Test 2

Phys 111, Fall 2010, Section 1

Name: ______________________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

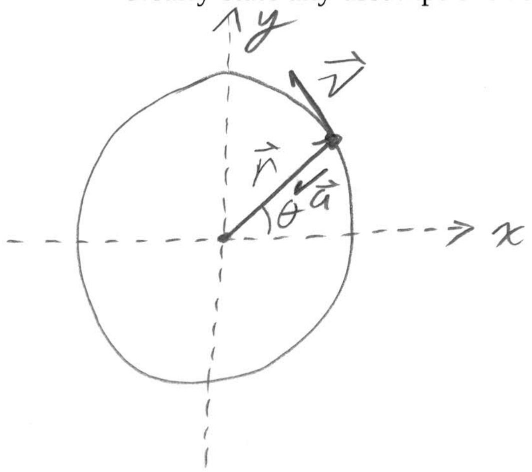
SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

1) Starting with the unit vector expression for the position, r, of a particle constrained to move in a circle, derive an expression for the magnitude of the velocity vector and the magnitude of the acceleration vector assuming that the particle is moving in **uniform circular motion.**

Include a picture with the position vector, r, the velocity vector, v, the acceleration vector, a, and the position angle, θ, clearly marked.

assume: r and $\frac{d\theta}{dt}$ are const.

Clearly state any assumptions required by the proof.



$$\vec{r} = r_x\hat{i} + r_y\hat{j}$$

$$\vec{r} = r\cos\theta\hat{i} + r\sin\theta\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = r\frac{d}{dt}(\cos\theta\hat{i} + \sin\theta\hat{j})$$

$$= r\left(-\frac{d\theta}{dt}\sin\theta\hat{i} + \frac{d\theta}{dt}\cos\theta\hat{j}\right)$$

$$\vec{v} = r\frac{d\theta}{dt}(-\sin\theta\hat{i} + \cos\theta\hat{j})$$

$$\boxed{v = r\omega} \quad \omega \equiv \frac{d\theta}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = r\omega\frac{d}{dt}(-\sin\theta\hat{i} + \cos\theta\hat{j})$$

$$= r\omega\frac{d\theta}{dt}(-\cos\theta\hat{i} - \sin\theta\hat{j})$$

$$\boxed{a = r\omega^2}$$

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2) Multiple Choice, 4 points each.

2.1) If you jumped out of a plane, you would begin speeding up as you fell downward. Eventually, due to wind resistance, you will fall at a constant velocity. At this point, the force due to wind resistance is

a) slightly smaller than the force of gravity
b) much smaller than the force of gravity
c) equal to the force of gravity
d) greater than the force of gravity

IF $v = const$, $\frac{dv}{dt} = 0$

so $F_R - mg = 0 \Rightarrow F_R = mg$

2.2) You are making a circular turn in your car when you hit a big patch of ice causing the frictional force between the tires and the road to go to zero. While the car is on the ice, it

a) moves in a straight line
b) continues on a circular path but with a larger radius
c) follows the original circular path
d) follows a curved path that is non-circular

The centripedal force is friction.

No central force, no circle, straight line

2.3) A merry-go-round is spinning with a fixed angular speed. As you walk from the center towards the edge

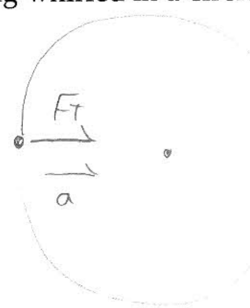
a) the force provided by static friction must increase so that you don't slide off.
b) the force provided by static friction must decrease so that you don't slide off.
c) The force provided by static friction remains constant and you don't slide off.

$a = r\omega^2 \Rightarrow F_f = mr\omega^2$

as $r\uparrow$, $F_f\uparrow$

2.4) A rock tied to a string is being whirled in a circle. In the absence of gravity, the tension in the string is

A) $F_T = mg$
B) $F_T = m\frac{v}{r}$
C) $F_T = mr\omega^2$
D) $F_T = r\omega^2$



$F_T = ma$, $a = r\omega^2$ -or- $a = v^2/r$

$F_T = mr\omega^2$ -or- $F_T = m\frac{v^2}{r}$

2.5) Consider a person standing in an elevator that is accelerating downward. The magnitude of the upward normal force exerted by the elevator floor on the person is
a) larger than
b) less than
c) equal to
the magnitude of the weight of the person.

$F_N < mg$

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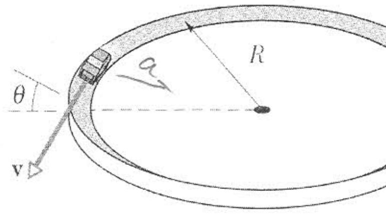
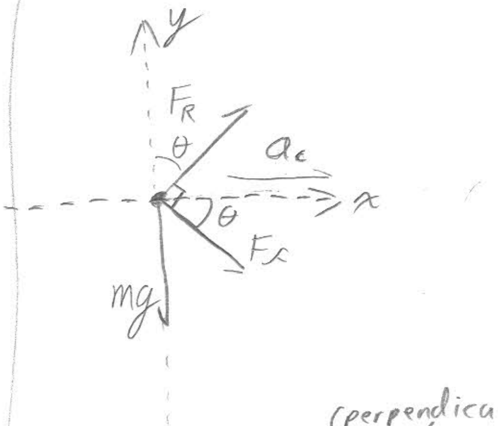
4) Curves on roadways are often banked so that the reaction force from the road provides part of the centripetal acceleration required to keep the car moving around the circle.



Assume the road is a circle of radius R, the car is going some velocity v, and the tires have some coefficient of static friction μ_s.

Find an expression for the minimum angle required to keep the car on the road in terms of R, v, and μ_s.

FBD - Side view



F_R is always $\perp$ (perpendicular) to road surface

* F_f is always $\parallel$ (parallel) to road surface.

NSL

x: $\sum F_x = ma_x$

$$F_R \sin\theta + F_f \cos\theta = m a_c$$ (centripetal)

$$\Rightarrow F_R \sin\theta + \mu_s F_R \cos\theta = m\frac{v^2}{R} \quad (1)$$

y: $\sum F_y = ma_y$ (no accel in y)

$$F_R \cos\theta - F_f \sin\theta - mg = 0$$

$$\Rightarrow F_R \cos\theta - \mu_s F_R \sin\theta - mg = 0 \quad (2)$$

From (1): $F_R(\sin\theta + \mu_s \cos\theta) = m\frac{v^2}{R}$

From (2): $F_R(\cos\theta - \mu_s \sin\theta) = mg$

Divide $\frac{1}{2}$: $\frac{F_R(\sin\theta + \mu_s\cos\theta)}{F_R(\cos\theta - \mu_s \sin\theta)} = \frac{m\, v^2/R}{mg}$

$$\Rightarrow gR\sin\theta + \mu_s gR\cos\theta = v^2\cos\theta - \mu_s v^2 \sin\theta$$

$$(gR + \mu_s v^2)\sin\theta = (v^2 - \mu_s gR)\cos\theta$$

$$\Rightarrow \boxed{\tan\theta = \frac{v^2 - \mu_s gR}{gR + \mu_s v^2}}$$

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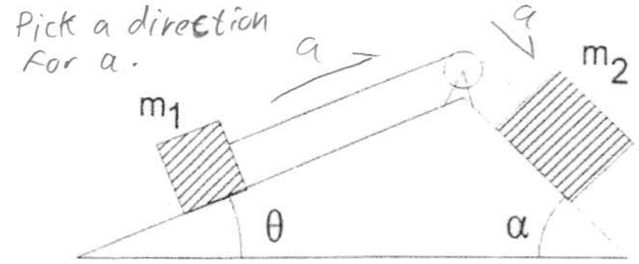
3) Consider the figure shown below. Assume the pulley is massless and frictionless.

$\theta = 30°$
$\alpha = 45°$
$\mu_k = 0.10$
$m_1 = 5.0$ kg
$m_2 = 15.0$ kg



a) Find an expression (no numbers) for the acceleration of the system.
b) Plug in the numbers and find the numeric answer.

Free Body Diagrams

a) Write NSL

m1:

x: $\sum F_x = ma_x$

y: $\sum F_y = ma_y$

① $F_T - F_{f1} - m_1 g \sin\theta = m_1 a$

② $F_{R1} - m_1 g \cos\theta = 0$

m2:

x: $\sum F_x = ma_x$

y: $\sum F_y = ma_y$

③ $m_2 g \sin\alpha - F_T - F_{f2} = m_2 a$

④ $F_{R2} - m_2 g \cos\alpha = 0$

continued
↓

* Rewrite friction terms using: $F_f = \mu_K F_R$

From (1): $F_T - \mu_K F_{R1} - m_1 g \sin\theta = m_1 a$ (5)

From (3): $m_2 g \sin\alpha - F_T - \mu_K F_{R2} = m_2 a$ (6)

* Solve (2) and (4) for F_R and subst. into (5) and (6)

Subst (2) → (5):

$$F_T - \mu_K m_1 g \cos\theta - m_1 g \sin\theta = m_1 a \quad (7)$$

Subst (4) → (6):

$$m_2 g \sin\alpha - F_T - \mu_K m_2 g \cos\alpha = m_2 a \quad (8)$$

* Solve (7) for F_T and subst into (8)

$$m_2 g \sin\alpha - \mu_K m_1 g \cos\theta - m_1 g \sin\theta - m_1 a - \mu_K m_2 g \cos\alpha = m_2 a$$

$$\Rightarrow g\left[m_2(\sin\alpha - \mu_K \cos\alpha) - m_1(\sin\theta + \mu_K \cos\theta)\right] = a(m_1 + m_2)$$

$$\boxed{a = g\,\frac{m_2(\sin\alpha - \mu_K\cos\alpha) - m_1(\sin\theta + \mu_K\cos\theta)}{(m_1 + m_2)}}$$

b)

$$a = 9.8\,\frac{(15)(\sin(45) - (0.10)\cos(45)) - (5.0)(\sin(30) + (0.10)\cos(30))}{5.0 + 15.0}$$

$$\boxed{a = 3.24\ m/s^2}$$

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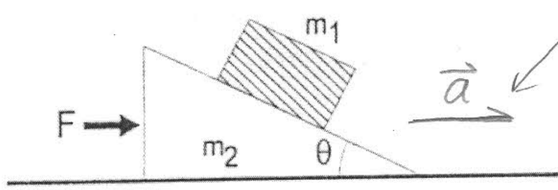
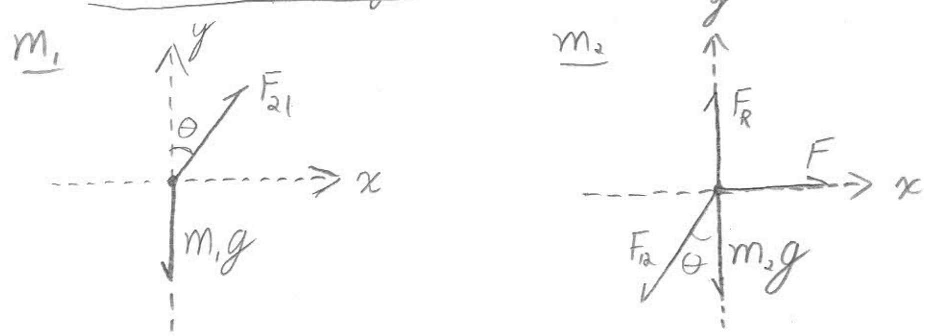
4) In the figure shown below all surfaces are frictionless. Find an expression for the the force applied to block m_2 so that block m1 does not slide down the ramp.

$\theta = 30°$
$m_1 = 2.0$ kg
$m_2 = 5.0$ kg



Both blocks are accelerating parallel to the floor. with the same accel.

Free Body diagrams



NSL

m_1

$x: \sum F_x = ma_x$

① $F_{21}\sin\theta = m_1 a$

$y: \sum F_y = ma_y$

② $F_{21}\cos\theta - m_1 g = 0$

m_2

$x: \sum F_x = ma_x$

③ $F - F_{21}\sin\theta = m_2 a$

④ $y: F_R - F_{12}\cos\theta - m_2 g = 0$

* To solve ③ for F, I'll eliminate a with eq ① and eliminate F_{21} with eq. ②.

continued ↓

Sample Test 2 Problem 4 continued

Divide $\frac{③}{①}$: $\frac{F - F_{21}\sin\theta}{F_{21}\sin\theta} = \frac{m_2 a}{m_1 a} \Rightarrow F - F_{21}\sin\theta = \frac{m_2}{m_1} F_{21}\sin\theta$

$$\Rightarrow F = F_{21}\sin\theta\left(1 + \frac{m_2}{m_1}\right) \quad ⑤$$

From ②: $F_{21} = \frac{m_1 g}{\cos\theta}$

$$\Rightarrow F = \frac{m_1 g}{\cos\theta}\sin\theta\left(1 + \frac{m_2}{m_1}\right)$$

$$\Rightarrow F = m_1\left(1 + \frac{m_2}{m_1}\right) g\tan\theta$$

$$\Rightarrow F = \left(m_1 + m_1\frac{m_2}{m_1}\right) g\tan\theta$$

$$\Rightarrow \boxed{F = (m_1 + m_2)\, g\tan\theta}$$

$$F = (2.0\,\text{kg} + 5.0\,\text{kg})(9.8\,\text{m/s}^2)\tan(30^\circ)$$

$$\boxed{F = 40\,\text{N}}$$