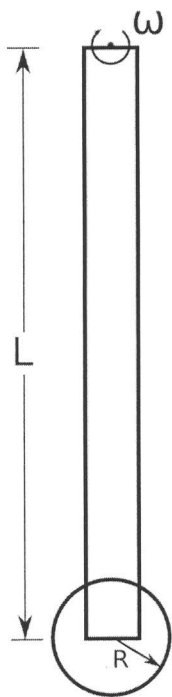


## Rotation – Set 3

1

A clock pendulum is constructed from a solid bar of length  $L$  and mass  $M$  with a disk of radius  $R$  and mass  $m$ . The bar is then hung from a pivot at one end and the disk is attached to the opposite end, as in the picture below.

- a) The moment of inertia of a uniform rod about an axis perpendicular to the rod through its center of mass is  $I_{cm} = \frac{1}{12}ML^2$ . Using the **Parallel Axis Theorem**, calculate the moment of inertia of the rod about one end.
- b) The moment of inertia of a disk about an axis perpendicular to its surface through its center of mass is  $I_{cm} = \frac{1}{2}mR^2$ . Using the **Parallel Axis Theorem**, calculate the moment of inertia of the disk when it's attached to the pendulum as shown in the picture below.
- c) Using the Principle of Superposition, calculate the moment of inertia of the combined rod-disk system.



$$a) I_R = I_{cm} + Md^2, \quad d = \frac{L}{2}$$

$$I_R = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \left(\frac{1}{12} + \frac{1}{4}\right)ML^2$$

$$\boxed{I_R = \frac{1}{3}ML^2}$$

$$b) I_D = I_{cm} + md^2, \quad d = L$$

$$\boxed{I_D = \frac{1}{2}mR^2 + mL^2}$$

$$c) I = I_R + I_D$$

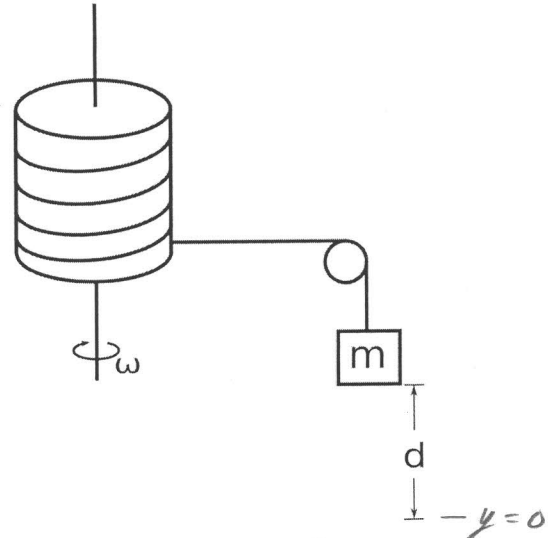
$$\boxed{I = \frac{1}{3}ML^2 + \frac{1}{2}mR^2 + mL^2}$$

## Rotation – Set 3

A solid cylinder of mass  $M$ , radius  $R$ , and moment of inertia  $I = \frac{1}{2}MR^2$  is allowed to rotate without friction about an axis through its center as shown. A massless string is wrapped around the cylinder, passes over a small massless frictionless pulley and is attached to a small mass  $m$ .

If the mass and the cylinder start from rest, what will the angular velocity of the cylinder be after the mass falls through a distance  $d$ ?

Using conservation of energy, find an expression for  $\omega_f$  in terms of  $d$ ,  $M$ ,  $m$ , and  $R$ .



$$U_I = mgd$$

$$U_F = 0$$

$$K_I = 0$$

$$K_F = \frac{1}{2}mV^2 + \frac{1}{2}I\omega_f^2 \leftarrow \text{Energy of Both objects.}$$

$$mgd = \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_f^2$$

$$V = R\omega$$

$$mgd = \frac{1}{2}mR^2\omega_f^2 + \frac{1}{4}MR^2\omega_f^2$$

$$mgd = \left(\frac{1}{2}m + \frac{1}{4}M\right)R^2\omega_f^2$$

$$\omega_f = \left[ \frac{m}{\frac{1}{2}m + \frac{1}{4}M} \frac{gd}{R^2} \right]^{\frac{1}{2}}$$

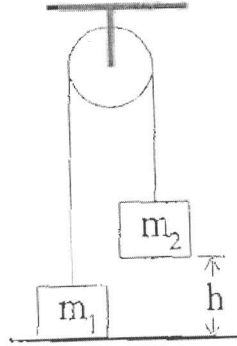
# Rotation – Set 3

Use work energy to solve the following problem.

Two masses are connected by a light string passing over a frictionless pulley. the Mass  $m_2$  is released from rest at a height of 4.0 m above the ground. You can treat the pulley as a solid disk,  $I = \frac{1}{2} MR^2$

Determine the speed of  $m_1$  as  $m_2$  hits the ground.

- $m_1 = 3.0 \text{ kg}$
- $m_2 = 5.0 \text{ kg}$
- $m_{\text{pulley}} = 0.5 \text{ kg}$
- $r_{\text{pulley}} = 0.1 \text{ m}$



$$U_I = m_2 g h$$

$$U_F = m_1 g h$$

$$K_I = 0$$

$$K_F = \frac{1}{2} (m_1 + m_2) v_F^2 + \frac{1}{2} I \omega_F^2$$

$\uparrow$   
 $m_1$  and  $m_2$  are moving at  $v$

$\uparrow$   
 Pulley is spinning

$$m_2 g h = m_1 g h + \frac{1}{2} (m_1 + m_2) v_F^2 + \frac{1}{2} I \omega_F^2$$

$$v = r \omega \Rightarrow \omega = \frac{v}{r}, \quad I = \frac{1}{2} m_p r^2$$

$$(m_2 - m_1) g h = \frac{1}{2} (m_1 + m_2) v_F^2 + \frac{1}{2} \left( \frac{1}{2} m_p r^2 \right) \frac{v_F^2}{r^2}$$

$$(m_2 - m_1) g h = \frac{1}{2} (m_1 + m_2) v_F^2 + \frac{1}{4} m_p v_F^2$$

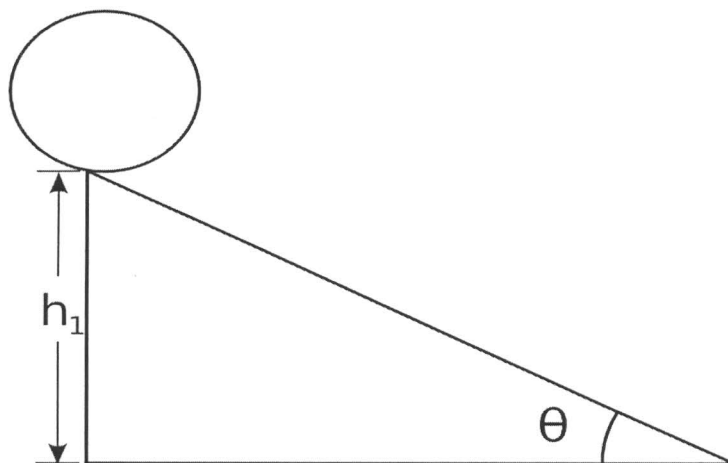
$$v_F = \left[ \frac{m_2 - m_1}{\frac{1}{2} m_1 + \frac{1}{2} m_2 + \frac{1}{4} m_p} g h \right]^{\frac{1}{2}} = \left[ \frac{5 - 3}{\frac{3}{2} + \frac{5}{2} + \frac{1}{8}} (9.8)(4.0) \right]^{\frac{1}{2}} = 4.4 \text{ m/s}$$

## Rotation – Set 3

4

A rolling object with a radius  $R$ , mass  $m$ , and moment of inertia  $I_{cm} = \frac{1}{2}mR^2$ , starts from rest at the top of an incline plane of height  $h$  that makes an angle  $\theta$  with the horizontal.

- What is the linear velocity of disk at the bottom?
- What is the angular velocity of the disk at the bottom?



a)  $U_I = mgh$

$$U_F = 0$$

$$K_I = 0$$

$$K_F = \frac{1}{2}m v_{cm}^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}m v_{cm}^2 + \frac{1}{2}I\omega^2$$

$$v = R\omega \Rightarrow \omega = \frac{v}{R}, \quad I = \frac{1}{2}mR^2$$

$$\Rightarrow mgh = \frac{1}{2}m v_{cm}^2 + \frac{1}{2}(\frac{1}{2}mR^2) \frac{v_{cm}^2}{R^2}$$

$$mgh = (\frac{1}{2} + \frac{1}{4}) v_{cm}^2 \Rightarrow v_{cm} = \left( \frac{4}{3}gh \right)^{1/2}$$

Rotation Set 3 P4 - continued

$$b) mgh = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

$$v = R\omega, \quad I = \frac{1}{2} m R^2$$

$$mgh = \frac{1}{2} m R^2 \omega^2 + \frac{1}{2} \cdot \frac{1}{2} m R^2 \omega^2 \Rightarrow gh = \frac{3}{4} R^2 \omega^2$$

$$\boxed{\omega = \left[ \frac{4}{3} \frac{gh}{R^2} \right]^{1/2}}$$

# Rotation – Set 3

A solid cylinder (radius =  $2R$ , mass =  $M$ ) rolls without slipping as it is pulled by a massless yoke attached to a string. The string goes over a frictionless pulley shaped as a solid disk (radius =  $R$ , mass =  $M$ ) and is attached to a hanging weight (mass =  $M$ ).

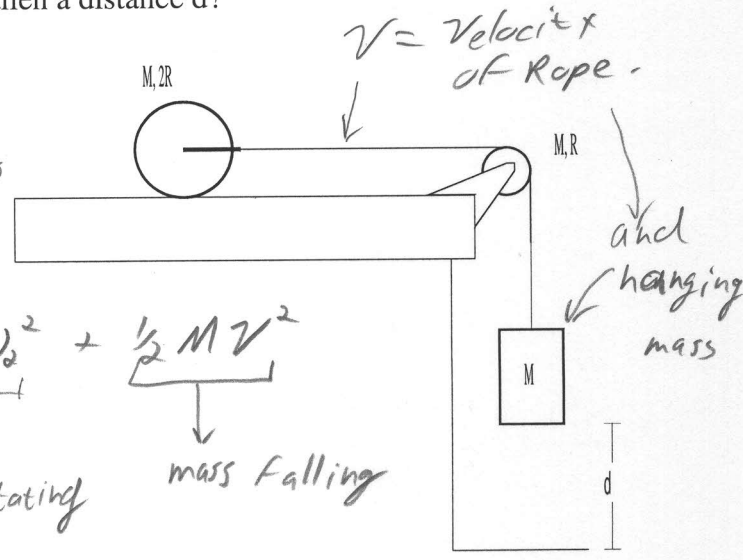
What is the velocity of the hanging weight after it has fallen a distance  $d$ ?

$$U_I = Mgd$$

$$U_F = 0$$

$$K_I = 0$$

$$K_F = \underbrace{\frac{1}{2} M V^2}_{\text{Disk 1 Rotating and translating}} + \underbrace{\frac{1}{2} I_{D1} \omega_1^2}_{\text{Disk 2 Rotating}} + \underbrace{\frac{1}{2} I_{D2} \omega_2^2}_{\text{Disk 2 Rotating}} + \underbrace{\frac{1}{2} M V^2}_{\text{mass Falling}}$$



Need to translate every thing to Velocity

using  $v = r\omega \Rightarrow \omega = \frac{v}{R}$

$$Mgd = \frac{1}{2} M v^2 + \frac{1}{2} \left( \frac{1}{2} M (2R)^2 \right) \frac{v^2}{(2R)^2} + \frac{1}{2} \left( \frac{1}{2} M R^2 \right) \frac{v^2}{R^2} + \frac{1}{2} M v^2$$

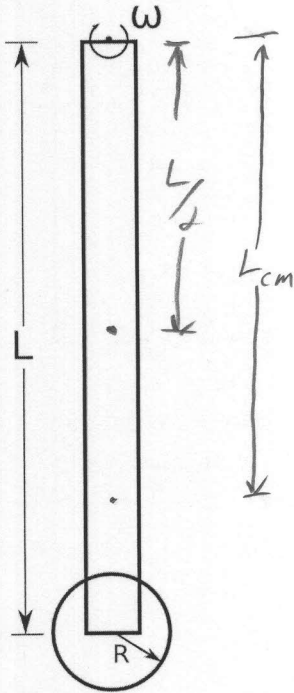
$$gd = \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2} \right) v^2$$

$$v = \sqrt{gd}$$

# Rotation – Set 3

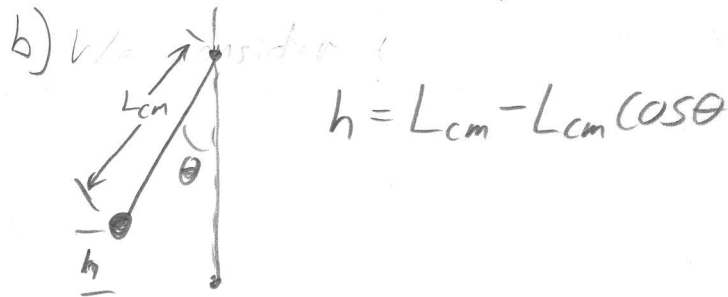
Consider, once again, the clock pendulum pictured below. The disk has a mass  $m$  and radius  $r$  and the bar has mass  $M$  and length  $L$ .

- a) Calculate the center of mass of the combined bar-disk system as measured from the axis of rotation.
- b) If the pendulum is pivoted so that it makes an angle  $\theta$  with the vertical, what will the angular velocity be when  $\theta=0$ ?



$$a) L_{cm} = \frac{1}{M+m} \sum m_i r_i = \frac{1}{M+m} (mL + M \frac{L}{2})$$

$$L_{cm} = \frac{\frac{1}{2}M + m}{M+m} L$$



$$h = L_{cm} - L_{cm} \cos \theta$$

Gravitational Potential Energy is dependant on the change in height of the center of mass.

$$U_I = mg(L_{cm} - L_{cm} \cos \theta) \quad U_F = 0$$

$$K_I = 0 \quad K_F = \frac{1}{2} I \omega^2$$

$$\omega^2 = \frac{2mgL_{cm}}{I} (1 - \cos \theta)$$

Rotation Set 3 P6, continued

$$\Rightarrow \omega^{\circ} = 2mg \frac{\frac{1}{2}M+m}{M+m} L \cdot \left( \frac{1}{3}ML^2 + \frac{1}{2}mR^2 + mL^2 \right)^{-1}$$



## Rotation – Set 3

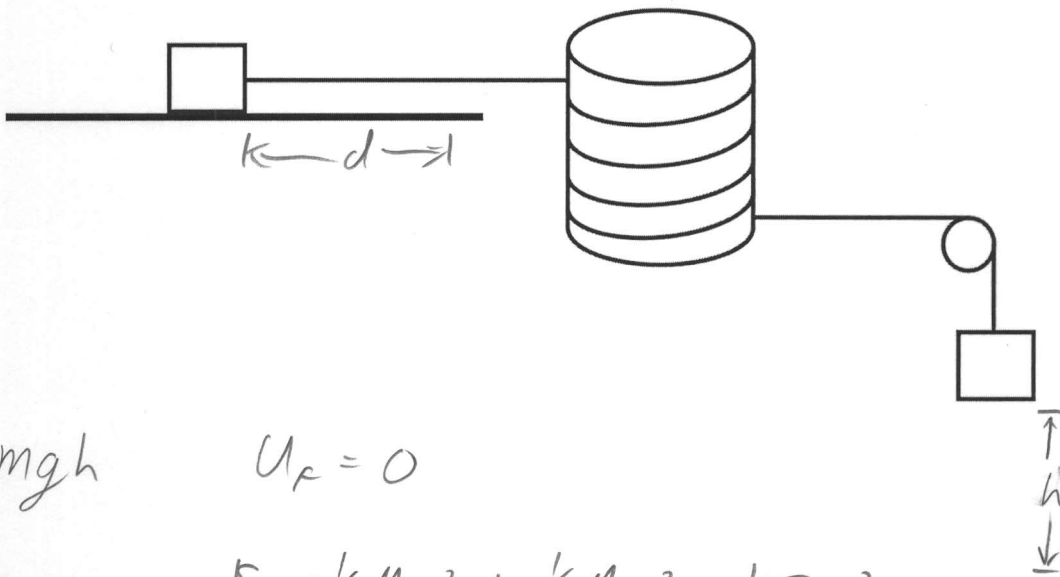
7

A block of mass  $M$  rests on a rough table with  $\mu_k = 0.3$ . A massless string is attached to the block, wrapped around a solid cylinder having a mass  $M$  and a radius  $R$ , runs over a massless frictionless pulley, and is attached to a second block of mass  $M$  that is hanging freely.

Using work/energy techniques, calculate the velocity of the blocks after they have moved a distance  $d$ .

$$I_{\text{cylinder}} = \frac{1}{2} MR^2$$

[ NOTE: Do NOT use torque/kinematics ]



$$U_I = mgh \quad U_F = 0$$

$$K_I = 0 \quad K_F = \frac{1}{2} Mv^2 + \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

$$W_F = -\mu_k mgd, \quad \underline{d = h}$$

$$mgd - \mu_k mgd = \frac{1}{2} Mv^2 + \frac{1}{2} Mv^2 + \frac{1}{2} \frac{1}{2} MR^2 \frac{v^2}{R^2}$$

$$mgd(1 - \mu_k) = \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{4}\right) v^2$$

$$v = \left[ \frac{4}{5} gd(1 - \mu_k) \right]^{1/2}$$