By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW CLEAR AND NEAT PICTURES SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, EXPLICITLY SHOW THE BASIC EQUATIONS YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

Moment of Inertia, discrete definition

Moment of Inertia, integral definition

Parallel Axis Theorem

Superposition

TABLE 10.2 Rotational Inertias





Thin ring or hollow cylinder about its axis $I = MR^2$





about its axis $I = \frac{1}{2}MR^2$



Solid sphere about diameter $I = \frac{2}{5}MR^2$

Flat plate about central axis

Flat plate about perpendicular axis

 $I = \frac{1}{12}M(a^2 + b^2)$



$$I = \int r^2 dm$$
$$I = I_{m} + Mc$$

2

 $I = \sum m_i r_i^2$

$$I_{T_{otal}} = \sum I_i$$

$$I = I_{cm} + Md$$
$$I = -\sum I$$

$$I = I_{cm} + M d$$

- 1) Derivations
- a) (10pts) Starting with the definition of linear Kinetic energy ($K = \frac{1}{2}mV^2$), show that rotational kinetic energy of a rigid body is $K = \frac{1}{2}I\omega^2$ where $I = \int r^2 dm$.
- b) (10pts) Starting with the definition of angular momentum ($L = m(\vec{r} \times \vec{V})$), show that the angular momentum of a rigid body is $L = I\omega$ where $I = \int r^2 dm$.

- 2) Multiple Choice, 4 points each.
- 2.1) A wad of clay with mass m_c is thrown at a **thin rod** whose length is *L* and whose mass is $2m_c$. The rod is allowed to rotate about a pivot a distance d = L/4 from its center as in the picture below. What is the moment of inertia of the clay, stick combination after the impact?



2.2) A disk, a hoop, a solid sphere, and a hollow sphere, all with the same mass and radius, are having a race down an incline plane. Rank them in the order that they will arrive at the bottom of the ramp, 1 = winner, 4 = loser.

____ Disk

____ Hoop

_____ Solid Sphere

- 2.3) Some children are riding on the outside edge of a merry-go-round. All of the children simultaneously move towards the center. Ignore friction in the rotation of the merry-go-round. When they move:
 - a) the moment of inertia of the system stays constant.
 - b) the angular momentum of the system stays constant.
 - c) the angular velocity of the system stays constant.
 - d) the merry-go-round slows down.

_____ Hollow Sphere

2.4) You are trying to turn a nut with a wrench. The same force is applied in each picture. Rank the pictures by torque, 1 = smallest. If any of the torques are the same, give them the same ranking.



- 2.5) Strings are wound around two identical pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force F. Both pucks start to move on a frictionless surface. Which puck arrives at the finish line first?
 - A) Puck 1
 - B) Puck 2
 - C) They arrive at the same time
 - D) There is not enough information to tell.



3) Mass m_2 is attached to a string that passes over a massless pulley. The other end of the string is attached to the central axle of a cylinder of mass m_1 and radius R. Assuming that $m_2 \gg m_1$, the cylinder rolls without slipping up a slope that makes an angle θ with the horizontal.

Assuming that the system starts from rest, use **Torque and Kinematics**, to find an expression for the velocity, v, of m_2 after it falls a distance h.



Extra Space

4) A uniform rod of mass *m* and length *l* has a point mass of mass *M* attached to its end. The rod is stood vertically on its end with the point mass at the top and allowed to fall over.

Assuming that the end on the ground doesn't slip, use the **Conservation of Energy** to find the angular velocity of the stick/mass combination as the top of the stick hits the ground.

Extra Space

5) A uniform 5-kg disk has an 80-cm diameter and is suspended from its edge so that it can swing freely. A small, dense 1-kg blob of Silly Putty is thrown horizontally at the disk with a speed of 7 m/s, and it sticks to the middle of the disk's edge as shown.



a. What is the angular speed about the point of suspension of the disk-blob combination immediately after the collision?

(For a disk, $I_{CM} = \frac{1}{2} M R^2$.)

[Think carefully about how to calculate *I*_{final}. What distance do you use?]

Extra Space