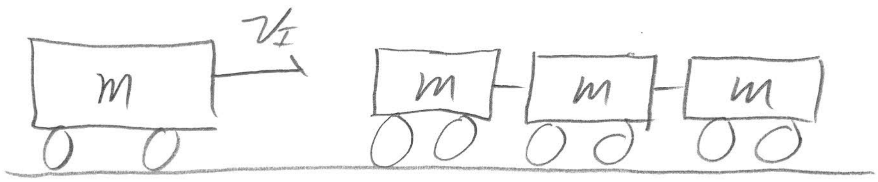
A 4000 kg railroad car inelastically collides with three other 4000 kg cars sitting at rest on a rough track. The four cars travel together down the rough track for 1.5 m before they stop. Assuming $\mu_k = 0.10$, what is the velocity of the first car at impact?



Two parts: Collide and Slide to a stop:

Collide

$$P_I = P_F$$

$$\cancel{m} V_I = 4\cancel{m} V_F$$

$$\boxed{V_F = \frac{1}{4} V_I} \quad (1)$$

Slide

$$U_I = 0 \qquad U_F = 0$$

$$K_I = \frac{1}{2} 4m V_F^2 \qquad K_F = 0$$

$$W_F = -\mu_k 4mgd$$

$$\frac{1}{2} \cancel{4m} V_F^2 = \mu_k \cancel{4m} gd$$

$$\boxed{V_F = (2\mu_k gd)^{1/2}} \quad (2)$$

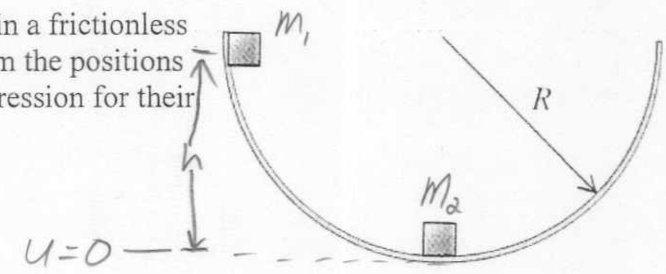
Put (1) and (2) together:

$$(2\mu_k gd)^{1/2} = \frac{1}{4} V_I \Rightarrow \boxed{V_I = (32 \mu_k gd)^{1/2}}$$

$$V_I = \left((32)(0.1)(9.8)(1.5)\right)^{1/2} = 6.8 \text{ m/s}$$

Momentum, Impulse, and Collisions

Two masses are released from rest in a frictionless hemispherical bowl or radius R from the positions shown in the figure. Derive an expression for their final height in the case of :



$h=R$

a) An elastic collision

b) An inelastic collision

c) How much bigger than the second mass does the first mass have to be so that the second mass gets out of the bowl.

a) Find Velocity of m_1 before collision

$U_i = mgh \quad K_i = 0$

$U_f = 0 \quad K_f = \frac{1}{2} m_1 V_{1i}^2 \Rightarrow V_{1i} = \sqrt{2gh}$

Find both velocities post collision

① $\cancel{\tfrac{1}{2}} m_1 V_{1i}^2 = \cancel{\tfrac{1}{2}} m_1 V_{1f}^2 + \cancel{\tfrac{1}{2}} m_2 V_{2f}^2$: conserve energy

② $m_1 V_{1i} = m_1 V_{1f} + m_2 V_{2f}$: conserve momentum

$$\frac{\cancel{m_1}(V_{1i}^2 - V_{1f}^2)}{\cancel{m_1}(V_{1i} - V_{1f})} = \frac{\cancel{m_2} V_{2f}^{\cancel{2}}}{\cancel{m_2} \cancel{V_{2f}}}$$: Divide ① by ②

$$\Rightarrow \frac{\cancel{(V_{1i} - V_{1f})}(V_{1i} + V_{1f})}{\cancel{(V_{1i} - V_{1f})}} = V_{2f} \Rightarrow V_{2f} = V_{1i} + V_{1f}$$

$$V_{1f} = V_{2f} - V_{1i}$$

Plug back into momentum equation

$m_1 V_{1i} = m_1 V_{1f} + m_2 V_{1i} + m_2 V_{1f}$

$$\boxed{V_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1i}}$$

$m_1 V_{1i} = m_1 V_{2f} - m_1 V_{1i} + m_2 V_{2f}$

$$\boxed{V_{2f} = \frac{2 m_1}{(m_1 + m_2)} V_{1i}}$$

Find h_1 and h_2

$U_i = 0 \quad K_i = \frac{1}{2} m v^2$
$U_f = mgh \quad K_f = 0$ $\Rightarrow \boxed{h = \frac{v^2}{2g}}$

$$h_1 = \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2} \frac{v_{1i}^2}{2g} \qquad h_2 = \frac{4m_1^2}{(m_1+m_2)^2} \cdot \frac{v_{1i}^2}{2g}$$

$$= \frac{(m_1 - m_2)^2}{(m_1+m_2)^2} \cdot \frac{\cancel{2g}h}{\cancel{2g}} \qquad = \frac{4m_1^2}{(m_1+m_2)^2} \frac{\cancel{2g}h}{\cancel{2g}}$$

$$\boxed{h_1 = \frac{(m_1 - m_2)^2}{(m_1+m_2)^2} R} \qquad \boxed{h_2 = \frac{4m_1^2}{(m_1+m_2)^2} R}$$

b) Use $v_{1i} = \sqrt{2gh}$ from part a

Find velocity of combined mass post collision.

$m_1 v_{1i} = (m_1 + m_2) v_f$: conserve momentum

$$v_f = \frac{m_1}{(m_1 + m_2)} v_{1i}$$

Find final height

$U_i = 0 \quad K_i = \frac{1}{2} m v_f^2$
$U_f = mgh_f \quad K_i = 0$ $\Rightarrow h_f = \frac{v_f^2}{2g} = \frac{m_1^2}{(m_1+m_2)^2} \frac{v_{1i}^2}{2g}$

$$= \frac{m_1^2}{(m_1+m_2)^2} \frac{\cancel{2g}h}{\cancel{2g}}$$

$$\boxed{h_f = \frac{m_1^2}{(m_1+m_2)^2} R}$$

c) If $h_2 \geq R$, the second mass will escape.

set $h_2 = R$

$$\cancel{R} = \frac{4m_1^2}{(m_1+m_2^2)}\cancel{R} \Rightarrow m_1 + m_2 = 2m_1$$

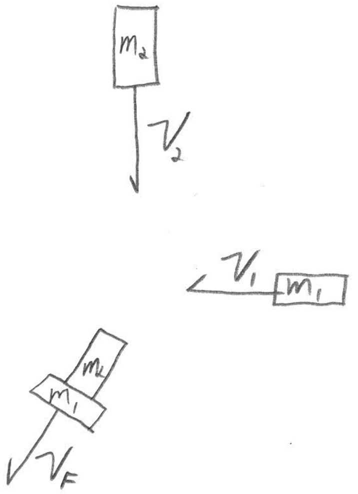
$$\Rightarrow \underline{m_2 = m_1}$$

So if m_1 is greater than m_2, m_2 will escape.

You are driving West along Summit Ave, lawfully doing the speed limit (50 km/hr) in your new car which (as you've read in the owners manual) has a mass of 1500 kg. Sleepy McSnoozer is driving South along Cleveland in his 1965 Ford pickup truck loaded with bags of cement. His truck (plus cement) weighs 2300 kg. Sleepy runs the red light and smashes into your car. The cars fuse together and skid to a stop.

Certain that Sleepy was speeding, you measure the skid mark and find that the length of the skid is L = 18 m. You look up the rubber/asphalt coefficient of friction and find that it is $\mu_k = 0.6$.

What was Sleepy's velocity? Was he speeding? The speed limit is 50 km/hr.



$\mu_K = 0.6$

$L = 18$

$m_1 = 1500\,kg$

$m_2 = 2300\,kg$

$$v_1 = 50\,\frac{km}{hr}\cdot 1\times10^3\,\frac{m}{km}\cdot\frac{1}{3600}\cdot\frac{hr}{s} = 13.9\,m/s$$

Two parts, collision and skid. Conserve momentum for collision, Conserve energy to do skid.

Collision

$P_I = P_F$

$x:\; m_1 v_1 = (m_1 + m_2)\, v_{Fx}$

$y:\; m_2 v_2 = (m_1 + m_2)\, v_{Fy}$

$$\boxed{① \; v_{Fx} = \frac{m_1}{(m_1+m_2)} v_1, \quad ② \; v_{Fy} = \frac{m_2}{(m_1+m_2)} v_2}$$

Skid

$U_I = 0 \qquad U_F = 0$

$K_I = \frac{1}{2}(m_1+m_2)|\vec{v}_F|^2 \qquad K_F = 0$

$W_f = -\mu_K (m_1+m_2) g L$

$\frac{1}{2}\cancel{(m_1+m_2)}|\vec{v}_F|^2 = 2\mu_K \cancel{(m_1+m_2)} g L$

$$\boxed{|\vec{v}_F|^2 = 2\mu_K g L \;③}$$

continued ↓

Systems of particles Set 3, P3 continued

Now: $|\vec{V}_F|^2$ is related to V_{Fx} and V_{Fy} by pythagoras.

$$④ \quad |\vec{V}_F|^2 = V_{Fx}^2 + V_{Fy}^2$$

Plugging ④ → ③ :

$$⑤ \quad V_{Fx}^2 + V_{Fy}^2 = 2 \mu_k g L$$

and plugging ① and ② → ⑤

$$\frac{m_1^2}{(m_1+m_2)^2} V_1^2 + \frac{m_2^2}{(m_1+m_2)^2} V_2^2 = 2 \mu_k g L$$

and solve for V_1:

$$\frac{m_1^2 V_1^2 + m_2^2 V_2^2}{(m_1+m_2)^2} = 2 \mu_k g L$$

$$m_1^2 V_1^2 + m_2 V_2^2 = 2 \mu_k g L (m_1 + m_2)^2$$

$$\Rightarrow \boxed{V_2 = \left[2 \mu_k g L (m_1 + m_2)^2 - m_1^2 V_1^2\right]^{\frac{1}{2}} \frac{1}{m_2}}$$

$$V_2 = \left[(2)(0.6)(9.8)(18)(1500 + 2300)^2 - (1500 \cdot 13.9)^2\right]^{\frac{1}{2}} \frac{1}{2300}$$

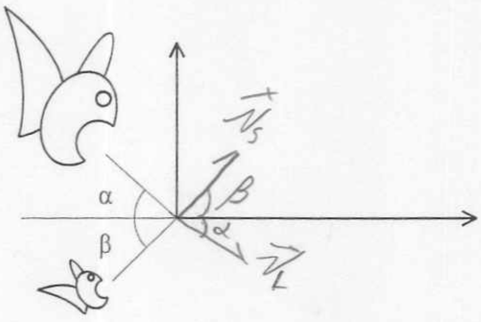
$$V_2 = 22.3 \,\tfrac{m}{s} \cdot 1 \times 10^{-3} \,\frac{km}{m} \cdot 3600 \,\tfrac{s}{hr} = 80 \,\tfrac{km}{hr}$$

Speeder!

1. A large fish will soon make a dish of a smaller fish. What is the velocity of the large fish and his dinner immediately after he eats? Give both the magnitude and direction of the final velocity with respect to the x-axis.

$m_{\text{large fish}} = 4.0$ kg
$v_{o\,\text{large fish}} = 1.0$ m/s
$\alpha_{\text{large fish}} = 25.0°$

$m_{\text{small fish}} = 0.20$ kg
$v_{o\,\text{small fish}} = 5.0$ m/s
$\beta_{\text{small fish}} = 50.0°$



Conserve momentum in both axis

$$x: m_L V_L \cos\alpha + m_s V_s \cos\beta = (m_L + m_s) V_F \cos\theta$$

$$y: -m_L V_L \sin\alpha + m_s V_s \sin\beta = (m_L + m_s) V_F \sin\theta$$

Divide y by x to eliminate V_F

$$\frac{-m_L V_L \sin\alpha + m_s V_s \sin\beta}{m_L V_L \cos\alpha + m_s V_s \cos\beta} = \frac{\cancel{(m_L+m_s)}\cancel{V_F}\sin\theta}{\cancel{(m_L+m_s)}\cancel{V_F}\cos\theta}$$

$$\tan\theta = \frac{-m_L V_L \sin\alpha + m_s V_s \sin\beta}{m_L V_L \cos\alpha + m_s V_s \cos\beta} \Rightarrow$$

$$\theta = \tan^{-1}\left[\frac{-(4.0)(1.0)\sin(25) + (0.2)(5)\sin(50)}{(4.0)(1.0)\cos(25) + (0.2)(5)\cos(50)}\right] = \boxed{-12°}$$

Plug back into x (or y) to get V_F

$$V_F = \frac{m_L V_L \cos\alpha + m_s V_s \cos\beta}{(m_L + m_s)\cos\theta} = \frac{(4)(1)\cos 25 + (0.2)(5)\cos(50)}{(4 + 0.2)\cos(-12)} = \boxed{1.0\ \text{m/s}}$$

Momentum, Impulse, and Collisions

Gayle runs at a speed of 4.0 m/s and dives onto a sled that is initially at rest on top of a frictionless snow covered hill sloping down at 30°. After she has traveled 10 m down the slope, her brother Billy hops on the sled; and together they travel on down the slope another 20 m. (Billy was at rest initially) What is their speed at the bottom of the hill?

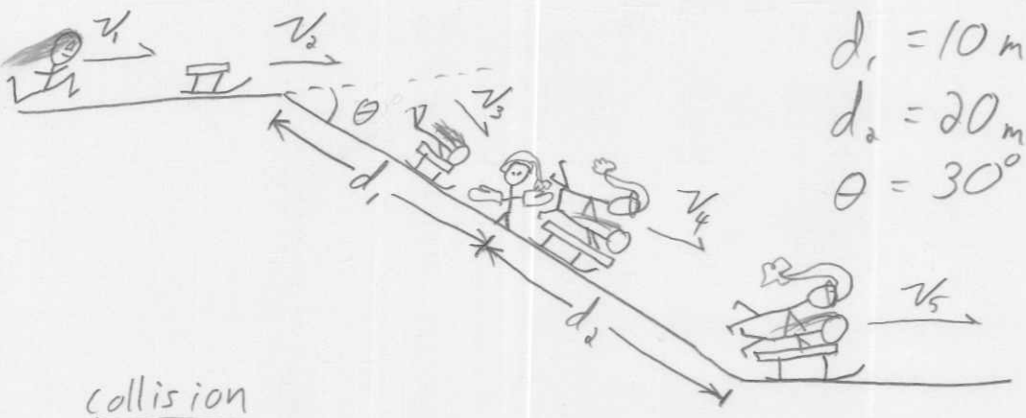
$m_{Gayle} = 50.0$ kg, $m_{Billy} = 30.0$ kg, $m_{sled} = 5.0$ kg

$v_1 = 4.0$ m/s

$d_1 = 10$ m

$d_2 = 20$ m

$\theta = 30°$



collision

1 $m_g v_1 = (m_g + m_s) v_2$: conserve momentum

$$v_2 = \frac{m_g}{(m_g + m_s)} v_1 = \frac{50}{50+5}(4) = \boxed{3.63 \text{ m/s}}$$

2 work-energy

$K_i = \frac{1}{2}(m_g + m_s) v_2^2$ $\quad$ $K_f = \frac{1}{2}(m_g + m_s) v_3^2$

$U_i = (m_g + m_s) g h = (m_g + m_s) g d_1 \sin\theta$ $\quad$ $U_f = 0$

$$\frac{1}{2}\cancel{(m_g + m_s)} v_2^2 + \cancel{(m_g + m_s)} g d_1 \sin\theta = \frac{1}{2}\cancel{(m_g + m_s)} v_3^2$$

$$v_3 = \left(v_2^2 + 2 g d_1 \sin\theta\right)^{1/2} = \left(3.63^2 + 2(9.8)(10)\sin(30)\right)^{1/2}$$

$$\boxed{= 10.5 \text{ m/s}}$$

3 collision #2

$(m_s+m_g)v_3 = (m_s+m_g+m_B)v_4$: conserve momentum

$$v_4 = \frac{m_s+m_g}{m_s+m_g+m_B}v_3 = \frac{50+5}{50+5+30}\cdot 10.5$$

$$= \underline{6.79\ m/s}$$

4 Work - Energy

$K_i = \frac{1}{2}(m_s+m_g+m_B)v_4^2$ $\qquad K_f = \frac{1}{2}(m_s+m_g+m_B)v_5^2$

$U_i = (m_s+m_g+m_B)g d_2 \sin\theta$ $\qquad U_f = 0$

$$\frac{1}{2}\cancel{(m_s+m_g+m_B)}v_4^2 + \cancel{(m_s+m_g+m_B)}g d_2 \sin\theta = \frac{1}{2}\cancel{(m_s+m_g+m_B)}v_5^2$$

$$v_5 = \left[v_4^2 + 2 g d_2 \sin\theta\right]^{1/2}$$

$$= \left[(6.79)^2 + (2)(9.8)(20)\sin(30)\right]^{1/2}$$

$$\boxed{= 15.6\ m/s}$$

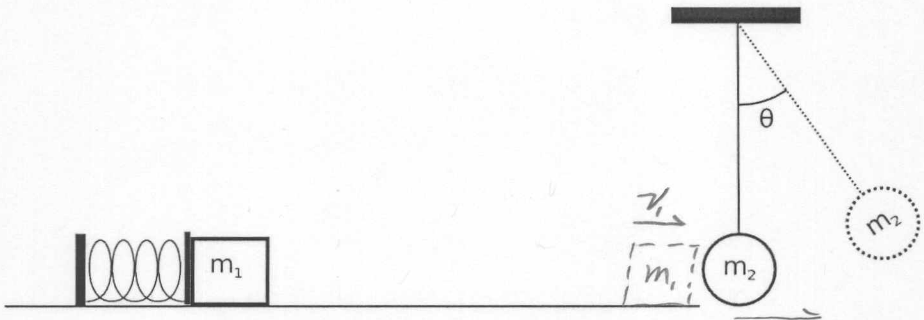
Test 3

Phys 111, Fall 2008, Section 1

3 20pts)

A block with a mass of $m_1 = 3.5$ kg is placed in front of a spring that has been compressed $d = 0.050$ m on a rough surface with $\mu_k = 0.30$. After the spring is released, the block travels $s = 0.75$ m to a hanging pendulum. The block collides elastically with a pendulum that has mass $m_2 = 10$ kg. It is hanging from a string with length $l = 1.4$m. It takes 30 J of work to compress the spring.

What is the maximum angle θ that the pendulum string will make with the vertical?



$\vec{v}_2$ - m_2 Vel. post collision

① Block leaves spring and travels to the pendulum:

$U_I = W_s \qquad U_F = 0 \qquad W_F = -\mu_k m_1 g(d+s)$

$K_I = 0 \qquad K_F = \frac{1}{2} m_1 v_1^2$

$$\boxed{W_s - \mu_k m_1 g(d+s) = \frac{1}{2} m v_1^2} \qquad v_1 = \left[\frac{2(30-(0.3)(3.5)(9.8)(0.755))}{3.5}\right]^{1/2}$$

$$\boxed{v_1 = 3.6 \text{ m/s}}$$

② m_1 and m_2 collide elastically

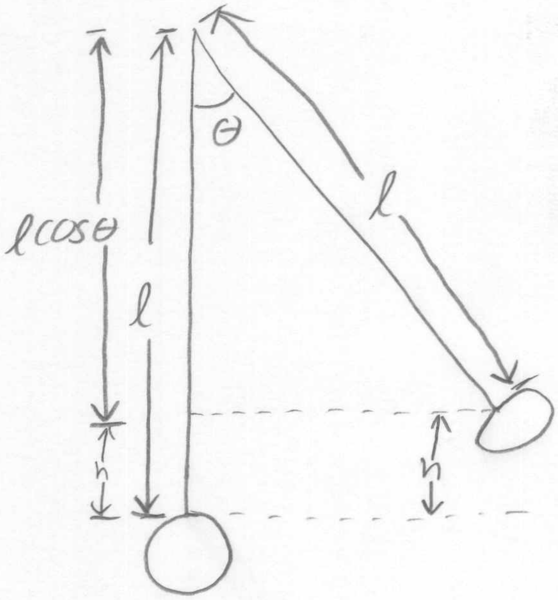
$$v_2 = \frac{m_2 - m_1}{m_2 + m_1}\cancelto{0}{v_{2I}} + \frac{2m_1}{m_2 + m_1} v_1$$

$$\boxed{v_2 = \frac{2m_1}{m_2 + m_1} v_1} \qquad \boxed{v_2 = \frac{2(3.5)}{(10+3.5)} 3.6 = 1.9 \text{ m/s}}$$

continued

↓

③ Pendulum swings up



$h = l - l\cos\theta$

$U_I = 0 \qquad U_F = mg(l - l\cos\theta)$

$K_I = \frac{1}{2} m v_2^2 \qquad K_F = 0$

$$\frac{1}{2}\cancel{m} v_2^2 = \cancel{m}g(l - l\cos\theta)$$

$$\frac{v_2^2}{2gl} = 1 - \cos\theta \Rightarrow \cos\theta = 1 - \frac{v_2^2}{2gl}$$

$$\boxed{\theta = \cos^{-1}\left(1 - \frac{v_2^2}{2gl}\right)}$$

$$\theta = \cos^{-1}\left(1 - \frac{1.9^2}{2(9.8)(1.4)}\right)$$

$$\boxed{\theta = 29^\circ}$$