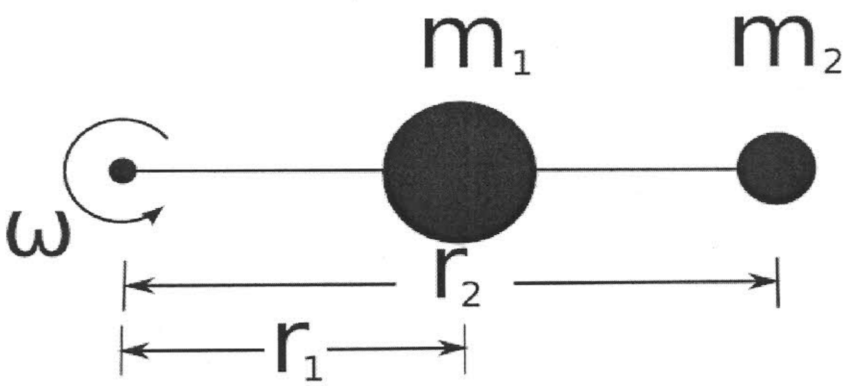
Rotation – Set 2

Consider a thin (essentially massless) bar with two masses attached to it as pictured below. The bar is rotating about the point shown in the diagram with an angular velocity ω.



a) Write an expression for the total kinetic energy of the system in terms of r_1, r_2, and ω. Simplify your expression as much as possible.

$$K_T = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

$$= \tfrac{1}{2} m_1 r_1^2 \omega^2 + \tfrac{1}{2} m_2 r_2^2 \omega^2$$

$$\boxed{K_T = \tfrac{1}{2}\left(m_1 r_1^2 + m_2 r_2^2\right)\omega^2}$$

b) Generalize the expression above to a system with *n* masses (use a summation symbol, Σ, in your expression).

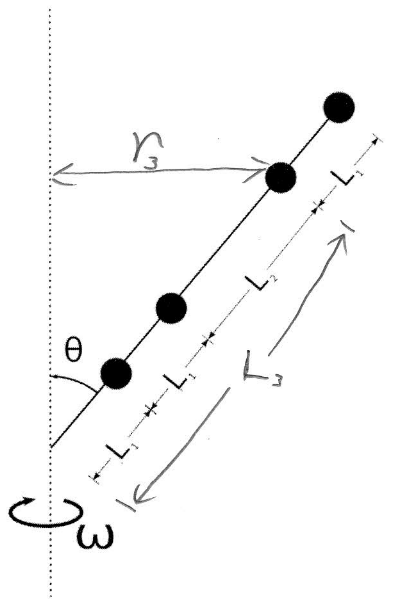
$$\boxed{K_T = \tfrac{1}{2}\left(\sum m_i r_i^2\right)\omega^2}$$

The term in parenthesis is the moment of inertia.

$I = \sum m_i r_i^2$, for discrete masses anyway...

Four point masses, each of mass m, are attached to a rigid massless rod that makes an angle θ with the axis of rotation. Let $L_2 = 2L_1$.

a) What is the moment of inertia of this system?
b) What is the kinetic energy of this system if it's rotating with angular velocity ω.



$r_3 = L_3 \text{SIN}\theta$

$r_3 = 4L_1 \text{SIN}\theta$

$$I = \sum m_i r_i^2$$

$$= mr_1^2 + mr_2^2 + mr_3^2 + mr_4^2$$

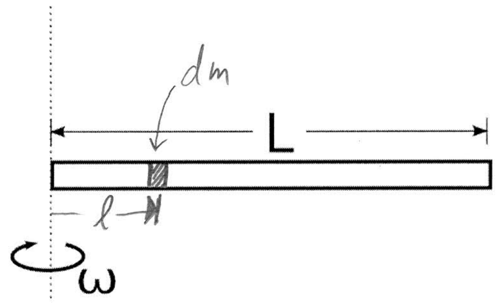
$$= m(r_1^2 + r_2^2 + r_3^2 + r_4^2)$$

$$= m\left(L_1^2 \text{SIN}^2\theta + (2^2)L_1^2 \text{SIN}^2\theta + (4^2)L_1^2\text{SIN}^2\theta + (5^2)L_1^2 \text{SIN}^2\theta\right)$$

$$= mL_1^2 \text{SIN}^2\theta\,(1 + 4 + 16 + 25)$$

$$\boxed{I = 46mL_1^2}$$

Calculate the moment of inertia of a uniform bar of length L and mass M about the axis of rotation shown.



$\lambda = \frac{M}{L}$

$I = \int r^2 dm, \qquad dm = \lambda \, d\ell$

$dm = \frac{M}{L} d\ell, \quad \underline{r = \ell}$

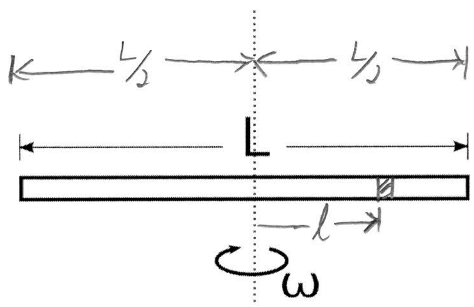
$I = \int_0^L \ell \frac{M}{L} d\ell = \frac{M}{L} \int_0^L \ell \, d\ell$

$= \frac{M}{L} \left(\frac{1}{3} \ell^3 \right) \Big|_0^L$

$= \frac{M}{\cancel{L}} \frac{1}{3} L^{\cancel{3}2}$

$\boxed{I = \frac{1}{3} M L^2}$

Calculate the moment of inertia of a uniform bar of length L and mass M about the axis of rotation shown.



$\lambda = \frac{M}{L}$

$I = \int r^2 dm, \qquad dm = \lambda \, dl$

$dm = \frac{M}{L} dl, \quad \underline{r = l}$

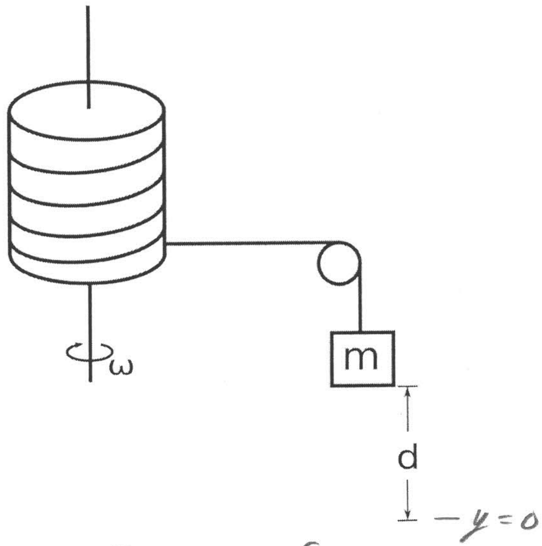
$$I = \int_{-L/2}^{L/2} l^2 \frac{M}{L} dl = \frac{M}{L} \int_{-L/2}^{L/2} l^2 dl = \frac{M}{L}\left(\tfrac{1}{3} l^3\right)\Big|_{-L/2}^{L/2}$$

$$= \frac{M}{L}\frac{1}{3}\left(\frac{L^3}{8} + \frac{L^3}{8}\right) = \frac{M}{\cancel{L}}\frac{1}{3}\frac{L^{\cancel{3}\,2}}{4}$$

$$\boxed{I = \frac{1}{12} M L^2}$$

Rotation – Set 3

A solid cylinder of mass *M*, radius *R*, and moment of inertia $I = \frac{1}{2}MR^2$ is allowed to rotate without friction about an axis through its center as shown. A massless string is wrapped around the cylinder, passes over a small massless frictionless pulley and is attached to a small mass *m*.



If the mass and the cylinder start from rest, what will the angular velocity of the cylinder be after the mass falls through a distance d?

Using conservation of energy, find an expression for ω_f in terms of *d*, *M*, *m*, and *R*.

$U_I = mgd \qquad U_F = 0$

$K_I = 0 \qquad K_F = \underline{\tfrac{1}{2}mv^2} + \underline{\tfrac{1}{2}I\omega_F^2}$ ← Energy of <u>Both</u> objects.

$$mgd = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_F^2$$

$$\underline{v = R\omega}$$

$$mgd = \frac{1}{2}mR^2\omega_F^2 + \frac{1}{4}MR^2\omega_F^2$$

$$mgd = \left(\frac{1}{2}m + \frac{1}{4}M\right)R^2\omega_F^2$$

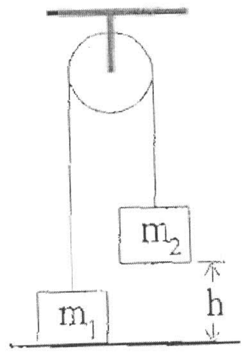
$$\boxed{\omega_F = \left[\frac{m}{\frac{1}{2}m + \frac{1}{4}M}\,\frac{gd}{R^2}\right]^{1/2}}$$

Rotation – Set 3

3

Use work energy to solve the following problem.
Two masses are connected by a light string passing over a frictionless pulley. the Mass m_2 is released from rest at a height of 4.0 m above the ground. You can treat the pulley as a solid disk. , $I = \frac{1}{2} M r^2$

Determine the speed of m_1 as m_2 hits the ground.



$m_1 = 3.0$ kg
$m_2 = 5.0$ kg
$m_{pulley} = 0.5$ kg
$r_{pulley} = 0.1$ m

$U_I = m_2 g h$ $\qquad$ $U_F = m_1 g h$

$K_I = 0$ $\qquad$ $K_F = \frac{1}{2}(m_1 + m_2) v_F^2 + \frac{1}{2} I \omega_F^2$

m_1 and m_2 are moving at v

Pulley is spinning

$$m_2 g h = m_1 g h + \frac{1}{2}(m_1 + m_2) v_F^2 + \frac{1}{2} I \omega_F^2$$

$$v = r\omega \Rightarrow \omega = \frac{v}{r}, \quad I = \frac{1}{2} m_p r^2$$

$$(m_2 - m_1) g h = \frac{1}{2}(m_1 + m_2) v_F^2 + \frac{1}{2}\left(\frac{1}{2} m_p \cancel{r^2}\right)\frac{v_F^2}{\cancel{r^2}}$$

$$(m_2 - m_1) g h = \frac{1}{2}(m_1 + m_2) v_F^2 + \frac{1}{4} m_p v_F^2$$

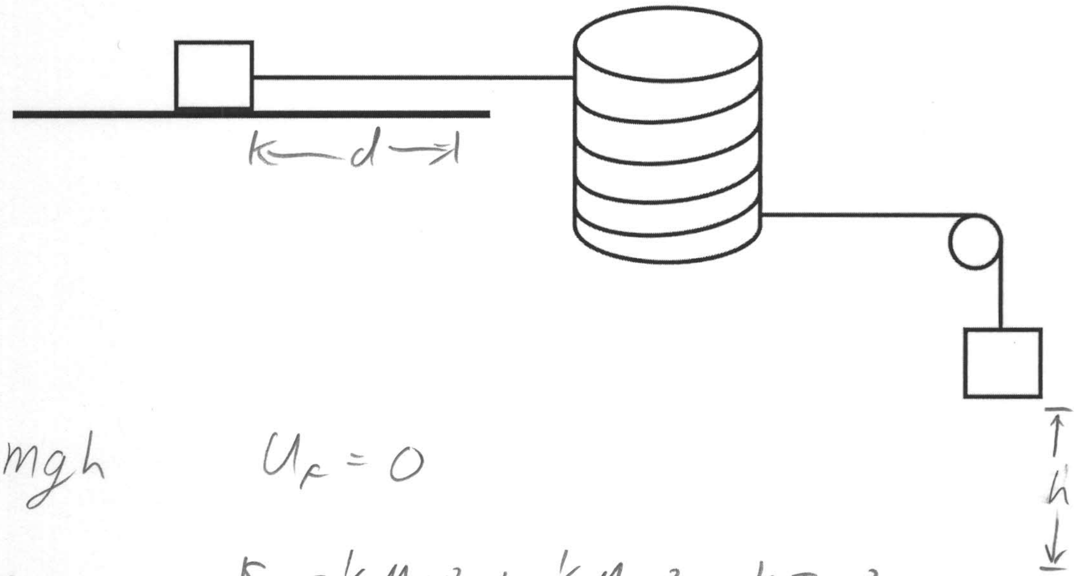
$$\boxed{v_F = \left[\frac{m_2 - m_1}{\frac{1}{2}m_1 + \frac{1}{2}m_2 + \frac{1}{4}m_p} g h\right]^{1/2}} = \left[\frac{5 - 3}{\frac{3}{2} + \frac{5}{2} + \frac{1}{8}}(9.8)(4.0)\right]^{1/2} = \boxed{4.4 \text{ m/s}}$$

A block of mass M rests on a rough table with $\mu_k = 0.3$. A massless string is attached to the block, wrapped around a solid cylinder having a mass M and a radius R, runs over a massless frictionless pulley, and is attached to a second block of mass M that is hanging freely.

Using work/energy techniques, calculate the velocity of the blocks after they have moved a distance d.

$$I_{cylinder} = \frac{1}{2}MR^2$$

[NOTE: Do NOT use torque/kinematics]



$U_I = mgh$ $U_F = 0$

$K_I = 0$ $K_F = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$W_F = -\mu_k mgd$, $\underline{d = h}$

$$\cancel{m}gd - \mu_k \cancel{m}gd = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\frac{1}{2}\cancel{M}\cancel{R^2}\frac{v^2}{\cancel{R^2}}$$

$$gd(1-\mu_k) = \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{4}\right)v^2$$

$$v = \left[\frac{4}{5}gd(1-\mu_k)\right]^{1/2}$$