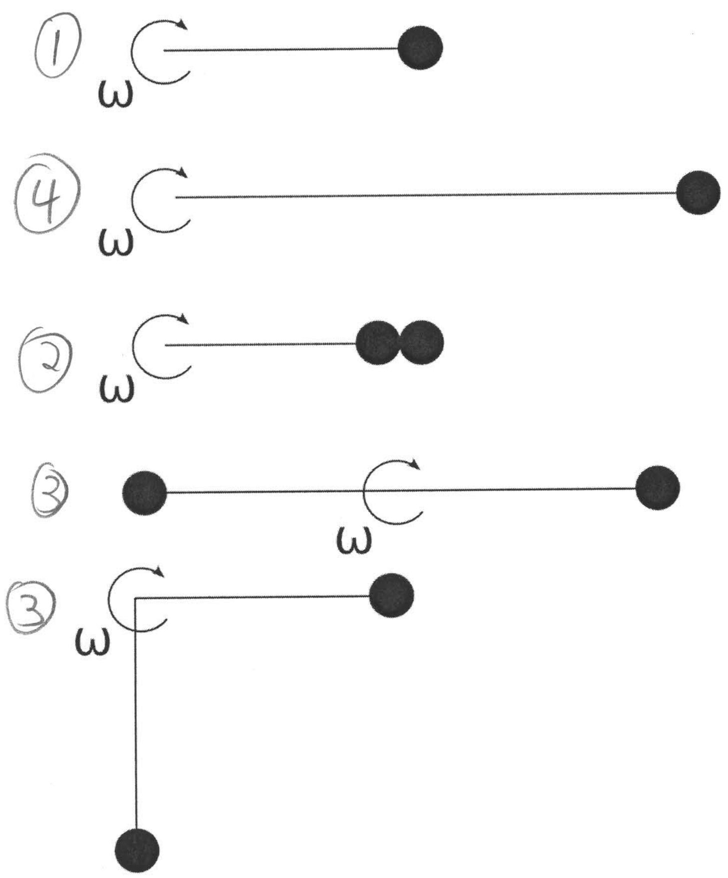
Rotation – Set 4

Below are several objects. Each circle is a point mass, and each point has the same mass. The connecting rods are massless. Rank them in order of moment of inertia, least to most. If any have the SAME moment, give them the same ranking number. **Explain your reasoning**.



$I = \sum m_i r_i^2$, So: The top object has a single mass close to the pivot

next, 2 < 3 because the mass doubles but 2 has one mass sightly closer. to the pivot

the 2 3's have twice the mass, but both at the same distance

last, 4 is a single mass twice as far but I goes as r^2, so r makes a bigger difference than m.

Circular disks A and B have the same mass and thickness, but the density of disk A is greater than the density of disk B. Which has the greater moment of inertia? **Explain your reasoning.**

To have the same mass but different density, the Volume of A must be smaller. If they have the same thickness, the radius of A must be smaller. So, A has its mass concentrated closer to the axis of rotation so $I_B > I_A$

Two bars have the same mass, but one is shorter than the other. Which has a larger moment of inertia? **Explain your reasoning.**

more mass further from the axis.

$I_2 > I_1$

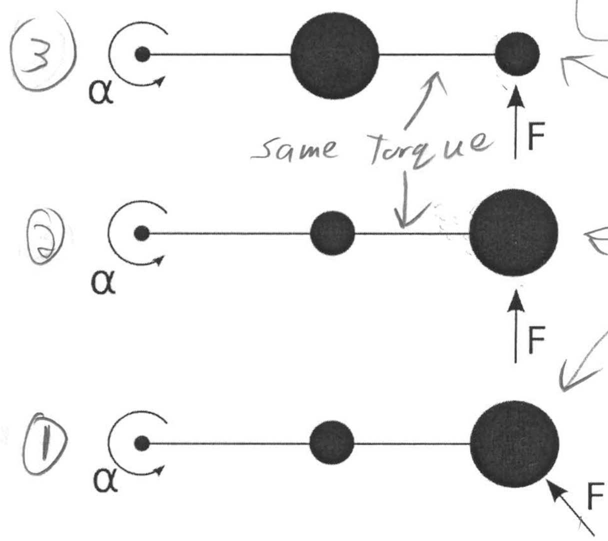
Below are the cross sections of five solids with identical masses. They all have equal widths at their widest points. Without looking them up or doing any calculations, rank the objects in order of moment of inertia, least to most. **Explain your reasoning**.

5 4 3 2 1

Goes in order according to which object has it's mass closest to the axis of rotation.

In the pictures below, two masses are connected by a massless rod and the system is allowed to rotate about the pivot shown. The large circle is more massive than the small circle. A force is applied to each system as shown in the diagram. Rank the three systems in order of their angular accelerations, least to most. **Explain your reasoning.**

$$\boxed{\alpha = \frac{\tau}{I} = \frac{r \times F}{I}}$$



smallest I

same I

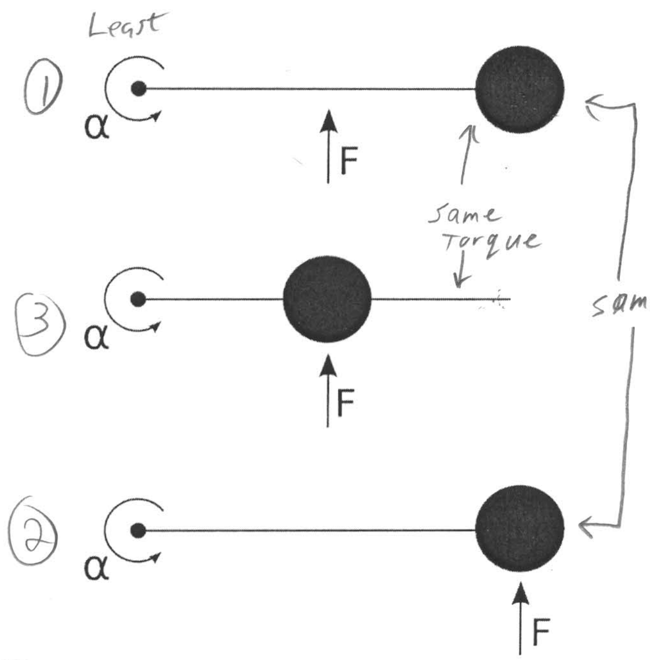
smallest Torque

constant τ, $I \uparrow$ $\alpha \downarrow$

const I, $\tau \uparrow$, $\alpha \uparrow$

In the pictures below, two masses are connected by a massless rod and the system is allowed to rotate about the pivot shown. A force is applied to each system as shown in the diagram. Rank the three systems in order of their angular accelerations, least to most. **Explain your reasoning.**

$$\boxed{\alpha = \frac{r \times F}{M L^2}}$$



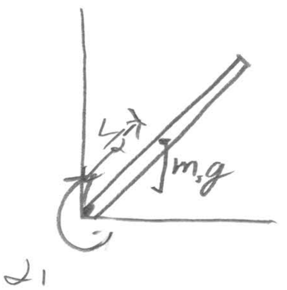
same I

Torque is proportional to R from pivot

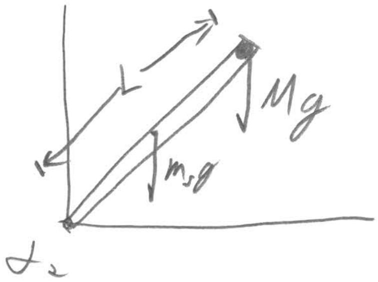
I is proportional to R^2 so moving the mass has a greater effect than moving the force.

Rotation – Set 4

Two meter sticks are stood in the corner where the floor meets the wall. One has a large mass attached to the end furthest from the corner. Which one hits the ground first? Discuss.



← Hits first



Torque increases by ML

I increases by ML^2

I increases more than Torque

so $\alpha \downarrow$

So: $\alpha_2 < \alpha_1$

A small 0.75 kg ball is attached to one end of a 1.25 m long massless rod. The other end of the rod is hung from a pivot under the influence of gravity. When the resulting pendulum is 30⁰ from vertical:

a) What is the magnitude of the torque about the pivot? (Draw a free body diagram)
b) What is the instantaneous angular acceleration (the acceleration at the moment when the pendulum is released)?
c) Can we use kinematics to find the angular velocity of the pendulum at the bottom of its swing? Why or why not?

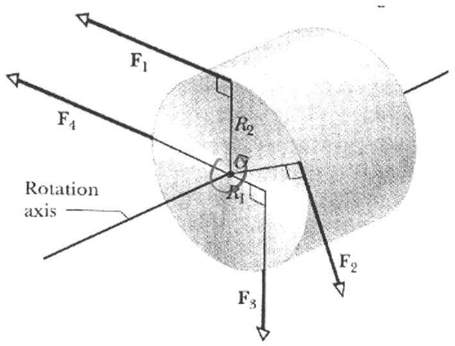
a) $\vec{T} = \vec{l} \times \vec{F} = (1.25)(0.75)(9.8)\,\mathrm{SIN}\,30 = 4.6\ N\cdot m$

b) $T = I\alpha \Rightarrow \alpha = \frac{T}{I} = \frac{4.6}{(0.75)(1.25)^2} = 3.9\ \mathrm{rad/s}$

c) No, because the torque is <u>not</u> constant. It depends on the angle θ

A cylinder with a mass of 2.0 kg can rotate about its central axis through the point 0. Forces are applied as in the figure below. $F_1 = 6.0$ N, $F_2 = 4.0$ N, $F_3 = 2.0$ N, $F_4 = 5.0$ N. Find the direction and the magnitude of the angular acceleration of the cylinder. (During rotation, the forces maintain the same angles relative to the cylinder.) $R_2 = 12\,cm = 0.12\,m$, $R_1 = 5\,cm = 0.05\,m$

$$\sum \vec{T} = I\vec{\alpha}$$



$$F_1R_2 - F_2R_2 - F_3R_1 + 0 = I\alpha$$

$$\alpha = \frac{1}{I}\left(F_1R_2 - F_2R_2 - F_3R_1\right), \quad I = \tfrac{1}{2}MR_2^2$$

$$\alpha = \frac{1}{\frac{1}{2}(2)(.12)^2}\left[(6.0)(.12) - (4.0)(.12) - (2.0)(.05)\right] = \boxed{9.7\ \mathrm{rad/s}}$$

A door has a mass of 50 kg and is 0.8 m wide. The moment of inertia is $I = 1/3\ MW^2$ where W is the width of the door. I push on the door with a constant force of F = 10 N in two places; in the middle of the door a distance W/2 from the hinge and at the knob, a distance W from the hinge.

a) Draw free body diagrams of the two cases.
b) What is the magnitude of the Torque for each case?
c) What is the magnitude of the angular accelerations for each case?
d) How much time does it take the door to rotate through 90^0 in each case?
e) How much force would I have to apply at W/2 so that the door rotated through 90^0 in the same amount of time as applying 10 N to the knob?

a)

b) $\vec{T} = \vec{r} \times \vec{F} = rF\sin\theta, \quad \theta = 90^\circ \Rightarrow \sin\theta = 1$

$T_1 = \frac{W}{2} F \qquad T_2 = WF$

$T_1 = (0.4)(10) \qquad T_2 = (0.8)(10)$

$= 4\ N\cdot m \qquad = 8\ N\cdot m$

c) $\alpha = \frac{T}{I}$

$\alpha_1 = \frac{4}{\frac{1}{3}MW^2} = \frac{4}{\frac{1}{3}(50)(0.8)^2} \qquad \alpha_2 = \frac{8}{\frac{1}{3}(50)(0.8)^2}$

$= 0.375\ \text{rad}/s \qquad = 0.75\ \text{rad}/s$

Rotation Set 4 P6 Continued

d) $\theta = \cancel{\theta_0}^{0} + \cancel{\omega_0 t}^{0} + \frac{1}{2}\alpha t^2$

$\theta = \frac{1}{2}\alpha t^2 \Rightarrow t = \left(\frac{2\theta}{\alpha}\right)^{1/2}$, $t_1 = \left(\frac{(2)(\pi/2)}{0.375}\right)^{1/2} = 2.9\,s$

$t_2 = \left(\frac{2\,\pi/2}{0.75}\right)^{1/2} = 2.0\,s$

e) To make the time equal, I need $\alpha_1 = \alpha_2$

And $\alpha = \frac{T}{I}$

so, $\frac{T_1}{I_1} = \frac{T_2}{I_2}$, But $I_1 = I_2$

$\Rightarrow T_1 = T_2 \Rightarrow r_1 F_1 = r_2 F_2$

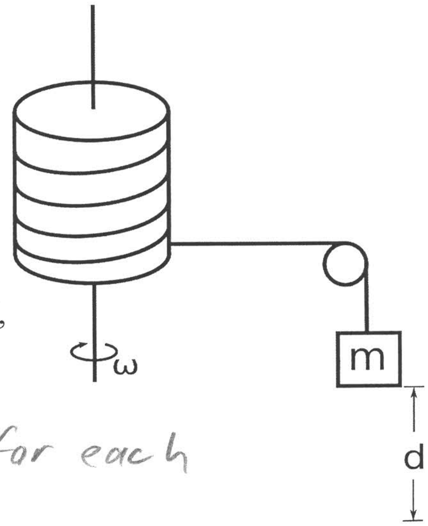
So: $F_1 = \frac{r_2}{r_1} F_2 \Rightarrow F_1 = \frac{W}{W/2} F_2$

$F_1 = 2F_2$

So: $\boxed{F_1 = 20\,N}$

Rotation – Set 4

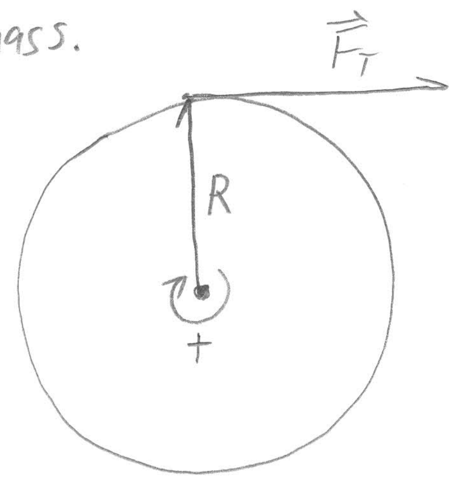
A solid cylinder of mass *M*, radius *R*, and moment of inertia $I = \frac{1}{2}MR^2$ is allowed to rotate without friction about an axis through its center as shown. A massless string is wrapped around the cylinder, passes over a small massless frictionless pulley and is attached to a small mass *m*.



If the mass and the cylinder start from rest, what will the angular velocity of the cylinder be after the mass falls through a distance d?

Using **Torque and Kinematics**, find an expression for ω_f in terms of *d*, *M*, *m*, and *R*.

1) Draw Free-Body Diagrams, one for each mass.



Cylinder, Top View
Defining clockwise rotation as positive.

Defining down as positive y to agree with positive rotation of the cylinder

2) Write Newton's 2nd law, Both torque version and translation version for each mass.

Cylinder, torque only
no translation

$$\sum \vec{\tau} = I\alpha$$

$$\sum R \cdot F \sin\theta = I\alpha$$

Hanging mass, translation only
no torque

$$\sum F = ma$$

continued ↓

Rotation Set 4, P7 Continued

Cylinder

$\sum RF\sin\theta = I\alpha$

$\theta = 90°, \sin\theta = 1$

$\Rightarrow RF_T = I\alpha, \quad I = \frac{1}{2}MR^2$

$\Rightarrow \cancel{R}F_T = \frac{1}{2}MR^{\cancel{2}}\alpha$

$\Rightarrow \boxed{F_T = \frac{1}{2}MR\alpha}$ ①

Hanging mass

$\sum F = ma$

$\boxed{mg - F_T = ma}$ ②

Combine ① and ② to eliminate F_T, solve for α

$mg - \frac{1}{2}MR\alpha = ma, \quad a = R\alpha$

$\Rightarrow mg - \frac{1}{2}MR\alpha = mR\alpha$

$\Rightarrow mg = (\frac{1}{2}M + m)R\alpha$

$\Rightarrow$ ③ $\boxed{\alpha = \frac{m}{\frac{1}{2}M + m}\frac{g}{R}}$ save this for later.

3) We have acceleration. Now we need to use kinematics to get ω, the angular velocity.

$\theta = \cancel{\theta_0}^0 + \cancel{\omega_0}^0 t + \frac{1}{2}\alpha t^2$

$\omega = \cancel{\omega_0}^0 + \alpha t$ eliminate t

$\Rightarrow t = \frac{\omega}{\alpha}$

$\theta = \frac{1}{2}\alpha\frac{\omega^2}{\alpha^2} \Rightarrow \boxed{\omega = [2\theta\alpha]^{1/2}}$

continued ↓

Rotation Set 4, P7 continued

So we have: $\omega = [2\theta\alpha]^{1/2}$

we are given d, so we need to write $d = R\theta$

$\Rightarrow \theta = \frac{d}{R}$

Then: $\omega = \left[2\frac{d}{R}\alpha\right]^{1/2}$

And we plug in α from eq. 3

$$\omega = \left[2\frac{d}{R}\frac{m}{\frac{1}{2}M + m}\frac{g}{R}\right]^{1/2}$$

$$\boxed{\omega = \left[\frac{m}{\frac{1}{2}M + m}\frac{2dg}{R^2}\right]^{1/2}}$$