SAMPLE TEST 1
PHYS 111 SPRING 2010

Name: Key

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS – YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS (IF NOT MORE) IMPORTANT THAN) YOUR FINAL ANSWER.

Total Score: ________

1) (20pts) When we solve kinematics problems, we use two basic equations (one for velocity and one for position) that arise directly from the definitions of velocity and acceleration. Starting with the definitions of velocity and acceleration, derive these equations using calculus and **list any assumptions that were made.**

Assuming constant acceleration

$$\underline{a=\frac{dv}{dt}} \Rightarrow \int_0^t a\,dt = \int_{v_0}^{v} dv \Rightarrow a\int_0^t dt = \int_{v_0}^{v} dv$$

$$\Rightarrow at = v - v_0$$

$$\Rightarrow \boxed{v = v_0 + at}$$

$$\underline{v = \frac{dx}{dt}} \Rightarrow \int_0^t v\,dt = \int_{x_0}^{x} dx \Rightarrow \int_0^t (v_0 + at)\,dt = \int_{x_0}^{x} dx$$

$$\Rightarrow \int_0^t v_0\,dt + \int_0^t at\,dt = \int_{x_0}^{x} dx$$

$$\Rightarrow v_0 t + \tfrac{1}{2}at^2 = x - x_0$$

$$\Rightarrow \boxed{x = x_0 + v_0 t + \tfrac{1}{2}at^2}$$

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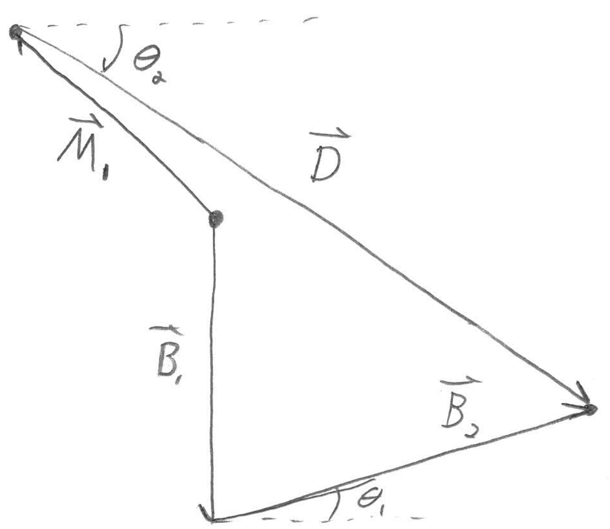
2) It's Sunday afternoon and you have just arrived at the park with your dog Bowser. Ignoring the leash law, Bowser takes off chasing rabbits and squirrels. You started out together but soon part ways. Bowser chases a rabbit due south for 25 yards and then chases a squirrel along a line 10° north of east for 30 yards. In the meantime, your leisurely stroll takes you to a point 10 yards north and 15 yards west of your starting point.

a) Find the x and y components of the displacement vector between you and Bowser.
b) Calculate the magnitude and direction of the vector you found in part a.



$\vec{B}_1 = 0\hat{i} - 25\hat{j}$ yards

$|\vec{B}_2| = 30$ yards

$\theta = 10°$

$\vec{M}_1 = -15\hat{i} + 10\hat{j}$ yards

$D_x = ?\ ,\ D_y = ?$

$$\vec{B}_1 + \vec{B}_2 = \vec{M}_1 + \vec{D} \Rightarrow \vec{D} = \vec{B}_1 + \vec{B}_2 - \vec{M}_1$$

a)

x: $D_x = B_{1x} + B_{2x} - M_{1x}$

$D_x = 0 + |\vec{B}_2|\cos\theta - M_{1x}$

$D_x = (30)\cos(10) + 15 = \boxed{45 \text{ yards}}$

y: $D_y = B_{1y} + B_{2y} - M_{1y}$

$= B_{1y} + |B_2| \sin\theta - M_{1y}$

$D_y = -25 + (30)\sin(10) + 10 = \boxed{-9.8 \text{ yards}}$

continued ↓

Test 1 problem 2 continued

b) $|\vec{D}| = (D_x^2 + D_y^2)^{1/2}$

$= (45^2 + -9.8^2)^{1/2}$

$= \boxed{46 \text{ yards}}$

$\tan\theta_2 = \left(\frac{D_y}{D_x}\right) \Rightarrow \theta_2 = \tan^{-1}\left(\frac{D_y}{D_x}\right)$

$\theta_2 = \tan^{-1}\left(\frac{-9.8}{45}\right) = \boxed{-12°}$

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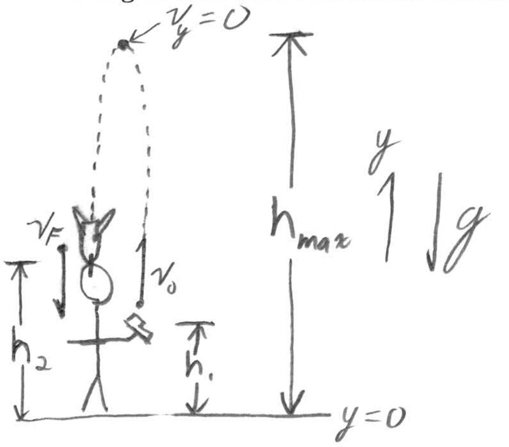
3) Wily coyote has purchased a new dart gun that he plans to use on roadrunner. Being somewhat uncoordinated he accidentally fires the gun straight up. The gun was 0.50 m above the ground when it fired, the dart reached a maximum height of 50.50 m above the ground, and Wily is 1.00 m tall. What is the darts velocity when it hits him in the head?

a) Find an expression for the Dart's INITIAL velocity. Plug in the numbers and find a numerical value.
b) Using the initial velocity, find an expression for the dart's FINAL velocity as it hits him in the head. Plug in numbers and find a numerical value.



$h_1 = 0.50\,m$

$h_2 = 1.00\,m$

$h_{max} = 50.50\,m$

$V_0 = ?,\quad V_F = ?$

a) We can find the dart's initial velocity by knowing it's maximum height and realizing that $V_y = 0$ at that point.

$$y_F = y_0 + V_{0y}t + \tfrac{1}{2}a_y t^2 \qquad V_y = V_{0y} + a_y t$$

$$h_{max} = h_1 + V_{0y}t - \tfrac{1}{2}gt^2 \qquad 0 = V_{0y} - gt$$

$$\Rightarrow t = \frac{V_{0y}}{g}$$ Plug into position eq.

$$h_{max} = h_1 + V_{0y}\frac{V_{0y}}{g} - \tfrac{1}{2}\cancel{g}\frac{V_{0y}^2}{g^{\cancel{2}}}$$

$$\Rightarrow h_{max} = h_1 + \frac{V_{0y}^2}{g} - \frac{1}{2}\frac{V_{0y}^2}{g}$$

$$\Rightarrow h_{max} = h_1 + \frac{1}{2}\frac{V_{0y}^2}{g} \Rightarrow \boxed{V_{0y} = \left[2g\,(h_{max} - h_1)\right]^{1/2}} = \left[2(9.8)(50.5 - 0.5)\right]^{1/2}$$

$$\boxed{V_{0y} = 31.3\ m/s}$$

continued ↓

b) Now we can assume we know V_{oy} and solve for V_F

$$V_F = V_{oy} + at \qquad y = y_0 + V_{oy}t + \tfrac{1}{2}at^2$$

$$① \; V_F = V_{oy} - gt \qquad ② \; h_2 = h_1 + V_{oy}t - \tfrac{1}{2}gt^2$$

awe rats... quadratic in t...

Best to just buck up!

Rewrite ② and apply the quadratic formula.

From ②: $0 = (h_1 - h_2) + V_{oy}t - \tfrac{1}{2}gt^2$

$$t = \frac{+V_{oy} \pm \sqrt{V_{oy}^2 + 2(h_1 - h_2)g}}{+g}$$

In this case, let's find numbers for t

$$t = \frac{1}{9.8}\left[31.3 \pm \left((31.3)^2 + (2)(0.5 - 1.0)(9.8)\right)^{1/2}\right]$$

$t = 1.6 \times 10^{-2}$, $\boxed{6.37 \text{ seconds}}$

Passing his head on the way <u>up</u>

on the way back down.

So! $V_F = 31.3 - (9.8)(6.37)$

$$\boxed{V_F = -31.1 \text{ m/s}}$$

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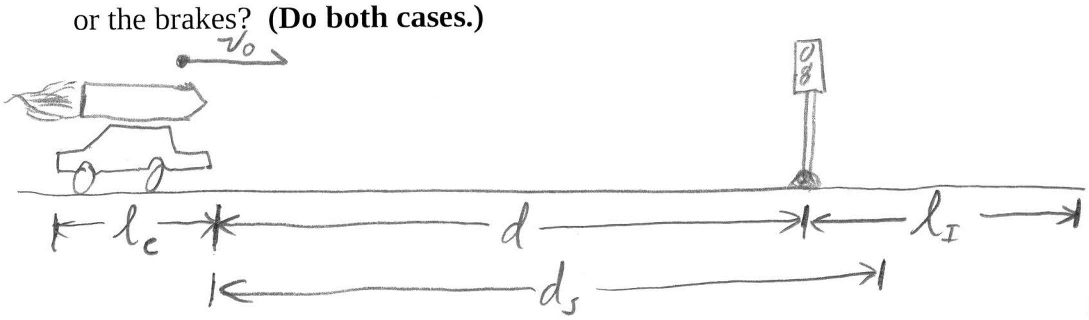
4) Wily coyote is driving his new *Acme* 0.50 m long rocket car at 20.0 m/s when he notices the light at the 20.0 m wide intersection 45.0 m ahead has just turned yellow. If he steps on the brakes his car will slow at a rate of -4.20 m/s^2. If he hits the gas the car will accelerate at a rate of 2.80 m/s^2.

a) The light will be yellow for 3.00 seconds and Wily's reaction time is 0.15 seconds. Wily needs to get home as quick as he can but he can't afford a ticket (he must clear the intersection) - should he hit the gas or the brakes? **(Do both cases.)**



$l_c = 0.5\,m$
$d = 45.0\,m$
$l_I = 20.0\,m$
$v_0 = 20\,m/s$
$a_1 = -4.2\,m/s^2$
$a_2 = 2.8\,m/s^2$
$t = 3.0\,s - 0.15\,s$
$t = 2.85\,s$

Brakes - Set $x=0$ at front of the car
where does wiley stop?

$x = x_0 + v_0 t + \frac{1}{2} a t^2$ $\qquad v = v_0 + at$

$d = 0 + v_0 t + \frac{1}{2} a_1 t^2$ $\qquad 0 = v_0 + a_1 t$

$\Rightarrow t = -\frac{v_0}{a_1}$

$d_s = 0 - v_0 \frac{v_0}{a_1} + \frac{1}{2} a_1 \frac{v_0^2}{a_1^2}$

$\boxed{d_s = -\frac{1}{2} \frac{v_0^2}{a_1}} \Rightarrow d_s = \frac{1}{2} \frac{(20.0)^2}{4.2} = \boxed{48\,m}$

He can't stop without entering the intersection!

Gas - set $x=0$ at the back of the car $\qquad \boxed{d + l_I = 65\,m}$
Where is wiley after 2.85s

$x = x_0 + v_0 t + \frac{1}{2} a t^2$

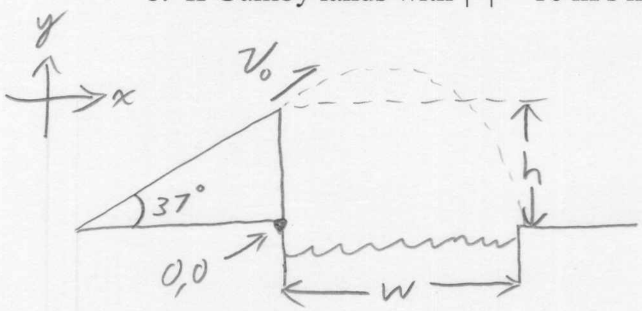
$d_g = 0 + v_0 t + \frac{1}{2} a t^2 \Rightarrow$ let $t = 2.85\,s$

$d_g = (20.0)(2.85) + \frac{1}{2}(2.8)(2.85)^2 = \boxed{68\,m}$ $\qquad$ $\boxed{\text{HIT THE GAS!}}$

4. Having recovered from an earlier crash, Gumby is ready to try new and more exciting stunts on his skateboard. After some prodding from the Blockheads, he decides to jump across a river. Gumby knows that the far bank is 3.0 m below the top of the ramp. The ramp is inclined at 37.0° above the x-axis, and he is moving at 15 m/s when he leaves it.

a. How wide of a river can Gumby jump if he puts the ramp on the edge of the riverbank?

b. If Gumby lands with $|\vec{v}| > 16$ m/s his leg will break. Does Gumby need crutches?



$h = 3.0\,m$

$V_0 = 15\,m/s$

$\theta = 37.0°$

Want: w, $|\vec{V}_f|$

x

$$x = x_0 + V_{0x}t + \cancel{\tfrac{1}{2}at^2}^{\,0}$$

$$w = V_0 \cos\theta \, t$$

y

$$y = y_0 + V_{0y}t + \tfrac{1}{2}at^2$$

$$0 = h + V_0 \sin\theta \, t - \tfrac{1}{2}gt^2$$

$$t = \frac{1}{g}\left[V_0 \sin\theta \oplus \left(V_0^2 \sin^2\theta + 2gh\right)^{1/2}\right]$$

$$* \boxed{w = \frac{V_0 \cos\theta}{g}\left[V_0 \sin\theta + \left(V_0^2 \sin^2\theta + 2gh\right)^{1/2}\right]} *$$

$$= \frac{15\cos(37)}{9.8}\left[15\sin 37 + \left(15^2 \sin^2 37 + (2)(9.8)(3)\right)^{1/2}\right]$$

$$= \underline{25\,m}$$

(b) Given: $W = 25m$

What is $|\vec{V}_F|$

x

$W = V_0 \cos\theta \, t \Rightarrow t = \dfrac{W}{V_0 \cos\theta}$

$$\boxed{V_{xF} = V_0 \cos\theta}$$

$V_{xF} = 15 \cos(37)$

$\underline{V_{xF} = 12.0 \text{ m/s}}$

y

$0 = h + V_0 \sin\theta \, t - \frac{1}{2} g t^2$

$V_{yF} = V_0 \sin\theta - gt$

$$\boxed{V_{yF} = V_0 \sin\theta - g \frac{W}{V_0 \cos\theta}}$$

$V_{yF} = 15 \cdot \sin 37 - \dfrac{(9.8)(25)}{15 \cos 37}$

$\underline{V_{yF} = -11.4 \text{ m/s}}$

$V_F = (12^2 + 11.4^2)^{1/2} = 16.5 \text{ m/s}$

Gumby needs cruthes