Sample Test 4
Phys 111 Spring 2010

Name: ____________________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

Show your work on all of the problems – your approach to the problem is as important as (if not MORE) important than) your final answer.

1) Derive the equations for the final velocities of particles undergoing an elastic collision in 1 dimension.

conserve momentum: $m_1 V_{1I} + m_2 V_{2I} = m_1 V_{1F} + m_2 V_{2F}$ ①

conserve energy: $\cancel{\tfrac{1}{2}} m_1 V_{1I}^2 + \cancel{\tfrac{1}{2}} m_2 V_{2I}^2 = \cancel{\tfrac{1}{2}} m_1 V_{1F}^2 + \cancel{\tfrac{1}{2}} m_2 V_{2F}^2$ ②

Get all m_1 on right and all m_2 on left

$m_1(V_{1I} - V_{1F}) = m_2(V_{2F} - V_{2I})$ ③

$m_1(V_{1I}^2 - V_{1F}^2) = m_2(V_{2F}^2 - V_{2I}^2)$ ④

Divide $\frac{④}{③}$: $\dfrac{\cancel{m_1}(V_{1I}^2 - V_{1F}^2)}{\cancel{m_1}(V_{1I} - V_{1F})} = \dfrac{\cancel{m_2}(V_{2F}^2 - V_{2I}^2)}{\cancel{m_2}(V_{2F} - V_{2I})}$

Remember: $(a^2 - b^2) = (a+b)(a-b)$: $\dfrac{\cancel{(V_{1I} - V_{1F})}(V_{1I} + V_{1F})}{\cancel{(V_{1I} - V_{1F})}} = \dfrac{\cancel{(V_{2F} - V_{2I})}(V_{2F} + V_{2I})}{\cancel{(V_{2F} - V_{2I})}}$

$$V_{1I} + V_{1F} = V_{2I} + V_{2F} \quad ⑤$$

Solve ⑤ for V_{2F}: $V_{2F} = V_{1I} + V_{1F} - V_{2I}$ ⑥

continued
↓

Sample Test 4, P1 continued

Put (6) → (1)

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 (v_{1I} + v_{1F} - v_{2I})$$

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 v_{1I} + m_2 v_{1F} - m_2 v_{2I}$$

$$v_{1F}(m_1 + m_2) = (m_2 - m_1) v_{1I} + 2 m_2 v_{2I}$$

$$\boxed{v_{1F} = \frac{m_2 - m_1}{m_1 + m_2} v_{1I} + \frac{2 m_2}{m_1 + m_2} v_{2I}}$$

And swap subscripts for v_{2F}

$$\boxed{v_{2F} = \frac{m_1 - m_2}{m_1 + m_2} v_{2I} + \frac{2 m_1}{m_1 + m_2} v_{1I}}$$

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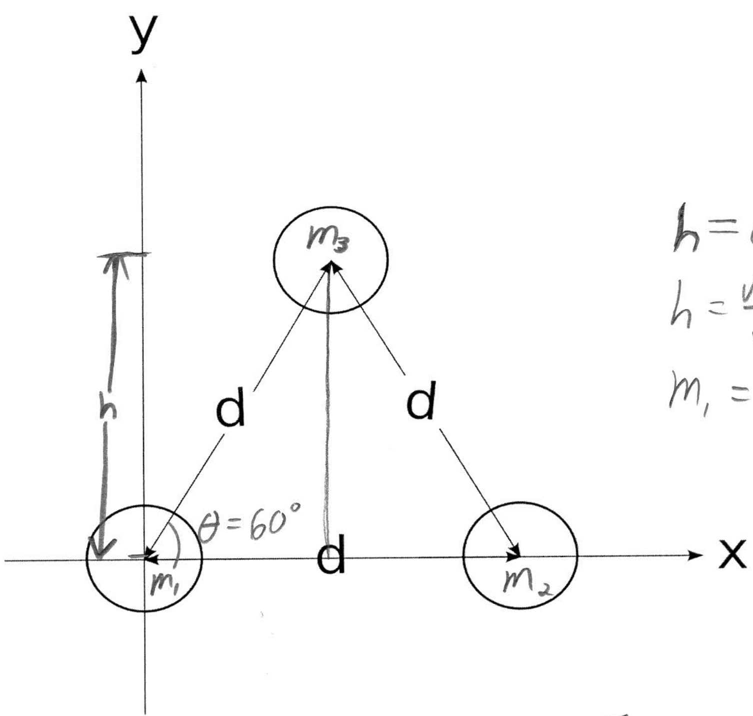
2. Three particles (point masses) are positioned at the vertices of an equilateral triangle. The two of the particles on the x-axis have the same mass. The y position of the center of mass is exactly halfway between the x-axis and the third particle. What is the mass of the third particle?



$h = d \sin\theta$

$h = \frac{\sqrt{3}}{2} d$

$m_1 = m_2$

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i}, \quad y_{cm} = \frac{h}{2} = \frac{\sqrt{3}}{4} d$$

$$\Rightarrow \frac{\sqrt{3}}{4} d = \frac{m_3 h}{m_1 + m_2 + m_3} \Rightarrow \frac{\cancel{\sqrt{3}}}{\cancel{4}_2} \cancel{d} = \frac{m_3 \frac{\cancel{\sqrt{3}}}{\cancel{2}} \cancel{d}}{2m_1 + m_3}$$

$$\Rightarrow \frac{1}{2} = \frac{m_3}{2m_1 + m_3} \Rightarrow m_1 + \tfrac{1}{2} m_3 = m_3$$

$$\Rightarrow \boxed{m_3 = 2m_1}$$

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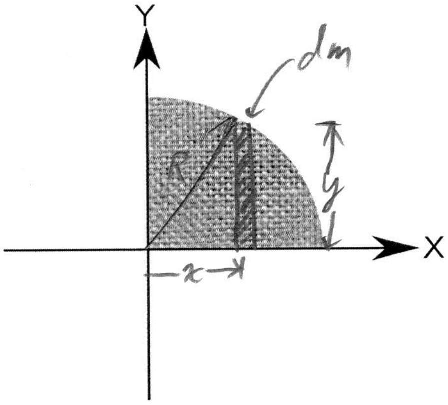
3. Find the location of the center of mass of a quarter of a circle of radius R.



$$\sigma = \frac{M}{A} = \frac{M}{\frac{1}{4}\pi R^2}$$

$$da = y\,dx$$

$$x_{cm} = \frac{1}{M}\int x\,dm, \quad dm = \sigma y\,dx = \frac{4M}{\pi R^2} y\,dx$$

By the pythagorean theorem: $R^2 = x^2 + y^2$

$$\Rightarrow y = \sqrt{R^2 - x^2}$$

$$\Rightarrow dm = \frac{4M}{\pi R^2}\sqrt{R^2 - x^2}\,dx$$

$$\Rightarrow x_{cm} = \frac{1}{\cancel{M}}\int_0^R x \frac{4\cancel{M}}{\pi R^2}\sqrt{R^2 - x^2}\,dx$$

$$\Rightarrow x_{cm} = \frac{4}{\pi R^2}\int_0^R x\sqrt{R^2 - x^2}\,dx$$

let $u = R^2 - x^2 \Rightarrow du = -2x\,dx$

when $x = R,\ u = 0$
$x = 0,\ u = R^2$

continued ↓

Sample test 4, P3 continued

$$\Rightarrow x_{cm} = \frac{2\cancel{\pi}}{\pi R^2}\int_{R^2}^{0} \cancel{x}\sqrt{u}\,\frac{1}{-2\cancel{x}}\,du$$

minus sign flips limits

$$\Rightarrow x_{cm} = \frac{2}{\pi R^2}\int_{0}^{R^2} u^{1/2}\,du$$

$$\Rightarrow x_{cm} = \frac{2}{\pi R^2}\left(\frac{2}{3}u^{3/2}\right)\Bigg|_{0}^{R^2} = \frac{4}{3}\frac{1}{\pi \cancel{R^2}}\left(R^{\cancel{2}}\right)^{\cancel{3}/\cancel{2}}$$

$$\boxed{x_{cm} = \frac{4}{3\pi}R}$$

And, by symmetry, $\boxed{y_{cm} = \frac{4}{3\pi}R}$

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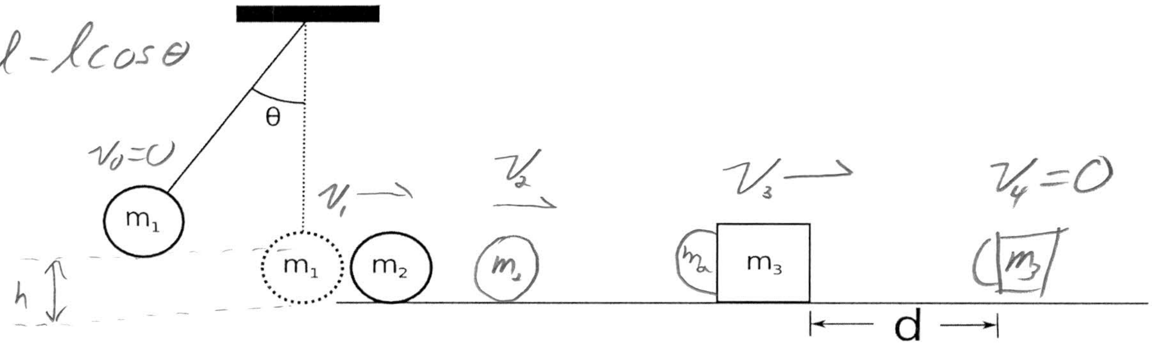
4. A mass $m_1 = 3$ kg is attached to a string of length $l = 4.0$ m to create a pendulum. The pendulum, initially making an angle θ with the vertical, is released from rest. At the bottom of its swing, it collides elastically with mass $m_2 = 5$ kg. Mass 2 rolls (no friction) and sticks to $m_3 = 5$ kg. The m_2, m_3 combination slides with $\mu_k = 0.3$ a distance $d = 0.2$ m before coming to rest.

What was the original value of θ?

$h = l - l\cos\theta$



Step ①: W/E for pendulum

$U_I = mg(l - l\cos\theta)$ $\qquad$ $U_F = 0$

$K_I = 0$ $\qquad$ $K_F = \frac{1}{2} m_1 v_1^2$

$$\Rightarrow \cancel{m_1} g l (1 - \cos\theta) = \frac{1}{2} \cancel{m_1} v_1^2$$

$$\boxed{v_1 = \sqrt{2 g l (1 - \cos\theta)}} \quad ①$$

Step ②: Collide to get v_2

$$\boxed{v_2 = \frac{2 m_1}{m_1 + m_2} v_1} \quad ②$$

Taken directly from our equation for elastic collisions.

Step ③: Collide to get v_3

$$m_2 v_2 = (m_2 + m_3) v_3$$ conserve momentum.

$$\boxed{v_3 = \frac{m_2}{m_2 + m_3} v_2} \quad ③$$

continued ↓

Sample test 4, P4 continued

Step 4: W/E to slide to a stop

$$U_I = U_F = 0$$

$$K_I = \tfrac{1}{2}(m_2 + m_3) v_3^2 \qquad K_F = 0$$

$$W_{NCF} = -\mu_k (m_2 + m_3) g d$$

$$\tfrac{1}{2} \cancel{(m_2 + m_3)} v_3^2 - \mu_k \cancel{(m_2 + m_3)} g d = 0$$

$$\boxed{v_3^2 = 2\mu_k g d} \quad 4$$

Put everything together:

3 → 4 $$\frac{m_2^2}{(m_2+m_3)^2} v_2^2 = 2\mu_k g d$$

2 → $$\frac{m_2^2}{(m_2+m_3)^2} \cdot \frac{4m_1^2}{(m_1+m_2)^2} v_1^2 = 2\mu_k g d$$

1 → $$\left(\frac{2m_1 m_2}{(m_2+m_3)(m_1+m_2)}\right)^2 \cancel{2}\cancel{g} l (1 - \cos\theta) = \cancel{2}\mu_k \cancel{g} d$$

Let's let: $R = \dfrac{2m_1 m_2}{(m_2+m_3)(m_1+m_2)}$

then: $R^2 l (1 - \cos\theta) = \mu_k d$

$$\Rightarrow R^2 l - R^2 l \cos\theta = \mu_k d$$

$$\Rightarrow R^2 l \cos\theta = R^2 l - \mu_k d \Rightarrow \boxed{\cos\theta = 1 - \frac{\mu_k d}{R^2 l}}$$

continued ↓

Sample test 4, p 4 continued

$$R = \frac{2 m_1 m_2}{(m_1 + m_2)(m_2 + m_3)} = \frac{(2)(3)(5)}{(8)(10)} = \boxed{0.375}$$

and: $$\theta = \cos^{-1}\left[1 - \frac{\mu_k d}{R^2 \ell}\right]$$

$$\theta = \cos^{-1}\left[1 - \frac{(0.3)(0.2)}{(0.375)^2(4)}\right] = \cos^{-1}(0.89)$$

$$\boxed{\theta = 26.7°}$$

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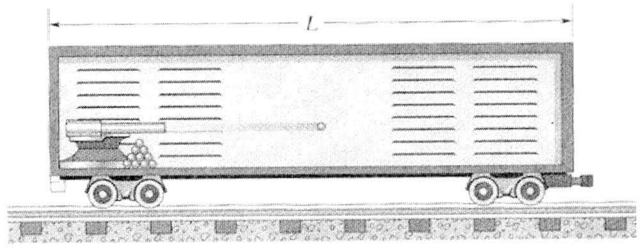
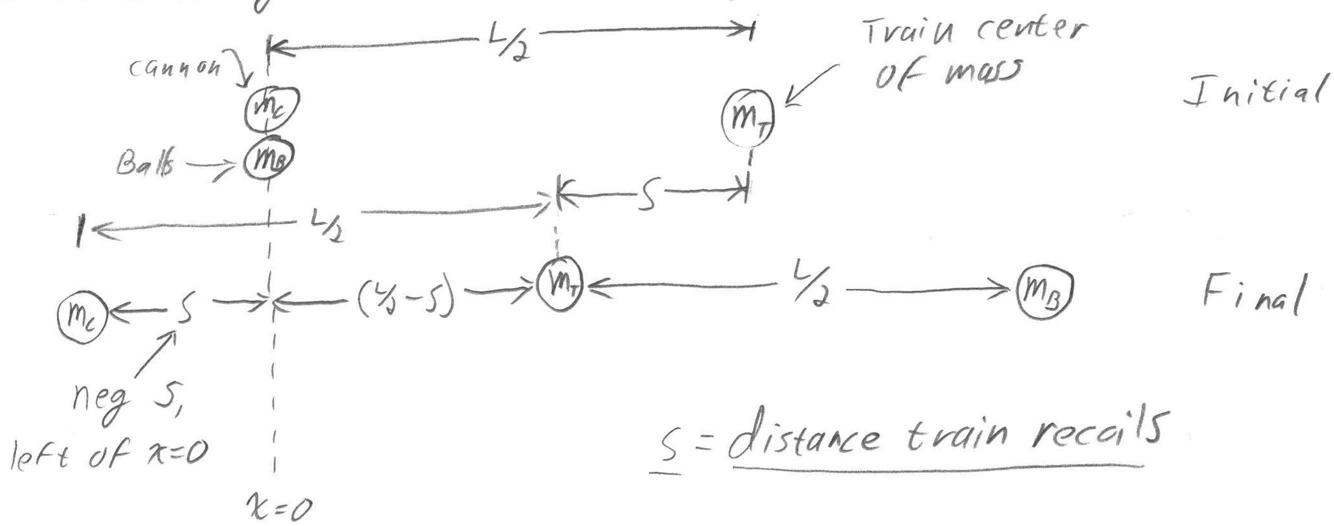
5. A cannon and 20 cannon balls are inside of a sealed railroad car of length L. The cannon fires balls to the right where they collide inelastically with the far wall and stop. The train car weighs 6000 kg, the cannon weighs 500 kg and each cannon ball weighs 20 kg. Assume that the track is frictionless, cannon is small compared to the car, ~~and that the cannon balls travel the entire distance L.~~



a) How far has the train car moved after ALL of the cannon balls have been fired?
b) What is the final velocity of the car after all of the cannonballs have hit the opposite wall?

Draw a diagram with relative positions first.



a) There are no external forces and $V_{cm} = 0$ so $x_{cm} = \text{Const.}$

After the cannonballs fire, in the positive x-direction, the train will recoil a distance $-s$, but x_{cm} will remain in the same spot.

So: $x_{cmI} = x_{cmF}$

continued ↓

Sample test 4, P5 continued

$$\frac{\sum m_i x_{iI}}{\cancel{\sum m_i}} = \frac{\sum m_i x_{iF}}{\cancel{\sum m_i}}$$

s is neg here...

$$\cancel{m_B \overset{0}{0}} + \cancel{m_C \overset{0}{0}} + m_T \cdot L/2 = -m_C s + (L/2 - s) m_T + (L - s) m_B$$

$$\Rightarrow m_T \cdot \frac{L}{2} = -m_C s + m_T \frac{L}{2} - m_T s + m_B L - m_B s$$

$$\Rightarrow L(\cancel{\tfrac{1}{2} m_T} - \cancel{\tfrac{1}{2} m_T} - m_B) = s(-m_C - m_T - m_B)$$

$$s = \frac{-m_B}{-m_C - m_T - m_B} L \Rightarrow \boxed{s = \frac{m_B}{m_C + m_T + m_B} L}$$

$$s = \frac{500}{6000 + 400 + 500} = \boxed{\frac{5}{69} L}$$

b) The system must conserve momentum because $\sum F_{ext} = 0$ and $P_I = 0$.

When the cannon fires, an impulse is delivered to the train and the ball.

At the opposite wall, an opposite (but equal) impulse is delivered to train and ball and they both stop.