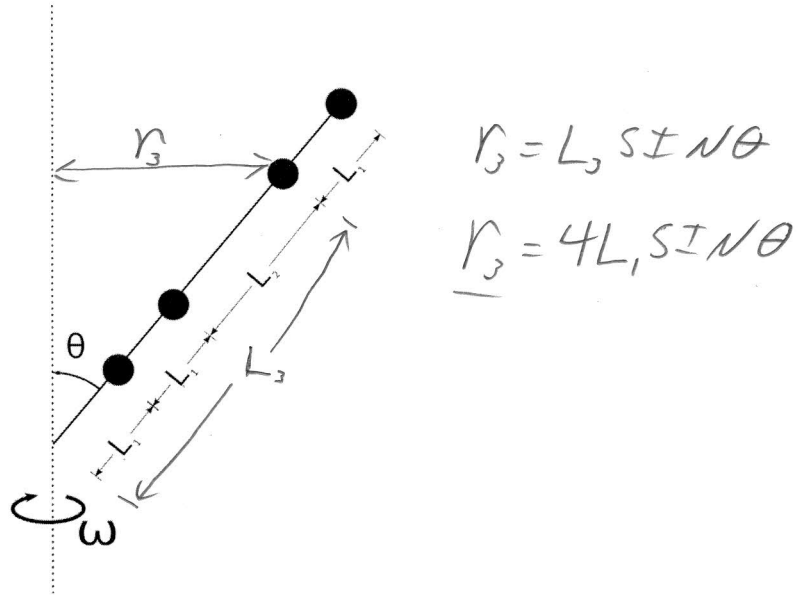
$$\boxed{I = \frac{1}{12} ML^2}$$

Rotation – Set 2

3

Four point masses, each of mass m , are attached to a rigid massless rod that makes an angle θ with the axis of rotation. Let $L_2 = 2L_1$.

- What is the moment of inertia of this system?
- What is the kinetic energy of this system if it's rotating with angular velocity ω .



$$I = \sum m_i r_i^2$$

$$= m r_1^2 + m r_2^2 + m r_3^2 + m r_4^2$$

$$= m (r_1^2 + r_2^2 + r_3^2 + r_4^2)$$

$$= m (L_1^2 \sin^2 \theta + (2^2) L_1^2 \sin^2 \theta + (4^2) L_1^2 \sin^2 \theta + (5^2) L_1^2 \sin^2 \theta)$$

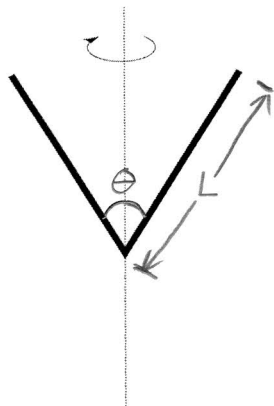
$$= m L_1^2 \sin^2 \theta (1 + 4 + 16 + 25)$$

$$\boxed{I = 46 m L_1^2}$$

Rotation – Set 2

4

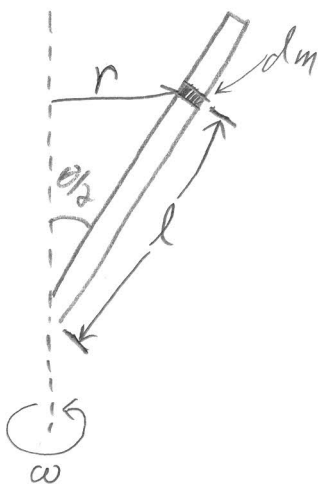
Calculate the moment of inertia of the bent rod of mass M shown in the figure below. The rotation axis is in the plane of the "V" bisecting it at the vertex. The rod is bent at an angle θ and each leg has a length L .



$$\lambda = \frac{M}{2L}$$

Each leg has Length L
and there are two legs.

The system is symmetric. For every dm on the right, there is one on an equal distance from the axis on the left.
So, we can calculate just one leg and multiply by 2.



$$I = \int r^2 dm$$

$r \Rightarrow$ distance from mass to axis of rot.

$dm \Rightarrow$ mass element of the rod

$l \Rightarrow$ is the variable of integration since we are summing pieces of dm along the rod

$$\text{So: } dm = \lambda dl \Rightarrow dm = \frac{M}{2L} dl$$

$$I = 2 \int_0^L r^2 \frac{M}{2L} dl$$

Since we're integrating in l , we must write r in terms of l .

Looking at the diagram: $r = l \sin\left(\frac{\theta}{2}\right)$

$$\text{so: } I = \int_0^L l^2 \sin^2\left(\frac{\theta}{2}\right) \frac{M}{L} dl$$

$$= \frac{M}{L} \sin^2\left(\frac{\theta}{2}\right) \int_0^L l^2 dl$$

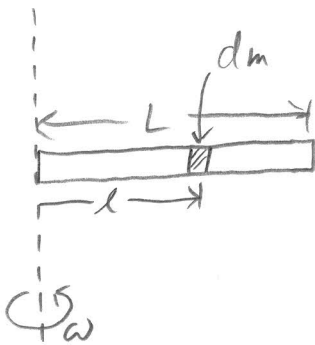
$$= \frac{M}{L} \sin^2\left(\frac{\theta}{2}\right) \frac{1}{3} L^3$$

$$\boxed{I = \frac{1}{3} ML^2 \sin^2\left(\frac{\theta}{2}\right)}$$

Rotation

A thin rod of length L has a non-uniform density profile of $\lambda = \lambda_0 \left[2 \frac{l^2}{L^2} + \frac{1}{3} \right]$.

What is the moment of inertia of this rod if it's about an axis perpendicular to the left end of the rod?



$$I = \int r^2 dm$$

$$dm = \lambda dl$$

$$\Rightarrow dm = \lambda_0 \left[2 \frac{l^2}{L^2} + \frac{1}{3} \right] dl$$

$r = l$ this time

$$I = \int_0^L l^2 \lambda_0 \left[2 \frac{l^2}{L^2} + \frac{1}{3} \right] dl$$

$$= \lambda_0 \int_0^L \left(2 \frac{l^4}{L^2} + \frac{l^2}{3} \right) dl$$

$$= \lambda_0 \left[\frac{2}{5} \frac{L^5}{L^2} + \frac{L^3}{9} \right] = \lambda_0 L^3 \left(\frac{2}{5} + \frac{1}{9} \right) = \frac{23}{45} \lambda_0 L^3$$

$$I = \frac{23}{45} \lambda_0 L^3$$

$$M = \int dm = \int_0^L \lambda_0 \left[2 \frac{l^2}{L^2} + \frac{1}{3} \right] dl = \lambda_0 \frac{2}{3} \frac{L^3}{L^2} + \frac{L}{3}$$

$$\boxed{M = \lambda_0 L}$$

$$\text{So: } \boxed{I = \frac{23}{45} ML^2}$$