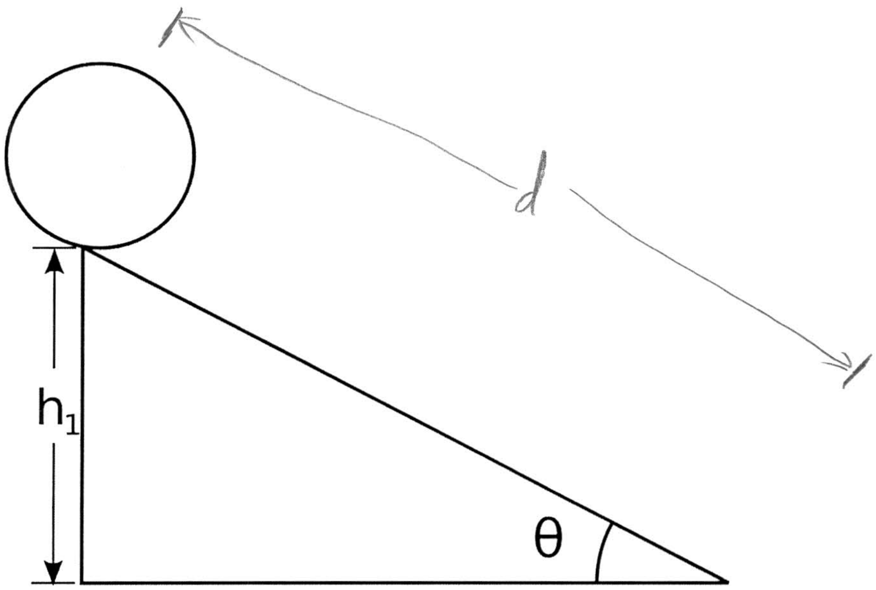
Rotation – Set 5

Use **Torque and Kinematics** to solve this problem.

A rolling object with a radius *R*, mass *m*, and moment of inertia I, starts from rest at the top of an incline plane of height *h* that makes an angle θ with the horizontal.

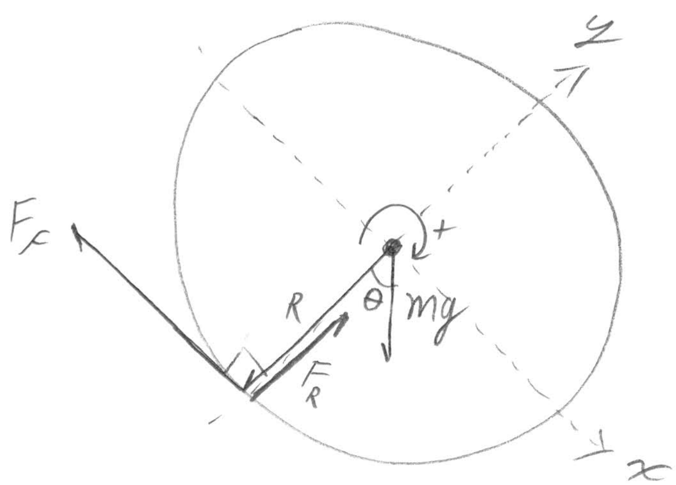
a) Find an expression for the linear and angular acceleration of the object in terms of I.
b) Using kinematics, find an expression for the linear and angular ~~acceleration~~ Velocity of the object in terms of I?

c) Assume that the object is a disk with $I=\frac{1}{2}mR^2$ and plug I into your velocity expressions. Verify that your answers are the as when you solved this problem using energy.



a)

Step 1 - FBD



In this problem, we need to consider both Rotation and translation.

So we have positive rotation as well as the x-y coordinates labeled

continued ↓

Rotation set 5, P1 continued

Step 2 - NSL

One object but both translation and rotation

Rotation

$$\sum \vec{T} = \vec{I}\alpha$$

$$\Rightarrow \sum RF\sin\theta = I\alpha$$

$$\Rightarrow \underbrace{RF_f \sin(90)}_{\sin 90 = 1} + \underbrace{\cancel{RF_R \sin(180)}}_{\sin(180)=0}{}^{\to 0} + \underbrace{\cancel{0 \cdot mg \sin\theta}}_{R=0}{}^{\to 0} = I\alpha$$

$$\Rightarrow RF_f = I\alpha \Rightarrow \boxed{F_f = \frac{I}{R}\alpha}$$

Translation

x: $\boxed{mg\sin\theta - F_f = ma}$ 2

y: $F_R - mg\cos\theta = 0$ ← not useful

Plug 1 → 2: $mg\sin\theta - \frac{I}{R}\alpha = ma$

Let $a = R\alpha$: $mg\sin\theta - \frac{I}{R^2}a = ma \Rightarrow mg\sin\theta = (m + I/R^2)a$

$$\boxed{a = \frac{m}{m + I/R^2} g\sin\theta}$$ 1

$$\boxed{\alpha = \frac{m}{m + I/R^2}\frac{g}{R}\sin\theta}$$ 2

Rotation set 5, P1 continued

b) kinematics

$$x = \cancel{x_0}^0 + \cancel{v_0 t}^0 + \frac{1}{2}at^2 \qquad v = \cancel{v_0}^0 + at$$

$$t = \frac{v}{a}$$

$$d = \frac{1}{2}\cancel{a}\frac{v^2}{a^{\cancel{2}}}$$

$$d = \frac{1}{2}\frac{v^2}{a} \Rightarrow \boxed{v = (2da)^{\frac{1}{2}}} \quad ③$$

Plug in a from eq. ①

$$v = \left[2d\frac{m}{m + I/R^2} g \sin\theta\right]^{\frac{1}{2}}$$

$$\boxed{v = \left[\frac{m}{m + I/R^2} 2gd\sin\theta\right]^{\frac{1}{2}} = \left[\frac{m}{m + I/R^2} 2gh\right]^{\frac{1}{2}}}$$

$$\omega = \frac{v}{R} \Rightarrow \boxed{\omega = \left[\frac{m}{m + I/R^2}\frac{2gh}{R^2}\right]^{\frac{1}{2}}}$$

$$\text{IF } I = \frac{1}{2}mR^2, \quad m + I/R^2 \Rightarrow m + \frac{\frac{1}{2}m\cancel{R^2}}{\cancel{R^2}} \Rightarrow \frac{3}{2}m$$

$$\text{so: } \frac{m}{m + I/R^2} \Rightarrow \frac{\cancel{m}}{\frac{3}{2}\cancel{m}} \Rightarrow \underline{2/3}$$

$$\text{and: } \boxed{v = \left[\frac{4}{3}gh\right]^{\frac{1}{2}}} \text{ and } \boxed{\omega = \left[\frac{4}{3}\frac{gh}{R^2}\right]^{\frac{1}{2}}}$$

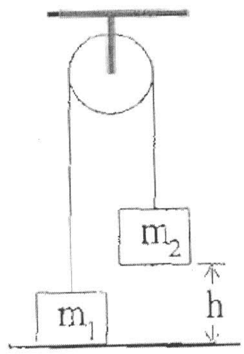
Rotation – Set 5

Use **Torque and Kinematics** to solve the following problem.

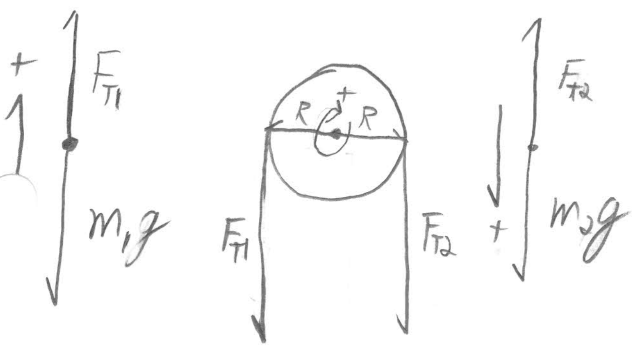
Two masses are connected by a light string passing over a frictionless pulley. the Mass m_2 is released from rest at a height of 4.0 m above the ground. You can treat the pulley as a solid disk.

Determine the speed of m_1 as m_2 hits the ground.

$m_1 = 3.0$ kg
$m_2 = 5.0$ kg
$m_{pulley} = 0.5$ kg
$r_{pulley} = 0.1$ m



FBD



Force

$$F_{T1} - m_1 g = m_1 a \quad (1)$$

$$m_2 g - F_{T2} = m_2 a \quad (2)$$

Torque

$$R F_{T2} - R F_{T1} = I\alpha \quad (3)$$

Solve (1) and (2) for F_{T1} and F_{T2} (they are <u>NOT</u> the same)

From (1): $F_{T1} = m_1(a+g)$

From (2): $F_{T2} = m_2(g-a)$

continued ↓

Rotation Set 5, P2 continued

① and ② → ③ :

$$R m_2 (g - a) - R m_1 (a + g) = I \alpha$$

want a for kinematics, so sub $\alpha = \frac{a}{R}$

and sub $I = \frac{1}{2} m_p R^2$

$$\Rightarrow \cancel{R} m_2 (g - a) - \cancel{R} m_1 (a + g) = \frac{1}{2} m_p R^{\cancel{2}} \frac{a}{\cancel{R}}$$

$$\Rightarrow m_2 g - m_2 a - m_1 a - m_1 g = \frac{1}{2} m_p a$$

$$\Rightarrow (m_2 - m_1) g = (m_1 + m_2 + \frac{1}{2} m_p) a$$

$$\Rightarrow \boxed{a = \frac{m_2 - m_1}{m_1 + m_2 + \frac{1}{2} m_p} g}$$

* Do kinematics, sub in a last

$$y = \cancel{y_0}^{0} + \cancel{v_0}^{0} t + \frac{1}{2} a t^2 \qquad v = \cancel{v_0}^{0} + a t$$

$$\Rightarrow t = \frac{v}{a}$$

$$h = \frac{1}{2} \cancel{a} \frac{v^2}{a^{\cancel{2}}}$$

$$\Rightarrow v = (2 y a)^{1/2} \Rightarrow \boxed{v = \left[2h \frac{m_2 - m_1}{m_1 + m_2 + \frac{1}{2} m_p} g \right]^{1/2}}$$

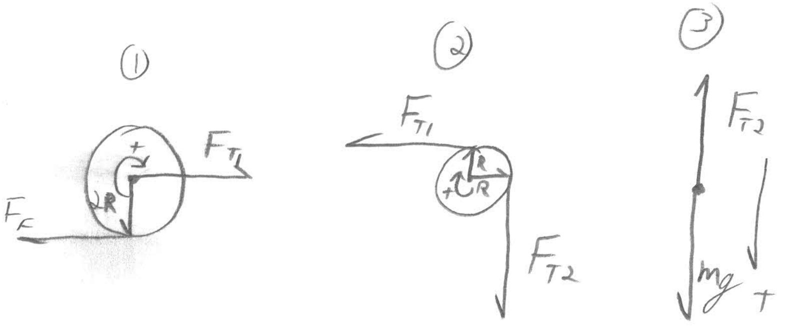
Rotation – Set 5

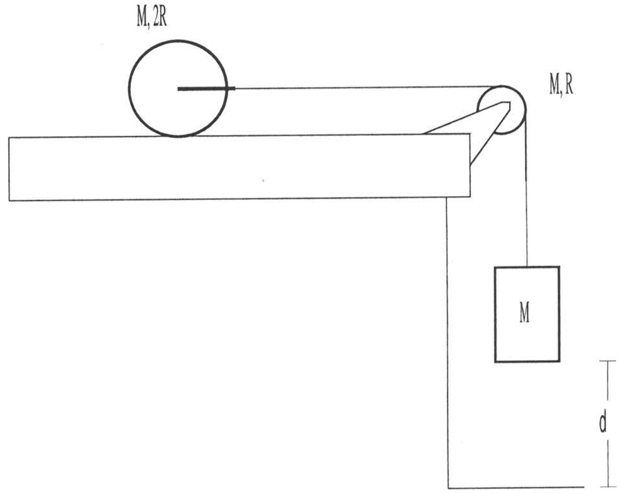
Use **Torque and Newton's Second Law** solve this problem.

A solid cylinder (radius = 2R, mass = M) rolls without slipping as it is pulled by a massless yoke attached to a string. The string goes over a frictionless pulley shaped as a solid disk (radius = R, mass = M) and is attached to a hanging weight (mass = M).

$$I_{cylinder} = \frac{1}{2} MR^2$$

What is the acceleration of the system?





① Is rotating AND translating

$$\cancel{-2(0)F_{T1}} + \cancel{2}RF_F = \frac{1}{\cancel{2}} m \cancel{4} R^{\cancel{2}} \alpha \rightarrow \text{Torque}$$

$$F_F = mR\alpha$$

$$F_{T1} - F_F = ma \rightarrow \text{Force}$$

Put these two together

$$F_{T1} - mR\alpha = ma$$

$$\Rightarrow F_{T1} - \frac{1}{2}ma = ma$$

$$\boxed{F_{T1} = \frac{3}{2}ma} \quad ①$$

continued ↓

Rotation Set 5, P3 continued

2 This pulley is only rotating

$$\cancel{R}F_{T2} - \cancel{R}F_{T1} = \frac{1}{2}mR^{\cancel{2}}\alpha \qquad a = r\alpha$$

$$\boxed{F_{T2} - F_{T1} = \frac{1}{2}ma} \quad 2$$

3 The mass is translating

$$mg - F_{T2} = ma$$

$$\Rightarrow \boxed{F_{T2} = m(g-a)} \quad 3$$

Plug 1 and 3 into 2

$$\cancel{m}(g-a) - \frac{3}{2}\cancel{m}a = \frac{1}{2}\cancel{m}a$$

$$g - a - \frac{3}{2}a = \frac{1}{2}a$$

$$g = \left(1 + \frac{3}{2} + \frac{1}{2}\right)a$$

$$\Rightarrow \boxed{a = \frac{1}{3}g}$$

Rotation – Set 5

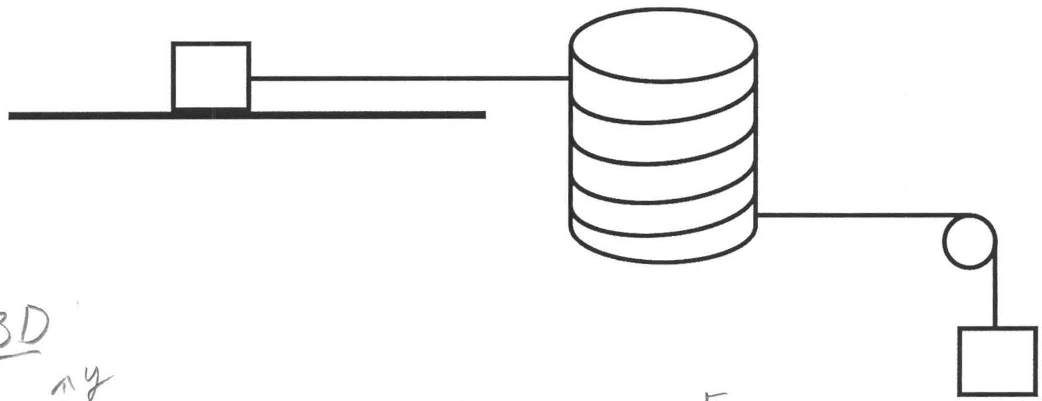
4

Use **Torque and Newton's Second Law** solve this problem.

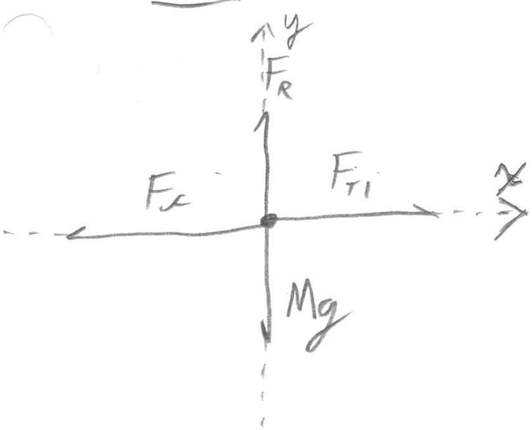
A block of mass M rests on a rough table with $\mu_k = 0.3$. A massless string is attached to the block, wrapped around a solid cylinder having a mass M and a radius R, runs over a massless frictionless pulley, and is attached to a second block of mass M that is hanging freely.

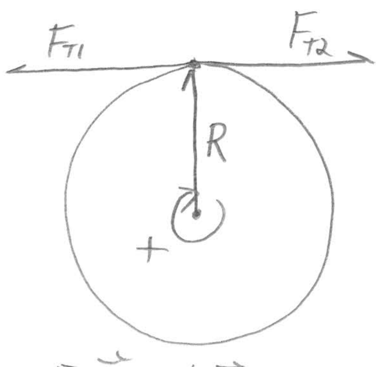
Find the acceleration of this system.

$$I_{cylinder} = \frac{1}{2} MR^2$$



FBD





NSL $\sum \vec{F} = ma$

x: $F_{T1} - F_f = Ma$

y: $F_R - Mg = 0$

$F_R = Mg$

$\Rightarrow \boxed{F_{T1} - \mu_k Mg = Ma}$ ①

$\sum \vec{\tau} = I\vec{\alpha}$

$\sum RF\sin\theta = I\alpha$

$RF_{T2} - RF_{T1} = \frac{1}{2} MR^2\alpha$

$\boxed{F_{T2} - F_{T1} = \frac{1}{2} MR\alpha}$ ②

$\sum \vec{F} = m\vec{a}$

$\boxed{-F_{T2} + Mg = Ma}$ ③

continued ↓

Rotation Set 5, P4 continued

Eliminate F_{T1} and F_{T2}:

From ①: $F_{T1} = \mu_K Mg + Ma$

From ③: $F_{T2} = Mg - Ma$

into ②: $\cancel{M}g - \cancel{M}a - \mu_K \cancel{M}g - \cancel{M}a = \frac{1}{2}\cancel{M}R\alpha$

$g(1-\mu_K) = 2a + \frac{1}{2}R\alpha \Rightarrow g(1-\mu_K) = 2R\alpha + \frac{1}{2}R\alpha$

$\Rightarrow g(1-\mu_K) = \frac{5}{2}R\alpha$

$\Rightarrow \boxed{\alpha = \frac{2}{5(1-\mu_K)}\frac{g}{R}}$

or $\boxed{a = \frac{2}{5(1-\mu_K)}g}$