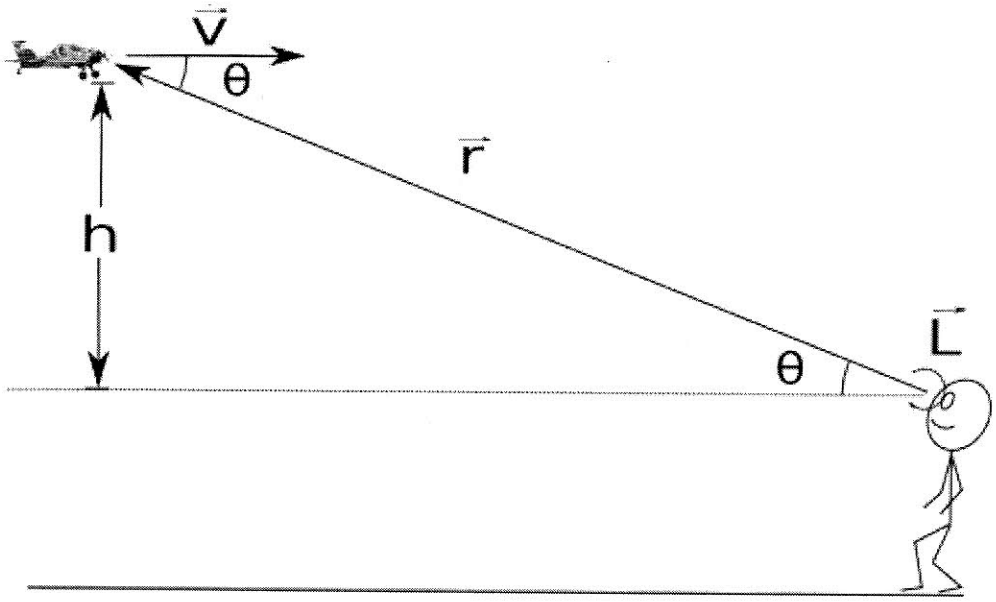
Show that the angular momentum of an airplane flying in a straight line at a constant velocity is constant.

Calculate L from the prospective of an observer on the ground (the observer's eye is the pivot point).



Angular momentum: $\vec{L} = \vec{r} \times m\vec{v}$

$|\vec{L}| = mrv\sin\theta$

or: $L = mvr\sin\theta$

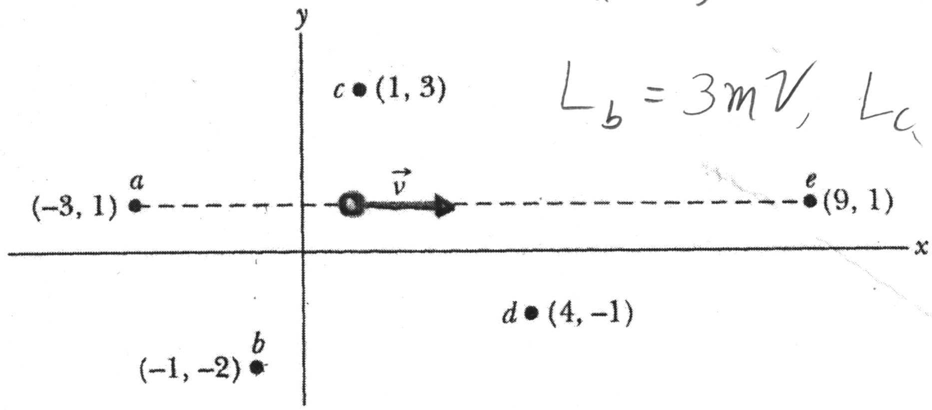
But, $r\sin\theta = h$

so: $\boxed{L = mvh}$ m, v, and h are all constant, so L is constant.

The figure below shows a particle moving at a constant velocity and five points along the trajectory. Rank the points according of the magnitude of the angular momentum of the particle measured about them, greatest first.

$L_a = 0, \; L_e = 0$

$L_b = 3mv, \; L_c = 2mv, \; L_d = 2mv$



$L_b > L_c = L_d > L_a = L_e$

The figure shows three particles of the same mass and speed moving as indicated by the velocity vectors. Points a, b, c, and d form a square and e is at the center of the square. Rank the points according to the magnitude of the net angular momentum of the system.

$\otimes$ +

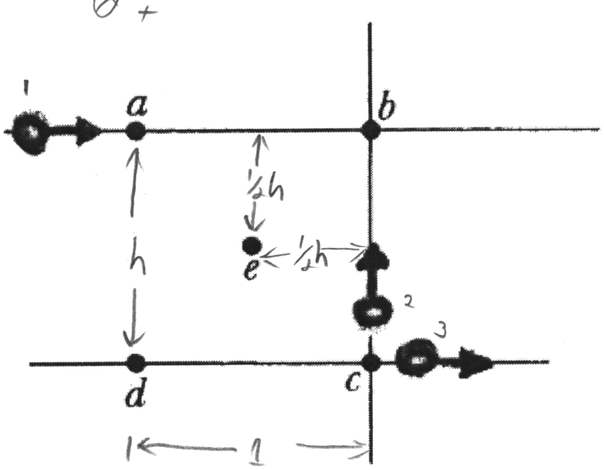
$L_a = 0 - mvh - mvh = -2mvh$

$L_b = 0 + 0 - mvh = -mvh$

$L_c = mvh + 0 + 0 = mvh$

$L_d = mvh - mvh + 0 = 0$

$L_e = \frac{1}{4}mvh - \frac{1}{2}mvh - \frac{1}{4}mvh = \frac{1}{2}mvh$



$\boxed{\text{ranking by magnitude: } L_a > L_b = L_c > L_e > L_d}$

Rotation – Set 6

The diameter of the Sun is approximately 100 Earth diameters and has a rotational period of about 25 days. If it ran out of nuclear fuel and suddenly collapsed to the diameter of the Earth, what would it's new rotational period be?

$$I_{sphere} = \frac{2}{5} M R^2$$

System will conserve angular momentum.

$$L_I = L_F \Rightarrow I_I \omega_I = I_F \omega_F$$

$$\Rightarrow \omega_F = \frac{I_I}{I_F} \omega_I$$

So: $I_I = \frac{2}{5} M_\odot (100 R_\oplus)^2$, $I_F = \frac{2}{5} M_\odot R_\oplus^2$

and $P = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{P}$

Therefore $\Rightarrow \frac{\cancel{2\pi}}{P_F} = \frac{\cancel{\frac{2}{5}} \cancel{M_\odot} (100)^2 \cancel{R_\oplus^2}}{\cancel{\frac{2}{5}} \cancel{M_\odot} \cancel{R_\oplus^2}} \cdot \frac{\cancel{2\pi}}{P_I}$

$$\Rightarrow P_F = \frac{P_I}{100^2} = \frac{25 \text{ days}}{1 \times 10^4} = 2.5 \times 10^{-3} \text{ days}$$

$$= \underline{3.6 \text{ minutes}}$$

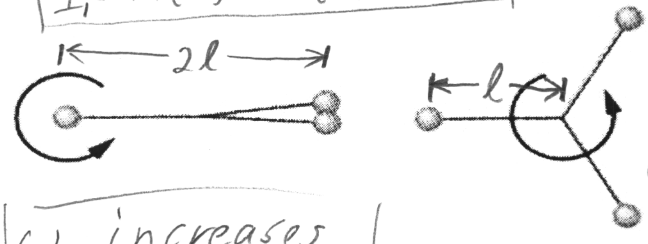
A Bola consists of three heavy balls connected to a common point by identical lengths of sturdy string. It is launched by holding one of the balls overhead and rotating the wrist, causing the other two balls to rotate in a horizontal circle. When it is released, its configuration changes from that shown in figure a to that shown in figure b.

$\boxed{I_1 = m(2l)^2 + m(2l)^2 = 8ml^2}$

$I_1\omega_1 = I_2\omega_2$

$\Rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1$

$\Rightarrow \omega_2 = \frac{8}{3}\omega_1$



(a) (b)

$\boxed{\omega \text{ increases.}}$

$I_2 = ml^2 + ml^2 + ml^2$

$\boxed{I_2 = 3ml^2}$

Does its angular momentum about it's axis of rotation increase, decrease, or stay the same? Does it's angular velocity about it's axis of rotation increase, decrease, or stay the same? Discuss.

I goes down so ω goes up

A beetle rides the rim of a horizontal disk rotating like a merry-go-round. If the beetle walks along the rim in the direction of the rotation, will the magnitudes of the following quantities increase decrease or remain the same?

Angular momentum of the system measured about the rotation axis.
a) increase b) decrease $\boxed{\text{c) remain the same}}$ d) not enough information

Angular velocity of the beetle measured about the rotation axis.
$\boxed{\text{a) increase}}$ b) decrease c) remain the same d) not enough information

Angular momentum of the beetle measured about the rotation axis.
$\boxed{\text{a) increase}}$ b) decrease c) remain the same d) not enough information

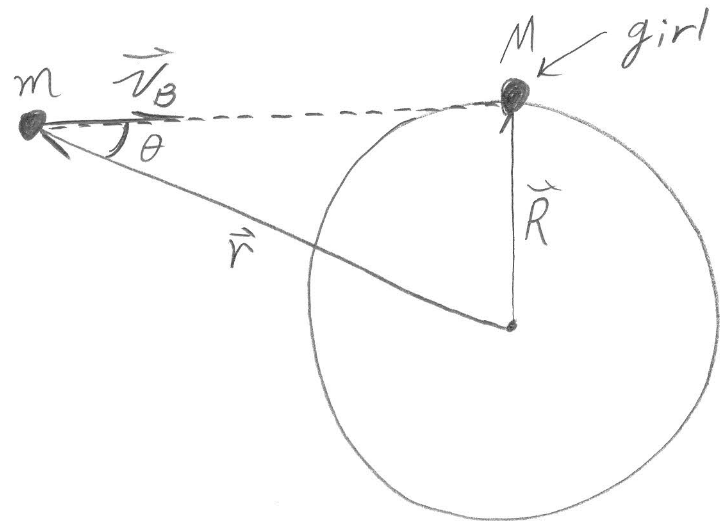
Angular velocity of the disk measured about the rotation axis.
b) increase $\boxed{\text{b) decrease}}$ c) remain the same d) not enough information

Angular momentum of the disk measured about the rotation axis.
a) increase $\boxed{\text{b) decrease}}$ c) remain the same d) not enough information

Rotation – Set 6

A child of mass M is on the outer edge of a merry-go-round that has a moment of inertia I. Her friend throws a baseball with a mass m and velocity v in a direction tangent to the edge of the merry-go-round that is caught by the girl.

Find an expression for the angular velocity of the child, merry-go-round, baseball combination after the impact.



Conserve angular momentum: $L_I = L_F$

L_I is a touch tricky because we do <u>not</u> have rigid body rotation.

The merry-go-round and girl initially have zero angular momentum since they are at rest.

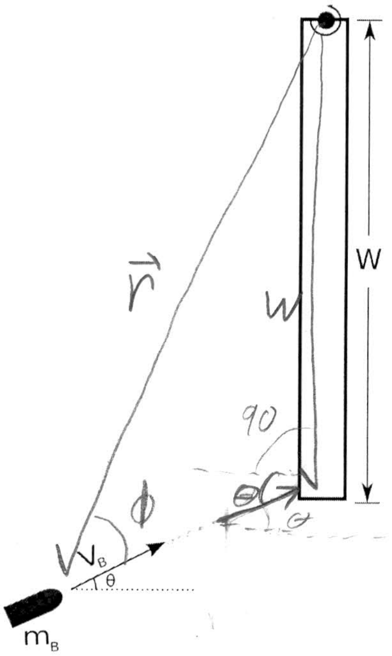
Baseball: $L_I = \vec{r} \times m\vec{v} = mvr\sin\theta$

But $r\sin\theta = R$ so $\boxed{L_I = mvR}$

$L_F = I_F\omega_F$, $I_F = I + MR^2 + mR^2 = I + (m+M)R^2$

So: $mvR = (I + (m+M)R^2)\omega_F \Rightarrow \boxed{\omega_F = \frac{m}{I/R^2 + m + M}\frac{v}{R}}$

A bullet of mass m is fired with a velocity v_b at an angle θ with respect to the horizontal towards a door of width W and mass M. The moment of inertia of the door about its center is $I_{cm} = 1/12\ MW^2$. The bullet impacts the door on the edge opposite the hinge as shown in the picture below. Find an expression for the angular velocity of the bullet door combination after the impact.



$$L_I = \vec{r} \times m\vec{v}$$

$$L_I = W m_B v_B \sin(90+\theta)$$

$$L_I = W m_B v_B \cos\theta$$

$$L_F = I_F \omega_F$$

$$I_F = I_{cm} + M\left(\frac{W}{2}\right)^2 + m_B W^2$$

$$\Rightarrow I_F = \frac{1}{12}MW^2 + \frac{1}{4}MW^2 + m_B W^2$$

$$\Rightarrow I_F = \left(\tfrac{1}{3}M + m_B\right)W^2$$

$$L_I = L_F$$

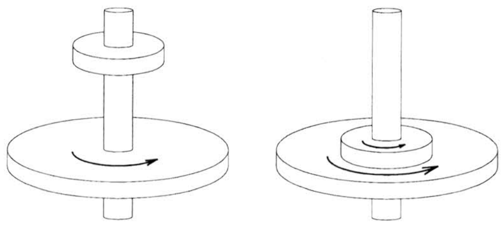
$$W m_B v_B \cos\theta = \left(\tfrac{1}{3}M + m_B\right)W^2 \omega_F$$

$$\boxed{\omega_F = \frac{m_B}{\frac{1}{3}M + m_B}\frac{v_B}{W}\cos\theta}$$

A disk with moment of inertia of I_1 rotates about a vertical, frictionless axle with an angular speed ω_o. A second disk, initially at rest, has a moment of inertia I_2 and is dropped onto the first disk. Because of friction between the two surfaces, the two disks eventually reach the same speed ω_f.

(a) Calculate ω_f.

(b) Show that the kinetic energy of the system decreases in this interaction and calculate the ratio of the final rotational energy to the initial rotational energy.



Conserve angular momentum.

$$L_I = L_F \Rightarrow I_I\omega_I = I_F\omega_F$$

$$I_I = I_1, \quad I_F = I_1 + I_2 \Rightarrow \text{Superposition.}$$

$$I_1\omega_o = (I_1 + I_2)\omega_F \Rightarrow \boxed{\omega_F = \frac{I_1}{I_1 + I_2}\omega_o}$$

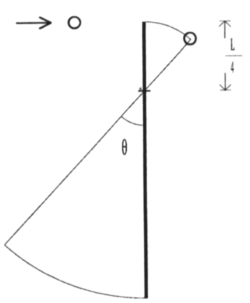
b) $K_I = \frac{1}{2}I_1\omega_o^2, \quad K_F = \frac{1}{2}I_F\omega_F^2$

$$\Rightarrow K_F = \frac{1}{2}\cancel{(I_1+I_2)}\frac{I_1^2}{(I_1+I_2)^{\cancel{2}}}\omega_o^2$$

$$\frac{K_F}{K_I} = \frac{\cancel{\frac{1}{2}}\frac{I_1^{\cancel{2}}}{(I_1+I_2)}\cancel{\omega_o^2}}{\cancel{\frac{1}{2}}\cancel{I_1}\cancel{\omega_o^2}} = \boxed{\frac{I_1}{I_1+I_2} < 1}$$

A chunk of clay of mass $m_{clay} = 0.500$ kg is moving at a velocity of 3.00 m/s when it hits and sticks to a long thin rod that is hung from a point ¼ of the way down from the top. The rod has a mass of $m_{rod} = 2m_{clay} = 1.00$ kg and a length of 1.00 m.

The moment of inertia a long thin rod about its *center of mass* is $I_{cm} = \frac{1}{12} mL^2$.



(a) What is the moment of inertia of the rod–clay combination? Calculate a number that you can use in the remainder of this problem.

(b) Where is the center of mass of the rod-clay combination as measured from the pivot point? Calculate a number that you can use in the remainder of this problem.

(b) What is ω of the resulting pendulum right after the collision?

(d) What is the maximum angle θ that the pendulum makes with the vertical?

a) $I_T = I_{ROD} + I_{clay}$

$$I_{ROD} = I_{CM} + m_{ROD}\left(\frac{L}{4}\right)^2, \quad I_{clay} = m_{clay} \cdot \left(\frac{L}{4}\right)^2$$

$$I_T = \frac{1}{12} m_{ROD} L^2 + \frac{1}{16} m_{ROD} L^2 + \frac{1}{16} m_{clay} L^2$$

$$= \frac{1}{12} 2 m_{clay} L^2 + \frac{1}{16} 2 m_{clay} L^2 + \frac{1}{16} m_{clay} L^2$$

$$= \left(\frac{1}{6} + \frac{1}{8} + \frac{1}{16}\right) m_{clay} L^2$$

$$\boxed{I_T = \frac{17}{48} m_{clay} L^2} = \frac{17}{48}(0.5)(1)^2 = \boxed{0.18 \text{ kg} \cdot \text{m}^2}$$

continued ↓

b) $l_{cm} = \sum m_i l_i \left(\frac{1}{m_T}\right)$

$$\Rightarrow l_{cm} = \left[m_{ROD}\frac{L}{4} - m_{clay}\frac{L}{4}\right]\left(\frac{1}{m_{clay} + m_{ROD}}\right)$$

$$\Rightarrow l_{cm} = \left(2\cancel{m_{clay}}\frac{L}{4} - \cancel{m_{clay}}\frac{L}{4}\right)\frac{1}{3\cancel{m_{clay}}}$$

$$\Rightarrow \boxed{l_{cm} = \frac{1}{12}L} \qquad \Rightarrow \boxed{l_{cm} = \frac{1}{12}}$$

c) Conserve angular momentum

$$L_I = \vec{r} \times m\vec{v} = m_c v \frac{L}{4}$$

$$L_F = I_T \omega_F$$

$$L_I = L_F$$

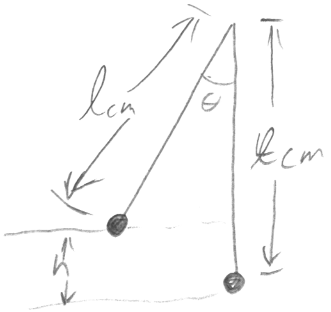
$$\frac{1}{\cancel{4}}\cancel{m_{clay}} v \cancel{L} = \frac{17}{\cancel{48}_{12}}\cancel{m_{clay}} L^2 \omega_F$$

$$\Rightarrow \boxed{\omega_F = \frac{12}{17}\frac{v}{L}} \qquad \Rightarrow \boxed{\omega_F = \frac{12}{17}\frac{3}{1} = 2.1\ \text{rad/s}}$$

continued ↓

d) Use work / Energy, NOT kinematics

We measure gravitational potential with respect to the center of mass.



$h = l_{cm} - l_{cm}\cos\theta$

$U_I = 0$ $\qquad$ $U_F = (m_{clay} + m_{Rod})\,g\,(l_{cm} - l_{cm}\cos\theta)$

$K_I = \frac{1}{2} I_T \omega_F^2$ $\qquad$ $K_F = 0$

$$\frac{1}{2} I_T \omega_F^2 = (m_{clay} + m_{ROD})\,g\,l_{cm}(1-\cos\theta)$$

$$\Rightarrow \frac{1}{2}\left[\frac{17}{48}\cancel{m_{clay}}\cancel{L}^2\right]\left[\frac{12}{17}\frac{V}{\cancel{L}}\right]^2 = (\cancel{m_{clay}} + 2\cancel{m_{clay}})\,g\,\frac{1}{12}L(1-\cos\theta)$$

$$\Rightarrow \frac{1}{\cancel{2}}\frac{\cancel{17}}{\underset{8}{\cancel{48}}}\frac{12^2}{17^{\cancel{2}}}V^2 = \frac{3}{\underset{\cancel{6}}{\cancel{12}}}L(1-\cos\theta)\,g$$

$$\Rightarrow \frac{\cancel{12}\cdot\cancel{12}}{\cancel{3}\cdot\cancel{8}\cdot 17}\cdot\frac{V^2}{gL} = (1-\cos\theta)$$

$$\Rightarrow \boxed{\cos\theta = 1 - \frac{6}{17}\frac{V^2}{gL}} \qquad \theta = \cos^{-1}\left[1 - \frac{6}{17}\frac{3^2}{(9.8)(1)}\right]$$

θ = Some number... :)