SAMPLE TEST 3
PHYS 111 SPRING 2010

Name: ______________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS – YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS (IF NOT MORE) IMPORTANT THAN) YOUR FINAL ANSWER.

1) Starting with the definition of work, derive the **Work Energy Theorem**.

$W = \int \vec{F} \cdot d\vec{s}$, $\quad \vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$W = \int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\Rightarrow W = \int (F_x\,dx + F_y\,dy + F_z\,dz) \Rightarrow \boxed{W = \int F_x\,dx + \int F_y\,dy + \int F_z\,dz}$$

$$\boxed{\underline{W_{net}} = \Sigma W} = \Sigma\left(\int F_x\,dx + \int F_y\,dy + \int F_z\,dz\right)$$

$$\Rightarrow W_{net} = \Sigma \int F_x\,dx + \Sigma \int F_y\,dy + \Sigma \int F_z\,dz$$

$$\Rightarrow W_{net} = \int \Sigma F_x\,dx + \int \Sigma F_y\,dy + \int \Sigma F_z\,dz$$

Now: $\Sigma F_x = ma_x$ (NSL) so we can subst.

consider just the x-axis.

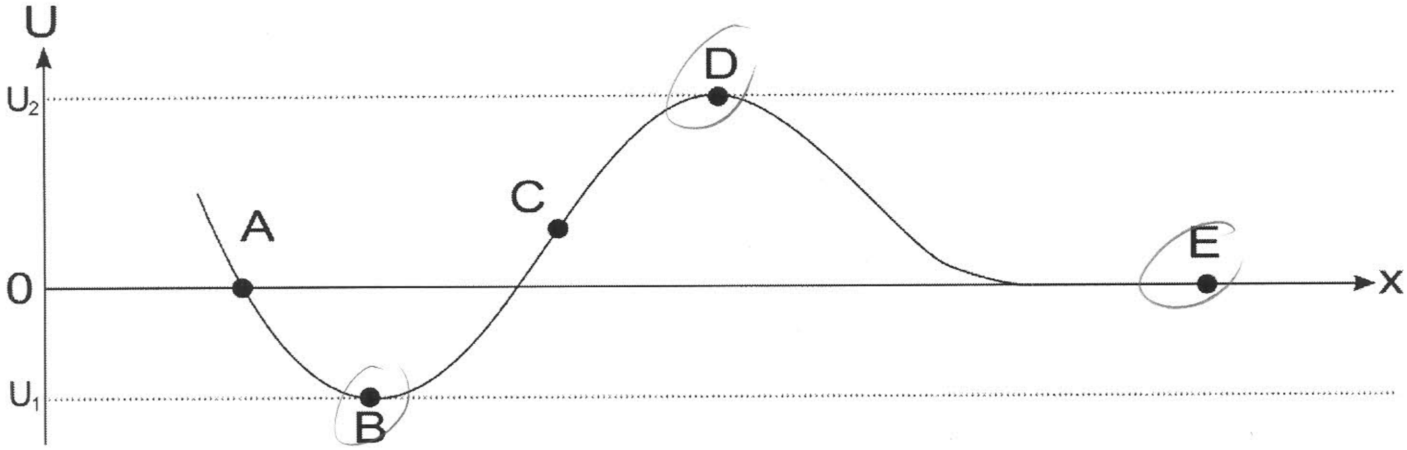
$$\int_{x_0}^{x} \Sigma F_x\,dx = \int_{x_0}^{x} ma_x\,dx = \int_{x_0}^{x} m\frac{dv_x}{dt}\,dx \overset{\text{change variable}}{=} \int_{v_{0x}}^{v_x} m\frac{dx}{dt}\,dv = \int_{v_{0x}}^{v_x} mv\,dv$$

$$= \tfrac{1}{2}mv_x^2 - \tfrac{1}{2}mv_{0x}^2 = \boxed{\Delta K_x}$$

So: $\boxed{W_{net} = \Delta K}$ QED.

SAMPLE TEST 3
PHYS 111 FALL 2012

2) Conceptual Questions, 6 points each.



2.1) Refer to the potential energy curve above. At which point(s) does the Force go to zero?

a) A, C
b) B, D
c) B, D, E
d) B

$F = -\frac{dU}{dx}$, so $F = 0$ at min/max points.

2.2) Your car travels at a *constant velocity* up a hill that makes an angle θ with the horizontal. After traveling a distance d, the **net work** done on the car is:

a) $W_{net} = mgd\sin(\theta)$
b) $W_{net} = -mgd\sin(\theta)$
c) $W_{net} = 0$
d) $W_{net} = mgd\sin(\theta) - N\mu_k d$

Constant Velocity $\Rightarrow \Delta K = 0$

and because $W_{net} = \Delta K$, $W_{net} = 0$

2.3) Two objects, one of mass m and the other of mass 2m, are dropped from the top of a building. When they hit the ground:

a) both will have the same kinetic energy.
b) the heavier one will have twice the kinetic energy of the lighter one.
c) the heavier one will have half the kinetic energy of the lighter one.
d) the lighter one will be moving at half the speed of the heavier one.

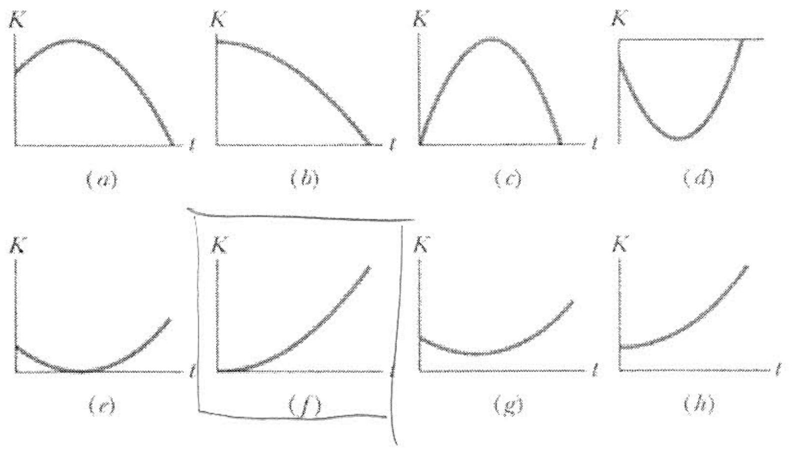
$mgh = K_F \Rightarrow K$ goes as m

so if m doubles, K doubles

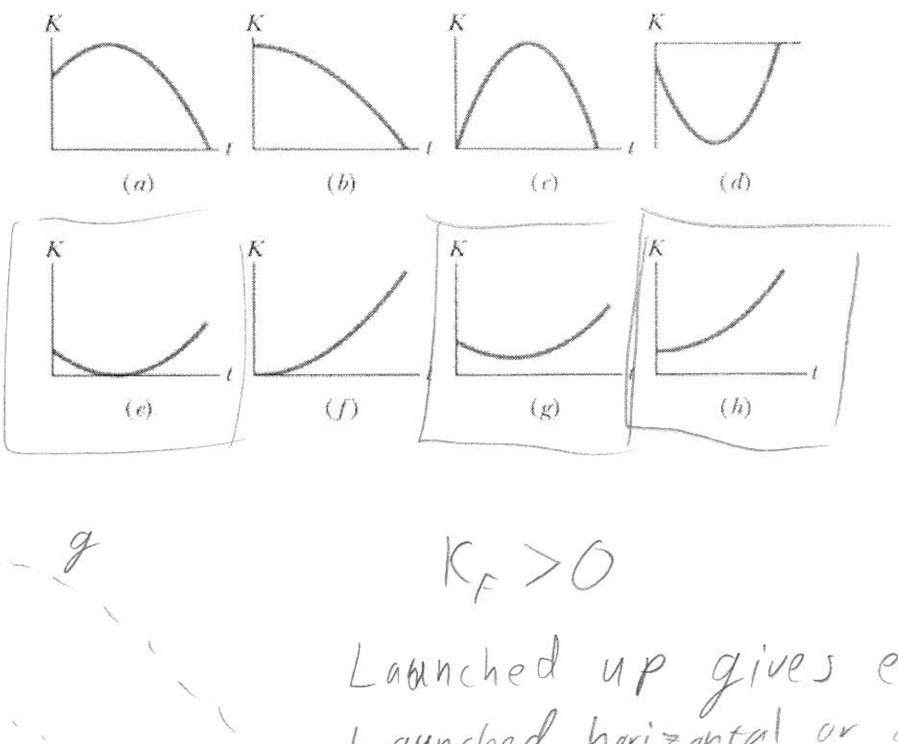
SAMPLE TEST 3
PHYS 111 FALL 2012

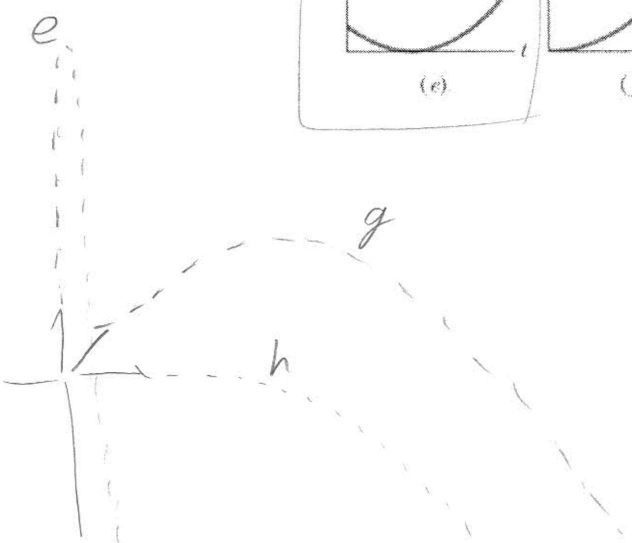
2.4) A glob of slime is dropped from rest from the edge of a cliff. Which graph(s) below could show how the kinetic energy changes as a function of time?

$v_0 = 0$
$\Rightarrow K_0 = 0$
and then increases...



2.5) A glob of slime is launched with an initial velocity from the top of a cliff. Which graph(s) below could show how the kinetic energy changes as a function of time?
(The direction of the initial velocity is intentionally left undefined.)

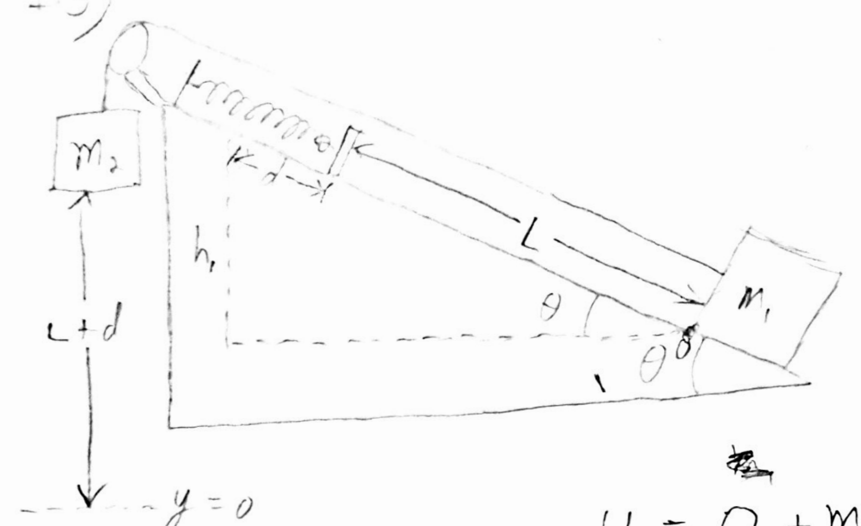




$K_f > 0$

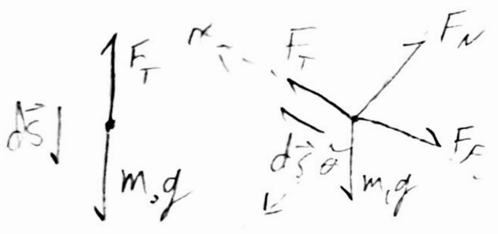
Launched up gives e and g
Launched horizontal or down gives h

#3)



Given	Want
h_1	d
m_2	
μ_k	
L	
k	
θ	

$$U_I = 0 + m_2 g(L+d) + 0 \qquad K_I = 0$$



$$U_F = m_1 g(L+d)\sin\theta + 0 + \tfrac{1}{2}kd^2 \qquad K_F = 0$$

$$F_N - m_1 g\cos\theta = 0$$

$$\Rightarrow F_N = m_1 g\cos\theta$$

$$W_{NCF} = \int_0^{L+d} \vec{F}_f \cdot d\vec{s} = -\int_0^{L+d} \mu_k m_1 g\cos\theta \, dx$$

$$= -\mu_k m_1 g\cos\theta(L+d)$$

$$m_2 g(L+d) - \mu_k m_1 g\cos\theta(L+d) = mg(L+d)\sin\theta + \tfrac{1}{2}kd^2$$

$$\Rightarrow \tfrac{1}{2}kd^2 - m_2 gd + \mu_k m_1 g\cos\theta d + m_1 gd\sin\theta - m_2 gL + \mu_k m_1 g\cos\theta L + m_1 g\sin\theta L$$

$$\Rightarrow \tfrac{1}{2}kd^2 + (\mu_k m_1 g\cos\theta + m_1 g\sin\theta - m_2 g)d + (\mu_k m_1 g\cos\theta + m_1 g\sin\theta - m_2 g)L = 0$$

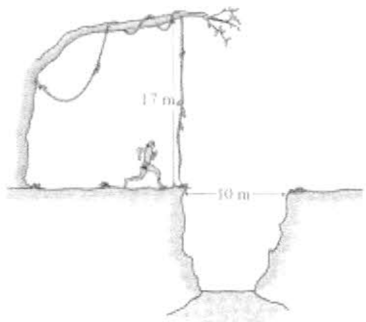
"Standard" form

SAMPLE TEST 3

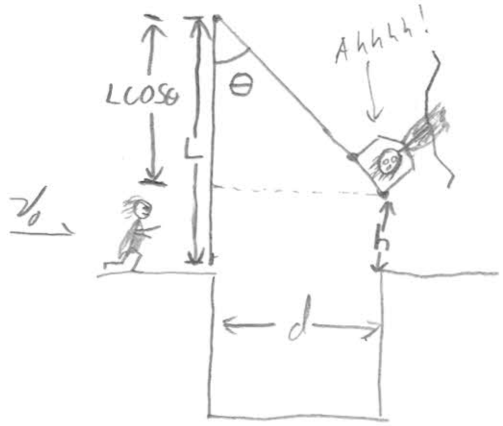
PHYS 111 SPRING 2010

Use work-energy techniques to solve the following problem.

Tarzan is late for a date with Jane and is running as fast as he can to meet her. On the way, he has to get over a 10m wide pit of dangerous croc-a-gators. A 17m vine is hanging vertically from a tree at one side of the pit. Tarzan is going to run up, grab the vine, swing across, and drop vertically to the ground on the other side.



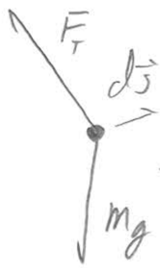
What must his minimum speed be to make it across?



$h = L - L\cos\theta$

$d = L\sin\theta$

$y=0$



$W_{F_T} = 0,\ \vec{F}_T \perp d\vec{s}$

some point mid swing

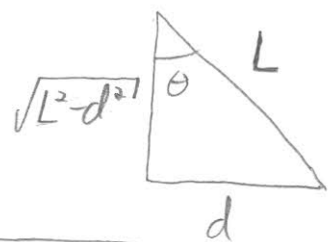
$U_I = 0$ $\qquad$ $U_F = mgL(1-\cos\theta)$

$K_I = \frac{1}{2}mv_0^2$ $\qquad$ $K_F = 0$ ← Stops to drop Vertically

$W_{NCF} = 0$

$$\frac{1}{2}\cancel{m}v_0^2 = \cancel{m}gL(1-\cos\theta)$$

$$v_0 = \left(2gL(1-\cos\theta)\right)^{1/2}$$



$$\cos\theta = \frac{\sqrt{L^2-d^2}}{L}$$

$$\boxed{v_0 = \left(2gL\left(1-\frac{\sqrt{L^2-d^2}}{L}\right)\right)^{1/2} = \left(2g\left(L-\sqrt{L^2-d^2}\right)\right)^{1/2}}$$

SAMPLE TEST 3
PHYS 111

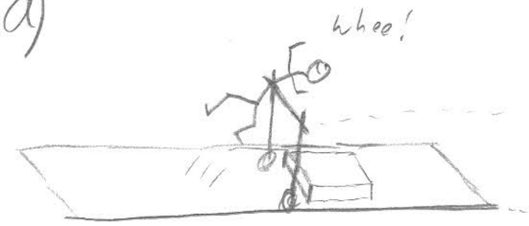
5) ***Use Work-Energy techniques to solve the following problem***

After getting your physics degree, you end up on the planning committee for the 2040 Olympic games. You are considering a new sport: Asteroid Jumping. You need to pick an asteroid that the high jumpers will not accidentally escape from after their jump (the initial velocity of their jump must be below the escape velocity of the asteroid).

a) On Earth, the absolute highest a high jumper could go is 3.0m. On the Earth's surface, what initial velocity is required to reach a height of 3.0m?

b) Asteroids are made out of rock that has a density of 2500 kg/m^3. Assuming spherical asteroids, find the radius of an asteroid with an escape velocity equal to the velocity from part a.
(HINT: The volume of a sphere is $\frac{4}{3}\pi r^3$ *, where r is the radius of the sphere.)*

a)



$U_I = 0$ $\quad K_I = \frac{1}{2} m V^2$

$U_F = mgh$ $\quad K_F = 0$

Given
$h = 3.0\,m$

Want
V

$\boxed{V = \sqrt{2gh}}$ again!

$V = \left((2)(9.8)(3.0)\right)^{1/2} = 7.7\,m/s$

b)

Given
$\rho = 2500\,kg/m^3$
$V_0 = 7.7\,m/s$

Want
r_A

$U_I = -\dfrac{G M_A m}{r_A}$ $\quad U_F = 0$

$K_I = \frac{1}{2} m V_0^2$ $\quad K_F = 0$

So: $-\dfrac{G M_A \cancel{m}}{r_A} + \frac{1}{2}\cancel{m} V_0^2 = 0$

Continued
↓

Extra Space for #5

$$\Rightarrow \frac{GM_A}{r_A} = \frac{1}{2}v_0^2$$

But: $M = \rho V$ where V = Volume

$$V = \frac{4}{3}\pi r^3$$

So: $M_A = \rho \frac{4}{3}\pi r_A^3$

And: $\dfrac{G\rho \frac{4}{3}\pi r_A^{\cancel{3}\,2}}{\cancel{r_A}} = \frac{1}{2}v_0^2$

$$\Rightarrow \boxed{r_A = \left[\frac{3v_0^2}{8G\rho\pi}\right]^{1/2}}$$

$$\Rightarrow r_A = \left[\frac{3(7.7)^2}{8(6.67\times10^{-11})(2500)(3.14)}\right]^{1/2}$$

$$\boxed{r_A = 6{,}500\ \text{m}}$$