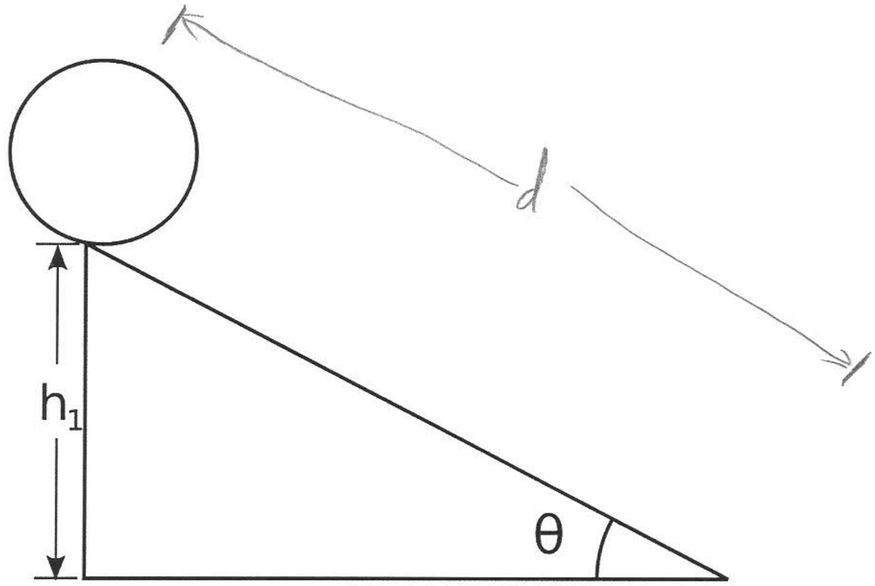
Use **Torque and Kinematics** to solve this problem.

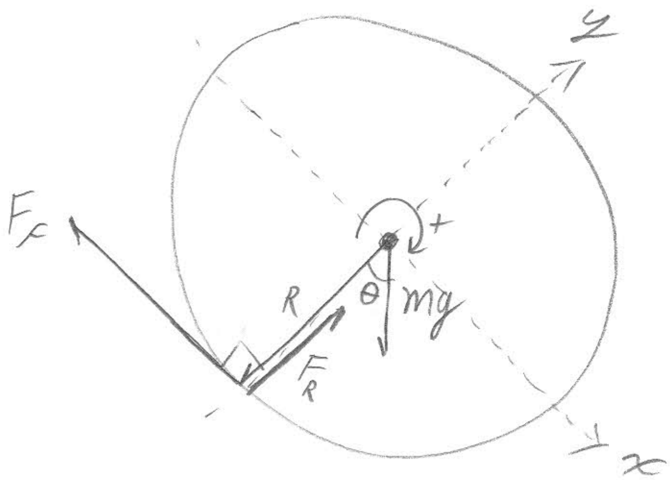
A rolling object with a radius R, mass m, and moment of inertia I, starts from rest at the top of an incline plane of height h that makes an angle θ with the horizontal.

a) Find an expression for the linear and angular acceleration of the object in terms of I.
b) Using kinematics, find an expression for the linear and angular ~~acceleration~~ Velocity of the object in terms of I?
c) Assume that the object is a disk with $I=\frac{1}{2}mR^2$ and plug I into your velocity expressions. Verify that your answers are the as when you solved this problem using energy.



a)

Step 1 – FBD



In this problem, we need to consider both Rotation and translation. So we have positive rotation as well as the x-y coordinates labeled

continued ↓

Rotation set 5, P1 continued

Step (2) - NSL

One object but both translation and rotation

Rotation

$\sum \vec{T} = \vec{I}\alpha$

$\Rightarrow \sum RF\sin\theta = I\alpha$

$\Rightarrow \underbrace{RF_x \sin(90)}_{\sin 90 = 1} + \underbrace{\cancel{RF_R \sin(180)}}_{\sin(180)=0}^{=0} + \underbrace{\cancel{0 \cdot mg \sin\theta}}_{R=0}^{=0} = I\alpha$

$\Rightarrow RF_x = I\alpha \Rightarrow \boxed{F_x = \frac{I}{R}\alpha}$

Translation

x: $\boxed{mg\sin\theta - F_x = ma}$ (2)

y: $F_R - mg\cos\theta = 0 \leftarrow$ not useful

Plug (1) → (2): $mg\sin\theta - \frac{I}{R}\alpha = ma$

Let $a = R\alpha$: $mg\sin\theta - \frac{I}{R^2}a = ma \Rightarrow mg\sin\theta = (m + I/R^2)a$

$$\boxed{a = \frac{m}{m + I/R^2} g\sin\theta} \quad (1)$$

$$\boxed{\alpha = \frac{m}{m + I/R^2}\frac{g}{R}\sin\theta} \quad (2)$$

Rotation set 5, P1 continued

b) kinematics

$x = \cancel{x_0}^{0} + \cancel{v_0 t}^{0} + \frac{1}{2}at^2$ $\qquad$ $v = \cancel{v_0}^{0} + at$

$d = \frac{1}{2}\cancel{a}\frac{v^2}{a^{\cancel{2}}}$ $\qquad$ $t = \frac{v}{a}$

$d = \frac{1}{2}\frac{v^2}{a} \Rightarrow \boxed{v = (2da)^{1/2}}$ 3

Plug in a from eq. 1

$$v = \left[2d\,\frac{m}{m + I/R^2}\,g\sin\theta\right]^{1/2}$$

$$\boxed{v = \left[\frac{m}{m + I/R^2}\,2gd\sin\theta\right]^{1/2} = \left[\frac{m}{m + I/R^2}\,2gh\right]^{1/2}}$$

$$\omega = \frac{v}{R} \Rightarrow \boxed{\omega = \left[\frac{m}{m + I/R^2}\,\frac{2gh}{R^2}\right]^{1/2}}$$

IF $I = \frac{1}{2}mR^2$, $m + I/R^2 \Rightarrow m + \frac{\frac{1}{2}m\cancel{R^2}}{\cancel{R^2}} \Rightarrow \frac{3}{2}m$

so: $\frac{m}{m + I/R^2} \Rightarrow \frac{\cancel{m}}{\frac{3}{2}\cancel{m}} \Rightarrow \underline{2/3}$

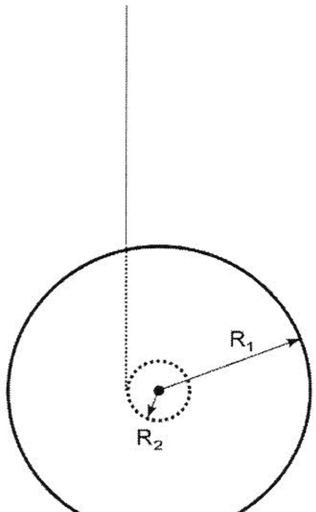
and: $\boxed{v = \left[\frac{4}{3}gh\right]^{1/2}}$ and $\boxed{\omega = \left[\frac{4}{3}\frac{gh}{R^2}\right]^{1/2}}$

4. The picture below represents the side view of a yo-yo. The inner dashed circle represents the axle that the string is wound around. The top of the string is held stationary and the yo-yo is allowed to fall, unwinding the string as it descends.

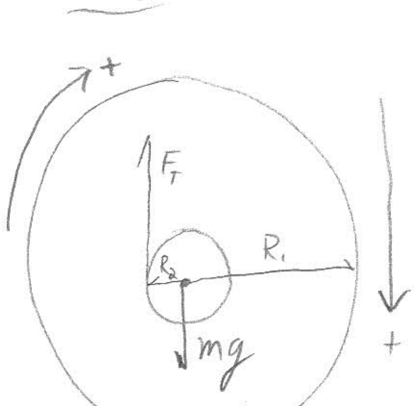
The moment of inertia of the yo-yo is: $I_{cm} = \frac{1}{2} M R_1^2$

Use **Torque/Kinematics** to answer the following question.

If the yo-yo starts from rest, what is its angular velocity after a length of string, *d*, is unwound?



FBD



NSL

$\Sigma F = ma$

$\boxed{mg - F_T = ma}$ ①

$\Sigma T = I\alpha$

$\boxed{F_T R_2 - mg\cancel{(0)} = I\alpha}$ ②

We need the linear acceleration, a, of the yo-yo. Let's eliminate F_T from equations ① and ②, then solve for a:

From ①: $F_T = mg - ma$

into ②: $\boxed{(mg - ma)R_2 = I\alpha}$ ③

Now, α and a are related through $\underline{R_2}$ (not R_1).

$a = R_2 \alpha$ (why? Think about it ...)

So: $(mg - ma) = \frac{I}{R_2}\frac{a}{R_2} \Rightarrow mg = \left(m + \frac{I}{R_2^2}\right) a \Rightarrow \boxed{a = \frac{m}{m + I/R_2^2} g}$

Plug in I: $a = \frac{m}{m + \frac{1}{2} m \frac{R_1^2}{R_2^2}} \Rightarrow a = \frac{1}{1 + \frac{1}{2}\frac{R_1^2}{R_2^2}} g$

continued ↓

Sample Test 5, p4 - continued

$$\boxed{a = \frac{R^2}{\frac{1}{2}R_1^2 + R_2^2}\, g} \quad 4$$

Now, kinematics will provide the <u>linear</u> velocity after traveling a distance <u>d</u>.

$$y = y_0 + V_0 t + \tfrac{1}{2} a t^2 \qquad V = V_0 + at$$

$$d = 0 + 0 + \tfrac{1}{2} a t^2 \qquad V = 0 + at$$

$$\Rightarrow \boxed{t = \frac{V}{a}}$$

$$\Rightarrow d = \tfrac{1}{2}\cancel{a}\frac{V^2}{a^{\cancel{2}}} \Rightarrow \boxed{V = (2da)^{1/2}}$$

Now; plug in a:

$$\boxed{V = \left[2d \frac{R_2^2}{\frac{1}{2}R_1^2 + R_2^2}\, g\right]^{1/2}}$$

And finally, convert to angular velocity: $V = R_2 \omega$

$$\Rightarrow \omega = \frac{1}{\cancel{R_2}}\left[2dg \frac{\cancel{R_2^{2}}}{\frac{1}{2}R_1^2 + R_2^2}\right]^{1/2}$$

$$\boxed{\omega = \left[\frac{2dg}{\frac{1}{2}R_1^2 + R_2^2}\right]^{1/2}}$$

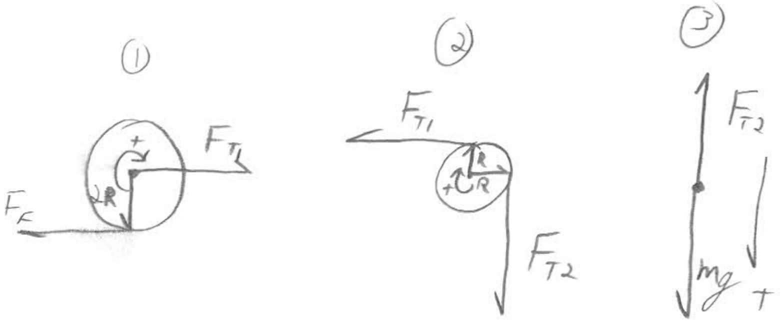
Rotation – Set 5

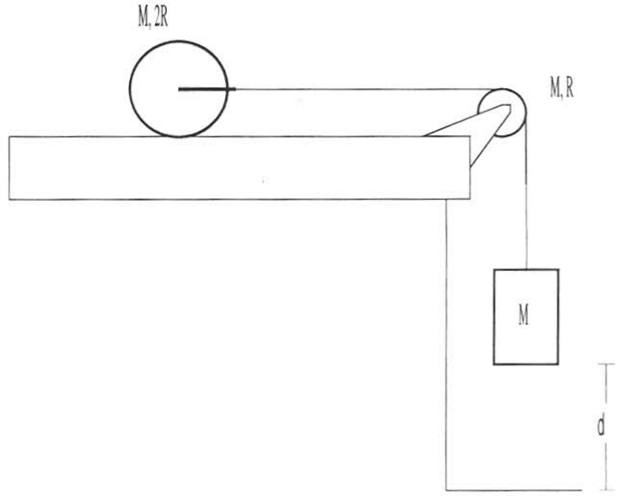
Use **Torque and Newton's Second Law** solve this problem.

A solid cylinder (radius = 2R, mass = M) rolls without slipping as it is pulled by a massless yoke attached to a string. The string goes over a frictionless pulley shaped as a solid disk (radius = R, mass = M) and is attached to a hanging weight (mass = M).

$$I_{cylinder} = \frac{1}{2} MR^2$$

What is the acceleration of the system?





1) Is rotating AND translating

$$\cancel{-2(0)F_{T1}}^{\,0} + \cancel{2}R F_f = \frac{1}{\cancel{2}} m \cancel{4}R^{\cancel{2}} \alpha \rightarrow \text{Torque}$$

$$F_f = mR\alpha$$

$$F_{T1} - F_f = ma \longrightarrow \text{Force}$$

Put these two together

$$F_{T1} - mR\alpha = ma$$

$$\Rightarrow F_{T1} - \frac{1}{2}ma = ma$$

$$\boxed{F_{T1} = \frac{3}{2}ma} \quad (1)$$

continued

↓

Rotation Set 5, P3 continued

② This pulley is only rotating

$$\cancel{R}F_{T2} - \cancel{R}F_{T1} = \tfrac{1}{2}mR^{\cancel{2}}\alpha \qquad a = r\alpha$$

$$\boxed{F_{T2} - F_{T1} = \tfrac{1}{2}ma} \quad ②$$

③ The mass is translating

$$mg - F_{T2} = ma$$

$$\Rightarrow \boxed{F_{T2} = m(g-a)} \quad ③$$

Plug ① and ③ into ②

$$\cancel{m}(g-a) - \tfrac{3}{2}\cancel{m}a = \tfrac{1}{2}\cancel{m}a$$

$$g - a - \tfrac{3}{2}a = \tfrac{1}{2}a$$

$$g = (1 + \tfrac{3}{2} + \tfrac{1}{2})a$$

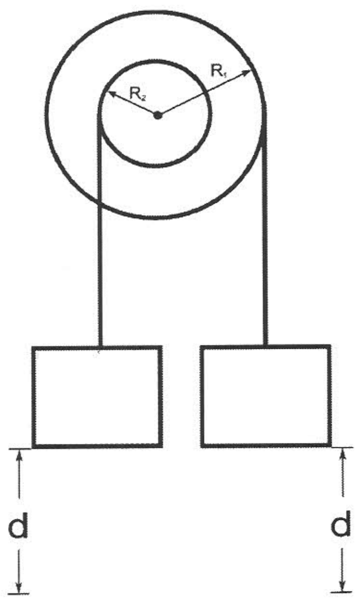
$$\Rightarrow \boxed{a = \tfrac{1}{3}g}$$

Rotation – Set 5

The picture below shows a modified atwood machine composed of a large YoYo mounted on a central axle so that it spins in place. Two blocks of equal mass are attached to the system by ropes. One rope is wound around the inner axle of the YoYo the outer disk. The mass of the YoYo is the same as the mass of the two blocks and $R_2 = \frac{1}{2} R_1$.

Assume that the moment of inertia of the pulley is $I = \frac{1}{2} M R_1^2$

a) If the masses are initially at rest, which way will the pulley rotate, clockwise or counter clockwise?
b) Using **Torque and Kinematics**, find an expression for the angular velocity of the pulleys after the mass attached to the large pulley has moved a distance d.
c) Using **Work/Energy** techniques, find an expression for the angular velocity of the pulleys after the mass attached to the large pulley has moved a distance d.



Given

M - same for all

d

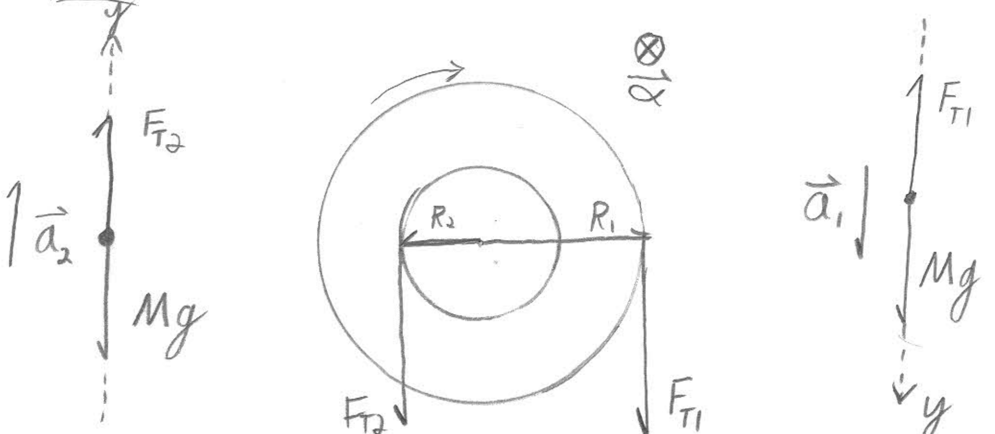
I_p

Want

v_F

a) The torque due to the right most mass will be greater than that due to the left most mass. Therefore, it will rotate CW.

b) FBD



Rotation Set 5, P1 continued.

* NSL

① $\boxed{F_{T2} - Mg = Ma_2}$ ② $\boxed{F_{T1}R_1 - F_{T2}R_2 = I\alpha}$ ③ $\boxed{Mg - F_{T1} = Ma_1}$

Because $R_1 \neq R_2$, $a_1 \neq a_2$. But, we can relate a_1 and a_2 to α:

$$\boxed{a_1 = R_1\alpha, \quad a_2 = R_2\alpha}$$

Then we from ①: $\underline{F_{T2} = Mg + MR_2\alpha}$

and from ③: $\underline{F_{T1} = Mg - MR_1\alpha}$

plug in to ②: $(Mg - MR_1\alpha)R_1 - (Mg + MR_2\alpha)R_2 = I\alpha$

$$\Rightarrow \cancel{M}gR_1 - \cancel{M}gR_2 = \cancel{M}R_1^2\alpha + \cancel{M}R_2^2\alpha + \frac{1}{2}\cancel{M}R_1^2\alpha$$

$$\Rightarrow \boxed{\alpha = \frac{R_1 - R_2}{3/2\,R_1^2 + R_2^2}\,g}$$

* Kinematics

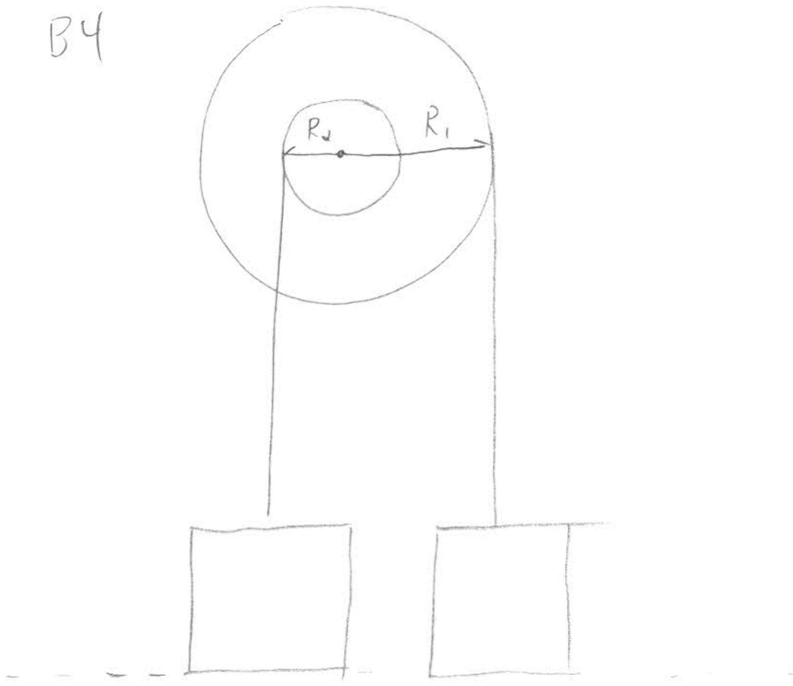
$$\theta = \cancel{\theta_0}^{0} + \cancel{\omega_0 t}^{0} + \frac{1}{2}\alpha t^2 \qquad \omega = \cancel{\omega_0}^{0} + \alpha t \Rightarrow t = \frac{\omega}{\alpha}$$

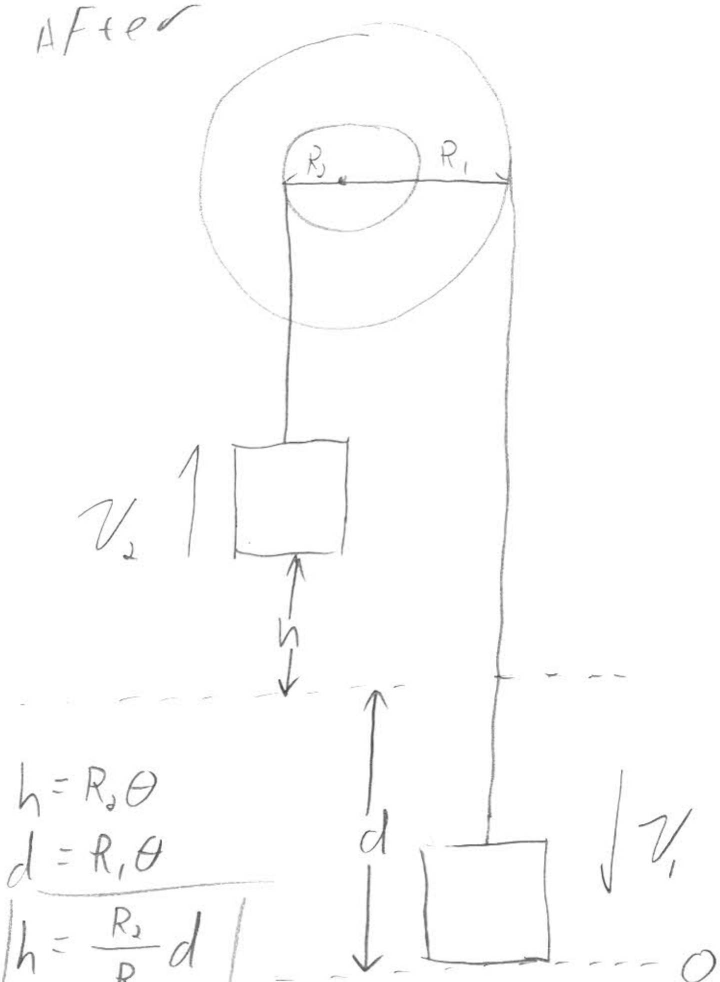
$$\frac{d}{R_1} = \frac{1}{2}\alpha t^2 \Rightarrow \frac{d}{R_1} = \frac{1}{2}\cancel{\alpha}\frac{\omega^2}{\cancel{\alpha^2}} \Rightarrow \omega = \left[\frac{2\alpha d}{R_1}\right]^{1/2}$$

$$\Rightarrow \boxed{\omega = \left[2d\,\frac{R_1 - R_2}{3/2\,R_1^2 + R_2^2}\,\frac{g}{R_1}\right]^{1/2}}$$

continued ↓

c) Work - Energy





$h = R_2\theta$

$d = R_1\theta$

$\Rightarrow \boxed{h = \frac{R_2}{R_1} d}$

$\Rightarrow \underline{v_1 = R_1\omega}, \ \underline{v_2 = R_2\omega}$

$U_I = Mgd + Mgd$

$U_F = Mg(d+h) + 0$

$K_I = 0$

$K_F = \frac{1}{2}Mv_1^2 + \frac{1}{2}Mv_2^2 + \frac{1}{2}I\omega^2$

$$\Rightarrow 2\cancel{M}gd = \cancel{M}gd + \cancel{M}g\frac{R_2}{R_1}d + \frac{1}{2}\cancel{M}R_1^2\omega^2 + \frac{1}{2}\cancel{M}R_2^2\omega^2 + \frac{1}{2}\frac{1}{2}\cancel{M}R_1^2\omega^2$$

continued

↓

Rotation Set 5, P1 continued.

$$\Rightarrow \left(2 - 1 - \frac{R_2}{R_1}\right) g d = \left(\frac{3}{4} R_1^2 + \frac{1}{2} R_2^2\right) \omega^2$$

$$\Rightarrow \left(1 - \frac{R_2}{R_1}\right) g d = \frac{1}{2}\left(\frac{3}{2} R_1^2 + R_2^2\right) \omega^2$$

$$\Rightarrow \left[\frac{R_1 - R_2}{R_1}\right] 2 g d = \left(\frac{3}{2} R_1^2 + R_2^2\right) \omega^2$$

$$\Rightarrow \boxed{\omega = \left[2d \, \frac{R_1 - R_2}{\frac{3}{2} R_1^2 + R_2^2} \, \frac{g}{R_1}\right]^{1/2}}$$

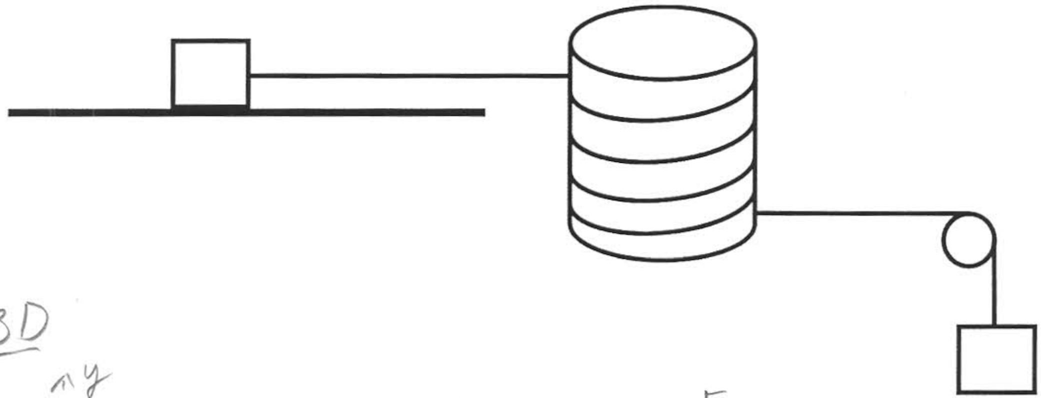
Rotation – Set 5

Use **Torque and Newton's Second Law** solve this problem.

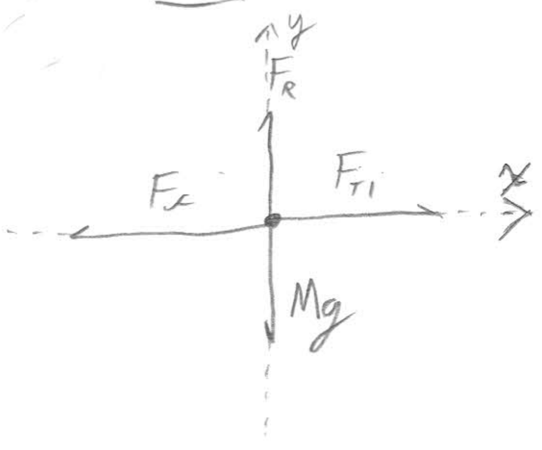
A block of mass M rests on a rough table with $\mu_k = 0.3$. A massless string is attached to the block, wrapped around a solid cylinder having a mass M and a radius R, runs over a massless frictionless pulley, and is attached to a second block of mass M that is hanging freely.

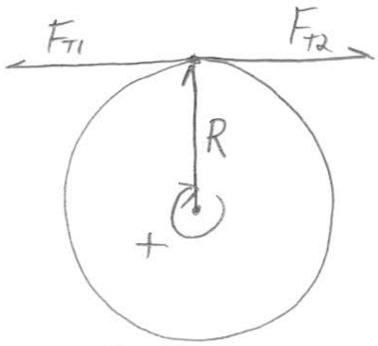
Find the acceleration of this system.

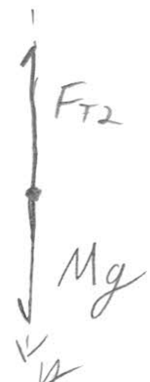
$$I_{cylinder} = \frac{1}{2}MR^2$$



FBD







NSL $\sum \vec{F} = ma$

x: $F_{T1} - F_f = Ma$

y: $F_R - Mg = 0$

$F_R = Mg$

$\Rightarrow \boxed{F_{T1} - \mu_k Mg = Ma}$ (1)

$\sum \vec{\tau} = I\vec{\alpha}$

$\sum RF\sin\theta = I\alpha$

$RF_{T2} - RF_{T1} = \frac{1}{2}MR^2\alpha$

$\boxed{F_{T2} - F_{T1} = \frac{1}{2}MR\alpha}$ (2)

$\sum \vec{F} = m\vec{a}$

$\boxed{-F_{T2} + Mg = Ma}$ (3)

continued ↓

Rotation Set 5, P4 continued

Eliminate F_{T1} and F_{T2}:

From ①: $F_{T1} = \mu_K Mg + Ma$

From ③: $F_{T2} = Mg - Ma$

into ②: $\cancel{M}g - \cancel{M}a - \mu_K \cancel{M}g - \cancel{M}a = \frac{1}{2}\cancel{M}R\alpha$

$g(1-\mu_K) = 2a + \frac{1}{2}R\alpha \Rightarrow g(1-\mu_K) = 2R\alpha + \frac{1}{2}R\alpha$

$\Rightarrow g(1-\mu_K) = \frac{5}{2}R\alpha$

$\Rightarrow \boxed{\alpha = \frac{2}{5(1-\mu_K)}\frac{g}{R}}$

or $\boxed{a = \frac{2}{5(1-\mu_K)}g}$