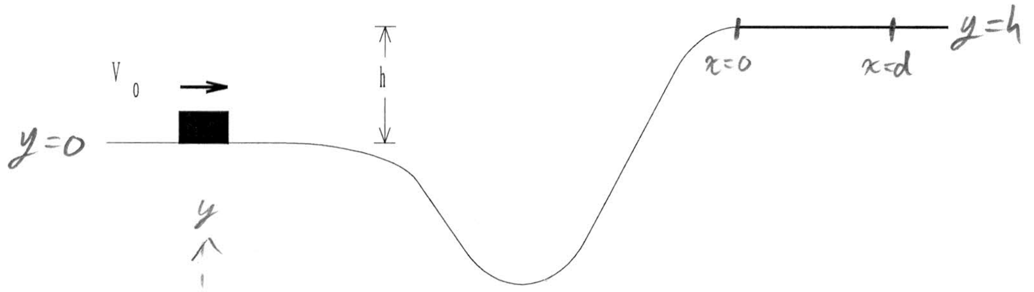
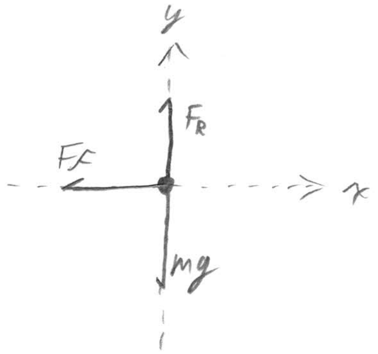
Energy Problems – Set 3

A block slides along the track shown below. The track is frictionless until the block reaches the level portion at the top of the hill, where the coefficient of friction is μ_k.



$U_g = mgy$



$$\vec{F}_f = -\mu_k F_R \hat{x} = -\mu_k mg \hat{x}$$

$U_I = 0$ $\qquad$ $U_F = mgh$

$K_I = \frac{1}{2} m v_0^2$ $\qquad$ $K_F = 0$

$$W_F = \int_0^d \vec{F}_f \cdot d\vec{s} = \int_0^d (-\mu_k mg \hat{x}) \cdot (dx \hat{x}) = -\mu_k mgd$$

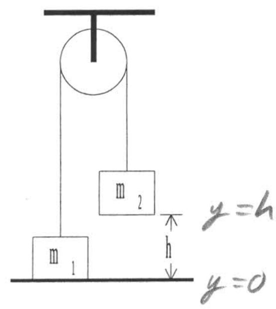
$$U_I + K_I + W_F = U_F + K_F$$

$$0 + \frac{1}{2} m v_0^2 - \mu_k mgd = mgh$$

$$\boxed{v_0 = \left[2g(\mu_k d + h)\right]^{1/2}}$$

Energy Problems – Set 3

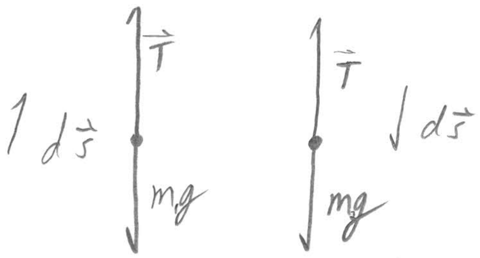
Use work-energy techniques to solve the following problem.



Two masses are connected by a light string passing over a light frictionless pulley. The mass m_2 is released from rest at a height of 4.0 m above the ground.

Determine the speed of m_1 just as m_2 hits the ground and the maximum height m_1 rises above the ground.

$m_1 = 3.0$ kg
$m_2 = 5.0$ kg



T and T do work
No pot. Func.

mg does work, conservative

consider Work done on the s[illegible]em by tension.

on mass 1 : $W_T = \int_0^h (\vec{T}\cdot d\vec{s}) = \int_0^h (T\hat{j})\cdot(dy\,\hat{j}) = \underline{Th}$

on mass 2 : $W_T = \int_0^h (\vec{T}\cdot d\vec{s}) = \int_0^h (T\hat{j})\cdot(-dy\,\hat{j}) = -\underline{Th}$

Net work on the <u>system</u> by T is <u>zero</u>

Now consider the <u>entire</u> system and conserve energy

a) $U_I = m_2 g h$ $\qquad U_F = m_1 g h$

$K_I = 0$ $\qquad K_F = \frac{1}{2} m_1 V^2 + \frac{1}{2} m_2 V^2$

Same velocity since they are linked by the rope.

continued ↓

EP3,6 - continued

$$m_2gh = m_1gh + \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2$$

$$\tfrac{1}{2}(m_1+m_2)v^2 = (m_2-m_1)gh$$

$$\boxed{v = \left[\frac{(m_2-m_1)}{m_1+m_2}2gh\right]^{1/2}} = \boxed{4.4\ m/s}$$

b) Find max height of m_1. The floor has interfered. Some energy ($\tfrac{1}{2}m_2v^2$) is lost.
consider just m_1

$U_I = m_1gh$ $\qquad$ $U_F = m_1gh_{max}$

$K_I = \tfrac{1}{2}m_1v^2$ $\qquad$ $K_F = 0$

$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$m_1gh + \tfrac{1}{2}m_1v^2 = m_1gh_{max} + 0$$

$$\Rightarrow h_{max} = h + \frac{v^2}{2g} = h + \frac{1}{\cancel{2g}}\frac{m_2-m_1}{m_1+m_2}\cancel{2g}h = h\left[1+\frac{m_2-m_1}{m_1+m_2}\right]$$

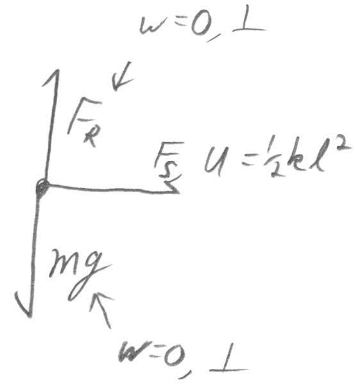
$$= h\left[\frac{\cancel{m_1}+m_2+m_2-\cancel{m_1}}{m_1+m_2}\right] = \boxed{\frac{2m_2}{m_1+m_2}h} = 5$$

yay!

A block of mass m is pushed against a spring of spring constant k and the spring is compressed a distance l. The block is released and slides across a frictionless surface for a short distance before encountering a surface with a coefficient of friction μ_k.

a. Use conservation of energy to find an expression for the velocity of the block after it leaves the spring.

b. Use conservation of energy to find an expression for how far it slides on the surface with friction before coming to a stop.



a) $U_I = \frac{1}{2}kl^2$ $U_F = 0$

$K_I = 0$ $K_F = \frac{1}{2}mv^2$

$\cancel{\frac{1}{2}}kl^2 = \cancel{\frac{1}{2}}mv^2 \Rightarrow \boxed{v = \sqrt{\frac{k}{m}}\,l}$

b) $U_I = 0$ $U_F = 0$

$K_I = \frac{1}{2}mv^2$ $K_F = 0$

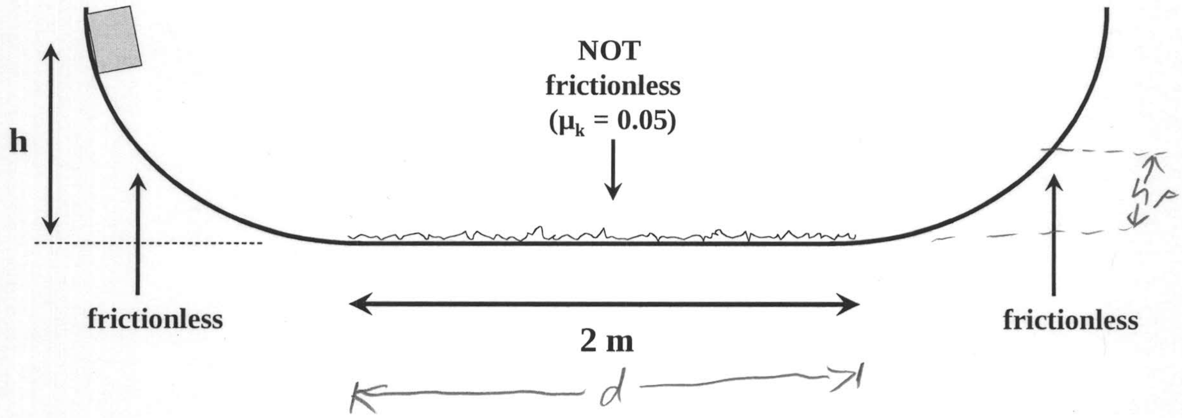
$W_F = \int_0^d \vec{F}_F \cdot d\vec{r} = \int_d^d (-\mu_k mg\hat{x})\cdot(dx\hat{x}) = -\mu_k mgd$

$\frac{1}{2}mv^2 - \mu_k mgd = 0 \Rightarrow \frac{1}{2}\frac{k}{m}l^2 = \mu_k gd$

$\boxed{d = \frac{kl^2}{2\mu_k mgd}}$

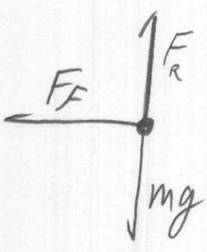
Energy Problems – Set 4

An 8.75-kg block starts at rest, at height h = 1.0 m, and slides down a frictionless ramp onto a horizontal plane where $\mu_k = 0.05$. If the block has enough energy after passing the plane, it will rise onto another frictionless ramp, and so forth.



(a) The block is released, makes its first trip to the right hand side, returns to the left hand side, and then returns once more to the right. On this second excursion to the right side, how high up the ramp does the block go?

This is a conservation of Energy problem with friction.
The simplest solution is to consider the two end points.
It starts a height h on the right and ends
a height h_F on the right after crossing the
friction 3 times.



F_R does no work, $\vec{F}_R \perp d\vec{s}$

F_F does work on the flat bottom, non-conservative

mg does work, conservative.

* $\underline{U_I = mgh}$ $\quad$ * $\underline{U_F = mgh_F}$ solve for h_F

* $\underline{K_I = K_F = 0}$

$\underline{W_F} = \int_0^d \vec{F}_F \cdot d\vec{s} = -\mu_k mgd$ $\underline{\text{for one trip}}$

continued ↓

EP4, 6 - Continued

Conserve energy accounting for 3 trips across the friction patch.

$$U_I + K_I + W_F = U_F + K_F$$

$$\cancel{m}\cancel{g}h + 0 + -3\mu_k \cancel{m}\cancel{g} d = \cancel{m}\cancel{g} h_F + 0$$

$$\Rightarrow \boxed{h_F = h - 3\mu_k d}$$

$$h_F = 1.0 - (3)(0.05)(2)$$

$$\boxed{h_F = 0.7\,m}$$

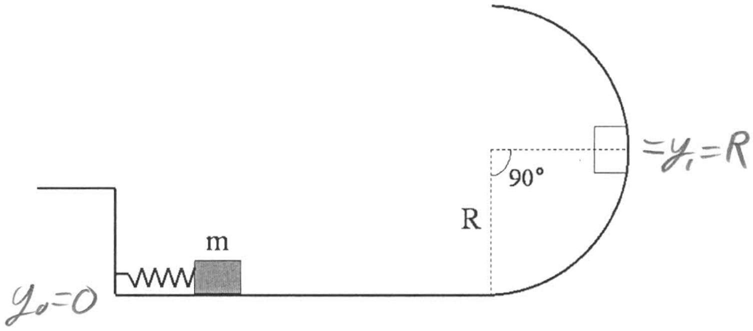
①

Energy Problems – Set 3

#5

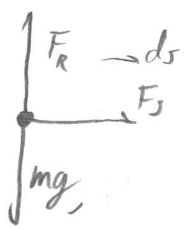
A mass *m* rests on a frictionless horizontal track while compressing a horizontal spring of spring constant *k*. The mass is released and it along a frictionless horizontal track before sliding up a frictionless circular surface of radius *R*.

a. Find an expression for the compression *d* such that the mass just comes to rest at a radius position of θ=90° as shown in the picture?



b. Now include friction in the problem. If the block stops at a radius position of θ=35°, how much work was done by the frictional force acting on the block?

a)



$W_{F_R}=0,\ \vec{F}_R \perp d\vec{s}$

$U_g = mgy$

$U_s = \frac{1}{2}kd^2$

$U_I = \frac{1}{2}kd_0^2 + \cancel{mgy_0}^{\,0}$ $U_F = \cancel{\frac{1}{2}kd_1^2}^{\,0} + mgy_1$

$K_I = 0$ $K_F = 0$

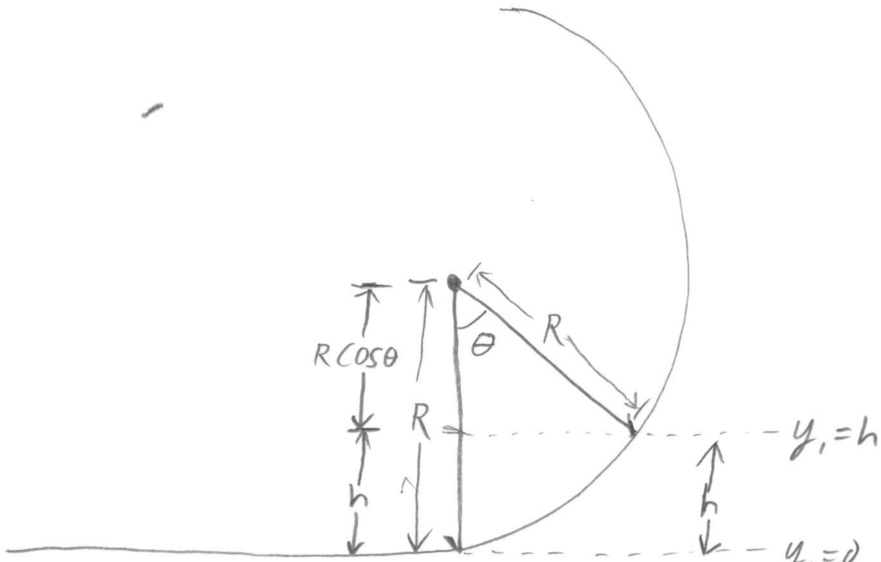
$U_I + K_I + W_{NCF} = U_F + K_F$

$\frac{1}{2}kd^2 + 0 + 0 = mgR + 0$

$$\boxed{d = \left[\frac{2mgR}{k}\right]^{1/2}}$$

EP3, ~~5~~ 5 - continued

b)



$h = R - R\cos\theta$

$h = R(1 - \cos\theta)$

With friction, we don't reach as high. We only reach $h = R(1-\cos\theta)$.

So, let's write the energy balance.

$U_I = \frac{1}{2}kd_0^2$ $\qquad U_F = mgh$

$K_I = 0$ $\qquad K_F = 0$

$$U_I + K_I + \underbrace{W_F}_{\text{Work done by Friction}} = U_F + K_F$$

$$\frac{1}{2}kd^2 + 0 + W_F = mgh + 0$$

$$\Rightarrow W_F = mgh - \frac{1}{2}kd^2$$

$$= mgh - \cancel{\tfrac{1}{2}}\cancel{k}\frac{\cancel{2}mgR}{\cancel{k}} = mgR(1-\cos\theta) - mgR$$

$$\boxed{W_F = -mgR\cos\theta}$$