Circular disks A and B have the same mass and thickness, but the density of disk A is greater than the density of disk B. Which has the greater moment of inertia? **Explain your reasoning.**

To have the same mass but different density, the Volume of A must be smaller. If they have the same thickness, the radius of A must be smaller. so, A has its mass concentrated closer to the axis of rotation so $I_B > I_A$

Two bars have the same mass, but one is shorter than the other. Which has a larger moment of inertia? **Explain your reasoning.**

more mass further from the axis.

$I_2 > I_1$

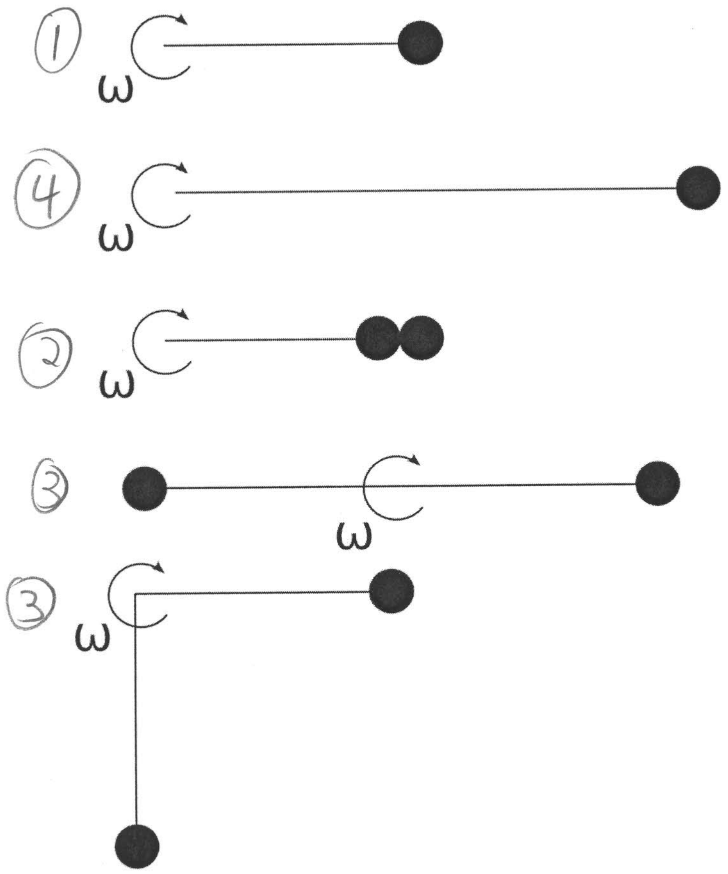
Below are the cross sections of five solids with identical masses. They all have equal widths at their widest points. Without looking them up or doing any calculations, rank the objects in order of moment of inertia, least to most. **Explain your reasoning**.

Hoop	Cube	Cylinder	Prism	Sphere
5	4	3	2	1

Goes in order according to which object has it's mass closest to the axis of rotation.

Rotation – Set 4

Below are several objects. Each circle is a point mass, and each point has the same mass. The connecting rods are massless. Rank them in order of moment of inertia, least to most. If any have the SAME moment, give them the same ranking number. **Explain your reasoning**.



$I = \sum m_i r_i^2$, So: The top object has a single mass close to the pivot

next, $2 < 3$ because the mass doubles but 2 has one mass sightly closer. to the pivot

the 2 3's have twice the mass, but both at the same distance

last, 4 is a single mass twice as far but I goes as r^2, so r makes a bigger difference than m.

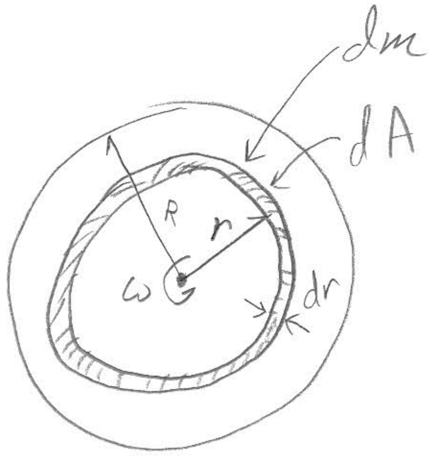
Rotation – Set 2

Consider a thin disk of mass M and radius R.

a) Calculate its moment of inertia, *I*, about an axis through its center of mass perpendicular to the surface of the disk.

b) Calculate its moment of inertia, *I*, about an axis through its center of mass parallel to the surface of the disk.

a)

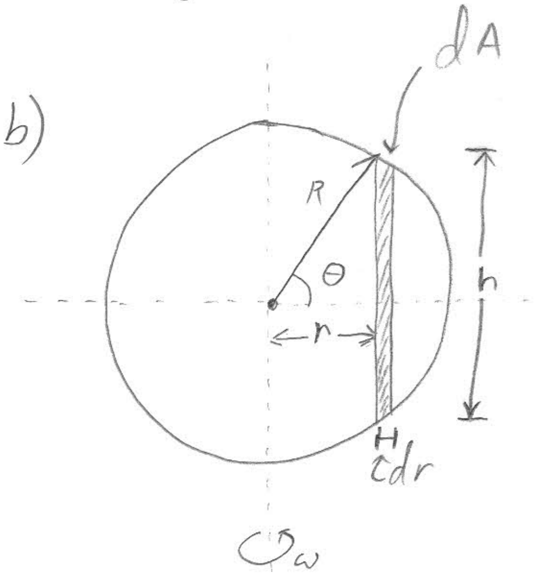


$$I = \int r^2 dm$$

$$dm = \sigma dA, \quad \sigma = \frac{M}{\pi R^2}, \quad dA = 2\pi r dr$$

$$I = \int_0^R r^2 \frac{M}{\cancel{\pi} R^2} 2\cancel{\pi} r dr = \frac{2M}{R^2}\int_0^R r^3 dr = \frac{\cancel{2}M}{\cancel{R^2}}\frac{1}{\cancel{4}_2}R^{\cancel{4}\,2} = \boxed{\frac{1}{2}MR^2}$$

b)



$$I = \int r^2 dm$$

$$dm = \sigma dA, \quad \sigma = \frac{M}{\pi R^2}, \quad dA = h dr$$

I like polar coordinates

$$h = 2R\sin\theta, \quad r = R\cos\theta,$$
$$dr = -R\sin\theta\, d\theta$$

$$\Rightarrow I = -\int_{\pi}^{0} R^{\cancel{2}}\cos^2\theta \frac{M}{\pi \cancel{R^2}} 2R\sin\theta\, R\sin\theta\, d\theta$$

continued ↓

Rotation Set 2 P8 continued

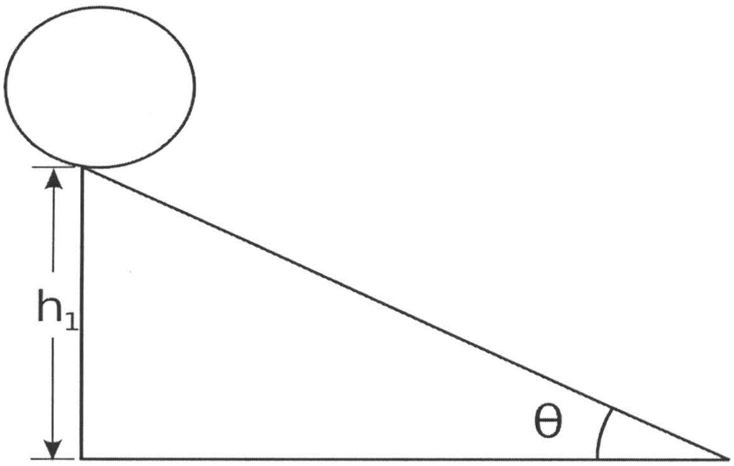
$$I = -\frac{2MR^2}{\pi}\int_{\pi}^{0} \cos^2\theta \sin^2\theta \, d\theta = +\frac{2MR^2}{\pi}\cdot\frac{\pi}{8} = +\boxed{\frac{1}{4}MR^2}$$

Rotation – Set 3

4

A rolling object with a radius R, mass m, and moment of inertia $I_{cm} = \frac{1}{2}mR^2$, starts from rest at the top of an incline plane of height h that makes an angle θ with the horizontal.

a) What is the linear velocity of disk at the bottom?
b) What is the angular velocity of the disk at the bottom?



a) $U_I = mgh \qquad U_F = 0$

$K_I = 0 \qquad K_F = \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I\omega^2$

$$mgh = \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I\omega^2$$

$$V = R\omega \Rightarrow \omega = \frac{V}{R}, \quad I = \frac{1}{2}mR^2$$

$$\Rightarrow \cancel{m}gh = \frac{1}{2}\cancel{m}V_{cm}^2 + \frac{1}{2}\left(\frac{1}{2}\cancel{m}\cancel{R^2}\right)\frac{V_{cm}^2}{\cancel{R^2}}$$

$$gh = \left(\frac{1}{2} + \frac{1}{4}\right)V_{cm}^2 \Rightarrow \boxed{V_{cm} = \left(\frac{4}{3}gh\right)^{1/2}}$$

continued

Rotation Set 3 P4 - continued

b) $mgh = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I \omega^2$

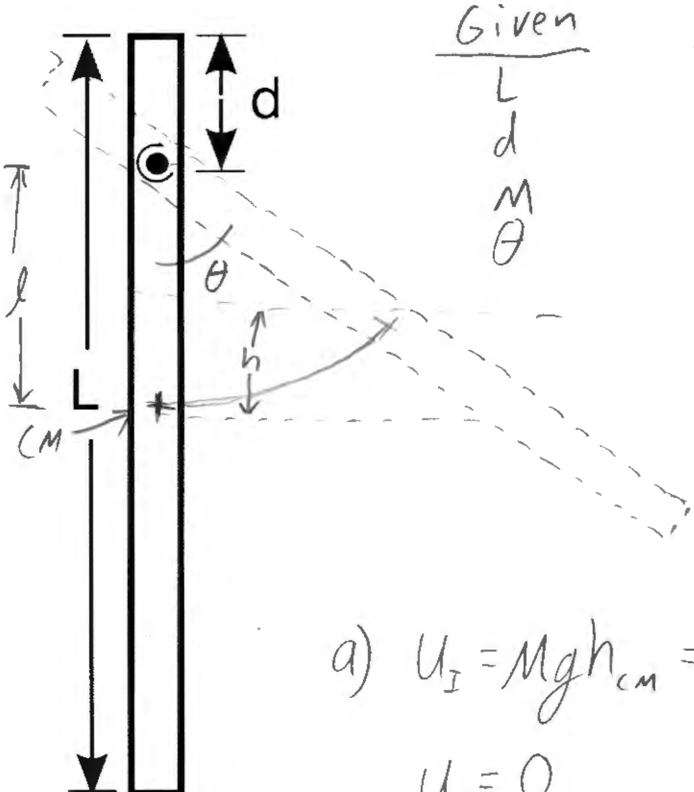
$V = R\omega \,, \quad I = \frac{1}{2} m R^2$

$mgh = \frac{1}{2} m R^2 \omega^2 + \frac{1}{2} \cdot \frac{1}{2} m R^2 \omega^2 \Rightarrow gh = \frac{3}{4} R^2 \omega^2$

$$\boxed{\omega = \left[\frac{4}{3} \frac{gh}{R^2}\right]^{1/2}}$$

A pendulum is constructed from a solid bar of length L and mass M. The bar is then hung from a pivot a distance d from the top of the bar. The bar is held at an angle θ from the vertical and released.

a) What is the bar's angular velocity as it passes vertical?
b) What is the velocity of the bottom tip of the bar as it passes vertical?
c) What is the velocity of the upper end of the bar as it passes vertical?
d) What is the velocity of the center of mass of the bar as it passes vertical?



Given	Want
L	ω
d	v_1
M	v_2
θ	v_3

a) $U_I = Mgh_{cm} = Mg(\ell - \ell\cos\theta)$, $\ell = \frac{1}{2}L - d$

$U_F = 0$

$K_I = 0$

PAT

$K_F = \frac{1}{2}I\omega^2$, $I = I_{cm} + Md^2$, d is ℓ here

$= \frac{1}{12}ML^2 + M\ell^2$

$\Rightarrow \cancel{M}g\ell(1-\cos\theta) = \frac{1}{2}\left[\frac{1}{12}\cancel{M}L^2 + \cancel{M}\ell^2\right]\omega^2$

$\Rightarrow \boxed{\omega^2 = \dfrac{g\ell(1-\cos\theta)}{\frac{1}{24}L^2 + \frac{1}{2}\ell^2} \text{ where } \ell = \frac{1}{2}L - d}$

b, c, d) In general, $v = r\omega$ where r is the distance between the axis of rotation and the point in question.

so: b) $v = (L - d)\omega$

c) $v = d\omega$

d) $v = (\frac{1}{2}L - d)\omega$

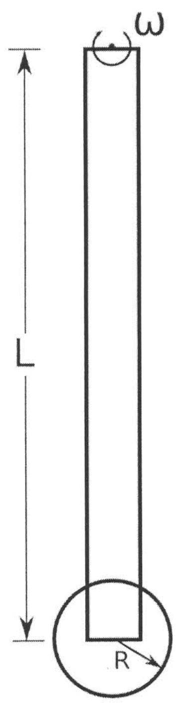
Rotation – Set 3

A clock pendulum is constructed from a solid bar of length L and mass M with a disk of radius R and mass m. The bar is then hung from a pivot at one end and the disk is attached to the opposite end, as in the picture below.

a) The moment of inertia of a uniform rod about an axis perpendicular to the rod through its center of mass is $I_{cm} = \frac{1}{12}ML^2$. Using the **Parallel Axis Theorem**, calculate the moment of inertia of the rod about one end.

b) The moment of inertia of a disk about an axis perpendicular to its surface through its center of mass is $I_{cm} = \frac{1}{2}mR^2$. Using the **Parallel Axis Theorem**, calculate the moment of inertia of the disk when it's attached to the pendulum as shown in the picture below.

c) Using the Principal of Superposition, calculate the moment of inertia of the combined rod-disk system.



a) $I_R = I_{cm} + Md^2, \; d = L/2$

$I_R = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \left(1/12 + 1/4\right)ML^2$

$\boxed{I_R = \frac{1}{3}ML^2}$

b) $I_D = I_{CM} + md^2, \; d = L$

$\boxed{I_D = \frac{1}{2}mR^2 + mL^2}$

c) $I = I_R + I_D$

$\boxed{I = \frac{1}{3}ML^2 + \frac{1}{2}mR^2 + mL^2}$

0

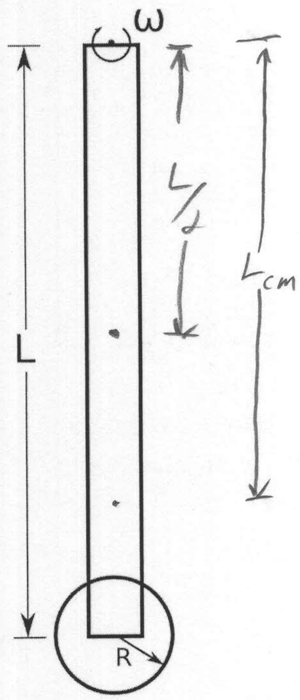
Rotation – Set 3

6

Consider, once again, the clock pendulum pictured below. The disk has a mass m and radius r and the bar has mass M and length L.

a) Calculate the center of mass of the combined bar-disk system *as measured from the axis of rotation.*

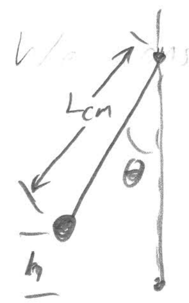
b) If the pendulum is pivoted so that it makes an angle θ with the vertical, what will the angular velocity be when θ=0?



a) $L_{cm} = \frac{1}{M+m} \sum m_i r_i = \frac{1}{M+m}\left(mL + M\frac{L}{2}\right)$

$$\boxed{L_{cm} = \frac{\frac{1}{2}M + m}{M+m} L}$$

b)



$h = L_{cm} - L_{cm}\cos\theta$

Gravitational Potential Energy is dependant on the change in height of the center of mass.

$U_I = mg(L_{cm} - L_{cm}\cos\theta)$ $\quad U_F = 0$

$K_I = 0 \quad K_F = \frac{1}{2} I \omega^2$

$$\omega^2 = \frac{2mgL_{cm}}{I}(1-\cos\theta)$$

continued ↓

Rotation Set 3 P6, continued

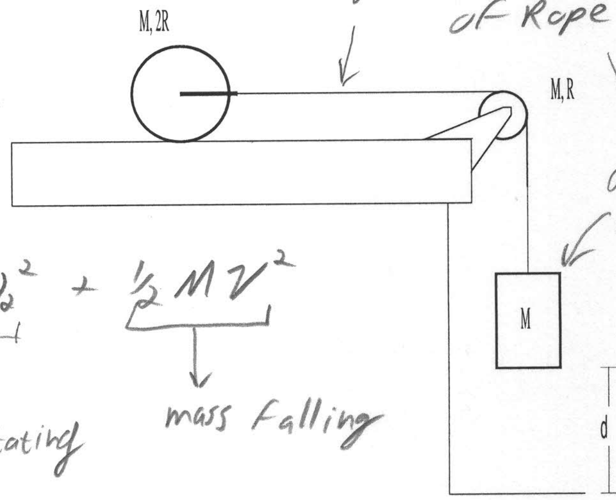
$$\Rightarrow \boxed{\omega^2 = 2mg\,\frac{\frac{1}{2}M+m}{M+m}L\cdot\left(\tfrac{1}{3}ML^2+\tfrac{1}{2}mR^2+mL^2\right)^{-1}(1-\cos\theta)}$$

Rotation – Set 3

A solid cylinder (radius = 2R, mass = M) rolls without slipping as it is pulled by a massless yoke attached to a string. The string goes over a frictionless pulley shaped as a solid disk (radius = R, mass = M) and is attached to a hanging weight (mass = M).

What is the velocity of the hanging weight after it has fallen a distance d?

$V =$ Velocity of Rope. and hanging mass



$U_I = Mgd$ $\qquad U_F = 0$

$K_I = 0$

Different ωs

$$K_F = \underbrace{\tfrac{1}{2}MV^2 + \tfrac{1}{2}I_{D1}\omega_1^2}_{\text{Disk 1 Rotating and translating}} + \underbrace{\tfrac{1}{2}I_{D2}\omega_2^2}_{\text{Disk2 Rotating}} + \underbrace{\tfrac{1}{2}MV^2}_{\text{mass falling}}$$

Need to translate every thing to Velocity

using $V = r\omega \Rightarrow \omega = \frac{V}{R}$

$$\cancel{M}gd = \frac{1}{2}\cancel{M}V^2 + \frac{1}{2}\left(\frac{1}{2}\cancel{M}\cancel{(2R)^2}\right)\frac{V^2}{\cancel{(2R)^2}} + \frac{1}{2}\left(\frac{1}{2}\cancel{M}\cancel{R^2}\right)\frac{V^2}{\cancel{R^2}} + \frac{1}{2}\cancel{M}V^2$$

$$gd = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2}\right)V^2$$

$$\boxed{V = \sqrt{gd}}$$