SAMPLE TEST 5

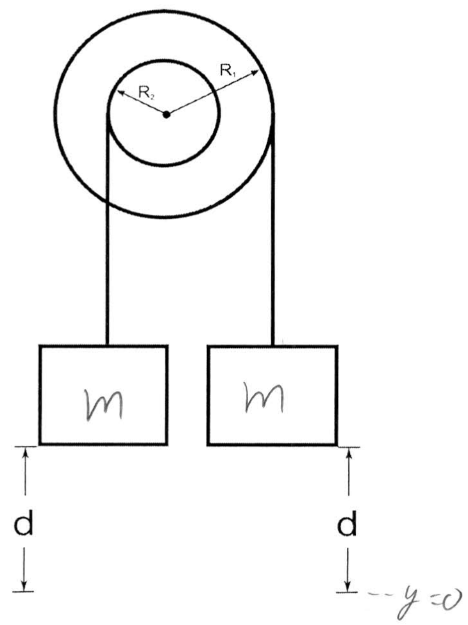
PHYS 111, FALL 2010, SECTION 1

2. The picture below shows a modified atwood machine composed of two pulleys of different radii that have been glued together so that their angular velocities will be the same. Two blocks of equal mass are attached to the system by ropes. One rope is wound around the small pulley and the other rope is wound around the large pulley. The mass of the pulley is the same as the mass of the two blocks, and $R_2 = \frac{1}{2}R_1$

Assume that the moment of inertia of the pulley is $I = \frac{1}{2}MR_1^2$

a) If the masses are initially at rest, which way will the pulley rotate, clockwise or counter clockwise?

b) Using Work/Energy techniques, find an expression for the angular velocity of the pulleys after the mass attached to the large pulley has moved a distance d.



a) Clock wise.

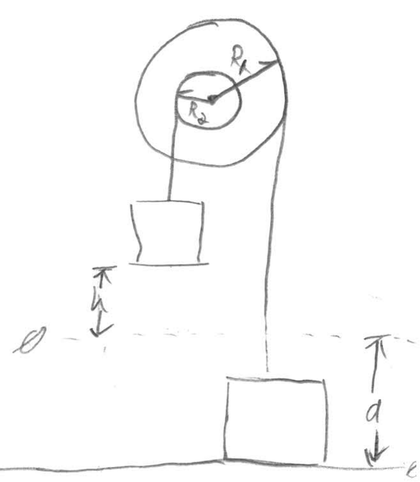
$T_1 = R_1 mg$, $T_2 = R_2 mg$

$R_1 > R_2 \Rightarrow T_1 > T_2$

b) $U_I = mgd$ $\quad U_F = mgh$

$K_I = 0$ $\quad K_F = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega^2$

$\Rightarrow \cancel{m}gd = \cancel{m}gh + \frac{1}{2}\cancel{m}(v_1^2 + v_2^2) + \frac{1}{2}\left[\frac{1}{2}\cancel{m}R_1^2\right]\omega^2$



$\Rightarrow \boxed{gd = gh + \frac{1}{2}(v_1^2 + v_2^2) + \frac{1}{4}R_1^2\omega^2}$ (1) $\quad h \neq d$

continued

we need to write h in terms of d using $s = r\theta$

$d = R_1\theta$ and $h = R_2\theta \Rightarrow h = \frac{1}{2}R_1\theta$, θ is the same for both

Divide and we have:

$$\frac{h}{d} = \frac{\frac{1}{2}\cancel{R_1}\cancel{\theta}}{\cancel{R_1}\cancel{\theta}} \Rightarrow \boxed{h = \tfrac{1}{2}d} \quad ②$$

We also need to write v_1 and v_2 in terms of ω using $v = r\omega$

$$③ \; \boxed{v_1 = R_1\omega}, \quad v_2 = R_2\omega \Rightarrow \boxed{v_2 = \tfrac{1}{2}R_1\omega} \; ④$$

Put it all together and solve for ω

②, ③, ④ $\rightarrow$ ①:

$$gd = g\tfrac{1}{2}d + \tfrac{1}{2}\left(R_1^2\omega^2 + \left(\tfrac{1}{2}R_1\right)^2\omega^2\right) + \tfrac{1}{4}R_1^2\omega^2$$

$$\Rightarrow \left(1 - \tfrac{1}{2}\right)gd = \left(\tfrac{1}{2} + \tfrac{1}{8} + \tfrac{1}{4}\right)R_1^2\omega^2$$

$$\Rightarrow \tfrac{1}{\cancel{2}}gd = \tfrac{7}{\underset{4}{\cancel{8}}}R_1^2\omega^2 \Rightarrow \boxed{\omega = \left[\frac{4}{7}\frac{gd}{R_1^2}\right]^{1/2}}$$

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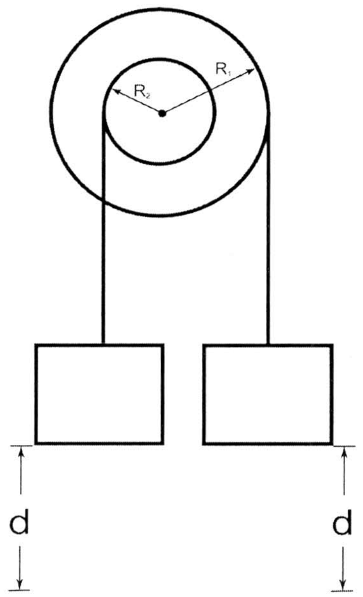
Phys 111, Fall 2010, Section 1

3. The picture below shows a modified atwood machine composed of two pulleys of different radii that have been glued together so that their angular velocities will be the same. Two blocks of equal mass are attached to the system by ropes. One rope is wound around the small pulley and the other rope is wound around the large pulley. The mass of the pulley is the same as the mass of the two blocks.

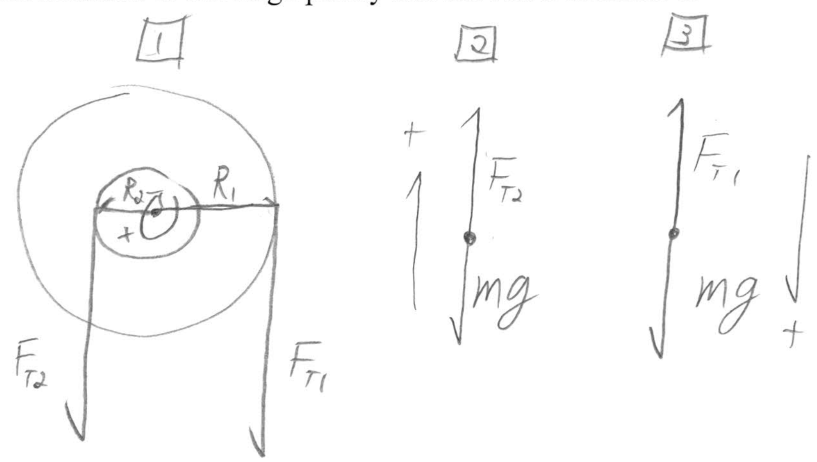
Assume that the moment of inertia of the pulley is $I = \frac{1}{2} M R_1^2$

a) Using Newton's Second Law (the rotational version and/or the translational as appropriate), find an expression for the angular acceleration of the pulleys.

b) Using your expression for acceleration from part a and kinematics, find an expression for the angular velocity of the pulleys after the mass attached to the large pulley has moved a distance d.



a)



Watch these accelerations... they aren't the same...

[1]

$\Sigma \tau = I\alpha$

① $\boxed{R_1 F_{T1} - R_2 F_{T2} = I\alpha}$

[2]

$\Sigma F = m a_2$

$F_{T2} - mg = m a_2$

② $\boxed{F_{T2} = mg + m a_2}$

[3]

$\Sigma F = m a_1$

$mg - F_{T1} = m a_1$

③ $\boxed{F_{T1} = mg - m a_1}$

Subst. ② ③ → ①:

$\cancel{R_1}(\cancel{m}g - \cancel{m}a_1) - \frac{1}{2}\cancel{R_1}(\cancel{m}g + \cancel{m}a_2) = [\frac{1}{2}\cancel{m}R_1^{\cancel{2}}]\alpha$

④ $\boxed{g - a_1 - \frac{1}{2}g - \frac{1}{2}a_2 = \frac{1}{2}R_1\alpha}$

Continued ↓

Question asks for α, so write a_1 and a_2 in terms of α using $a = r\alpha$.

$$\boxed{a_1 = R_1 \alpha}, \quad a_2 = R_2 \alpha \Rightarrow \boxed{a_2 = \tfrac{1}{2} R_1 \alpha}$$

Put these into (4):

$$g - R_1\alpha - \tfrac{1}{2}g - \tfrac{1}{2}\tfrac{1}{2}R_1\alpha = \tfrac{1}{2}R_1\alpha$$

$$\Rightarrow \left(1 - \tfrac{1}{2}\right)g = \left(1 + \tfrac{1}{4} + \tfrac{1}{2}\right)R_1\alpha$$

$$\Rightarrow \tfrac{1}{\cancel{2}}g = \tfrac{7}{\cancel{4}_2}R_1\alpha$$

$$\Rightarrow \boxed{\alpha = \frac{2}{7}\frac{g}{R_1}}$$

b) Kinematics

$$\theta = \cancel{\theta_0}^{0} + \cancel{\omega_0}^{0} t + \tfrac{1}{2}\alpha t^2 \qquad \omega = \cancel{\omega_0}^{0} + \alpha t$$

$$d = R_1\theta \Rightarrow \theta = \frac{d}{R_1} \qquad \omega = \alpha t \Rightarrow t = \frac{\omega}{\alpha}$$

$$\Rightarrow \frac{d}{R_1} = \tfrac{1}{2}\alpha t^2$$

$$\Rightarrow \frac{d}{R_1} = \tfrac{1}{2}\cancel{\alpha}\frac{\omega^2}{\alpha^{\cancel{2}}} \Rightarrow \omega = \left[\frac{2d}{R_1}\alpha\right]^{1/2}$$

$$\Rightarrow \omega = \left[\frac{2d}{R_1}\frac{2}{7}\frac{g}{R_1}\right]^{1/2}$$

$$\Rightarrow \boxed{\omega = \left[\frac{4}{7}\frac{gd}{R_1^2}\right]^{1/2}}$$

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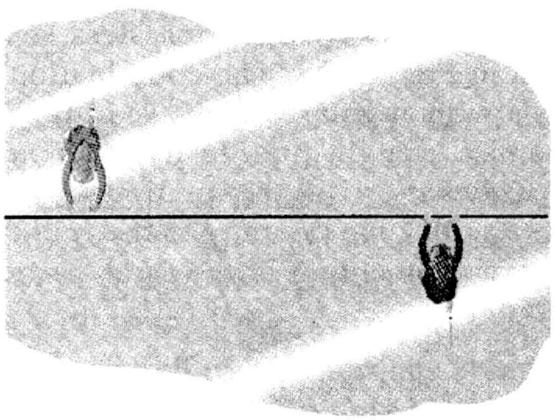
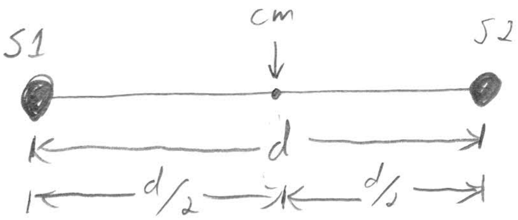
4) Two skaters, each with a mass of 50 kg, approach each other along parallel paths separated by 3.0 m. They have equal and opposite velocities of 1.4 m/s. The first skater is holding one end of a long pole with negligible mass. As the skaters pass, the second skater grabs the other end of the pole. Assume that the ice is completely frictionless.

a) What is the moment of inertia about the center of mass of the resulting skater-pole system?
b) What is the resulting angular velocity of the skater-pole system?



$m = 50\ kg$
$v = 1.4\ m/s$
$d = 3.0\ m$

a)



$I = \sum m_i r_i^2$

$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \boxed{\tfrac{1}{2} m d^2}$$

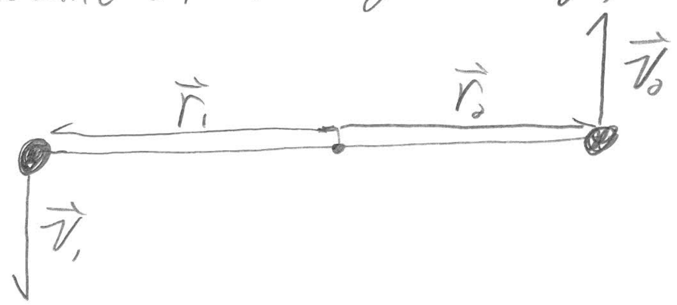
b) $L_I = L_F$

The skaters will rotate about the center of mass. But, prior to grabbing the pole, They are not a rigid body...

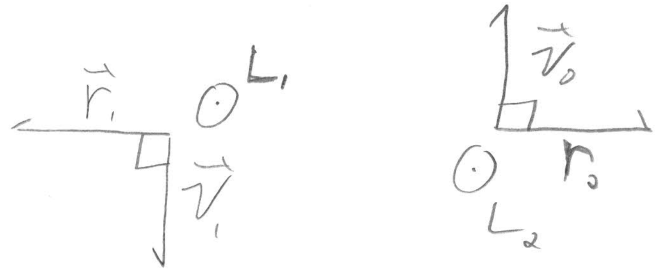
$$\vec{r}_1 \times m\vec{v}_1 + \vec{r}_2 \times m\vec{v}_2 = I\omega$$

continued
↓

At the instant of pole grabbing, it looks like this:



$\vec{r}_1 \times \vec{v}_1$ and $\vec{r}_2 \times \vec{v}_2$ give the same sign for $\vec{L}$



Both are out of the page.

So:

$$m r_1 v_1 + m r_2 v_2 = I\omega, \quad r_1 = r_2 = \frac{d}{2}$$
$$v_1 = v_2 = v$$

$$\frac{1}{2} m d v + \frac{1}{2} m d v = I\omega$$

$$m d v = I\omega \Rightarrow \omega = \frac{m d v}{I}$$

And plug in I from part a:

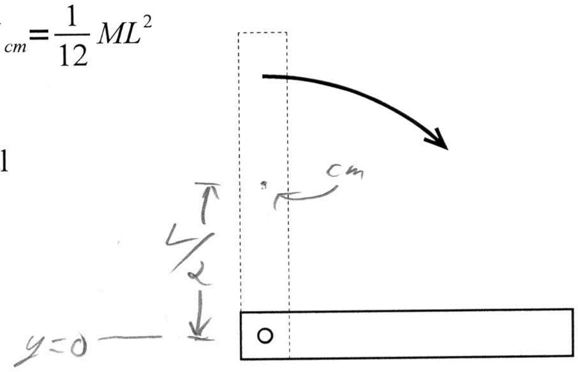
$$\omega = \frac{\cancel{m} d v}{\frac{1}{2}\cancel{m} d^{\cancel{2}}} \Rightarrow \boxed{\omega = 2\frac{v}{d}}$$

$$\boxed{\omega = 2\frac{1.4}{3} = 0.27 \text{ rad/s}}$$

5) A long, uniform rod of length L and mass M is pivoted about a horizontal, frictionless pin passing through one end of the rod. The rod is given a very slight push when it is in a vertical position.

The moment of inertia of a rod about its center of mass is $I_{cm} = \frac{1}{12} ML^2$

Find the angular velocity of the rod as it passes the horizontal



need to find I_{Rod}:

$I_{ROD} = I_{CM} + Md^2, \quad d = \frac{L}{2}$

$I_{ROD} = I_{cm} + \frac{1}{4}ML^2 \Rightarrow I_{ROD} = \frac{1}{12}ML^2 + \frac{1}{4}ML^2$

$\Rightarrow \boxed{I_{ROD} = \frac{1}{3}ML^2}$

Energy

$U_I = mg\frac{L}{2}$ $\qquad U_F = 0$

$K_I = 0$ $\qquad K_F = \frac{1}{2}I\omega^2$

$\cancel{m}g\frac{\cancel{L}}{\cancel{2}} = \cancel{\frac{1}{2}} \cdot \frac{1}{3}\cancel{m}L^{\cancel{2}}\omega^2$

$\boxed{\omega = \left[3\frac{g}{L}\right]^{\frac{1}{2}}}$