SAMPLE TEST 5
PHYS 111, FALL 2010, SECTION 1

1) Derivations

a) (10pts) Starting with the definition of linear Kinetic energy ($K=\frac{1}{2}mV^2$), show that rotational kinetic energy of a rigid body is $K=\frac{1}{2}I\omega^2$ where $I=\int r^2 dm$.

b) (10pts) Starting with the definition of angular momentum ($L=m|\vec{r}\times\vec{V}|$), show that the angular momentum of a rigid body is $L=I\omega$ where $I=\int r^2 dm$.

Proofs given in another post.

2) Multiple Choice, 4 points each.

2.1) A wad of clay with mass m_c is thrown at a **thin rod** whose length is L and whose mass is $2m_c$. The rod is allowed to rotate about a pivot a distance $d = L/4$ from its center as in the picture below. What is the moment of inertia of the clay, stick combination after the impact?

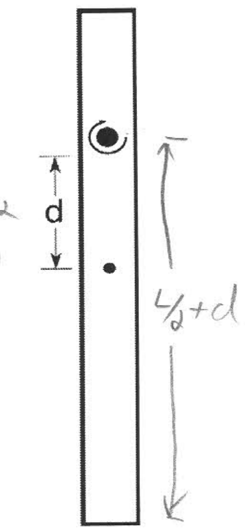
a) $\frac{7}{6}m_c L^2$

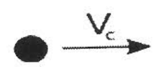
b) $\boxed{\left(\frac{1}{6}+\frac{1}{8}+\frac{9}{16}\right)m_c L^2}$

c) $\left(\frac{1}{12}+\frac{9}{16}\right)m_c L^2$

d) $\pi m_c L^2$

$$I = I_R + I_c$$
$$= \left[I_{cmR} + M_R d^2\right] + m_c\left(\frac{L}{2}+d\right)^2$$
$$= \left[\frac{1}{12}M_R L^2 + M_R\left(\frac{L}{4}\right)^2\right] + m_c\left(\frac{L}{2}+\frac{L}{4}\right)^2$$
$$= \frac{1}{\cancel{12}_{6}}\cancel{2}m_c L^2 + \cancel{2}m_c\frac{1}{\cancel{16}_{8}}L^2 + \frac{9}{16}m_c L^2$$
$$= \left[\frac{1}{6}+\frac{1}{8}+\frac{9}{16}\right]m_c L^2$$





2.2) A disk, a hoop, a solid sphere, and a hollow sphere, all with the same mass and radius, are having a race down an incline plane. Rank them in the order that they will arrive at the bottom of the ramp, 1 = winner, 4 = loser.

2 Disk 1/2

4 Hoop 1

1 Solid Sphere 2/5

3 Hollow Sphere 2/3

Because: $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I}$

τ is constant, α is inverse to I

so, rank in reverse I order

2.3) Some children are riding on the outside edge of a merry-go-round. All of the children simultaneously move towards the center. Ignore friction in the rotation of the merry-go-round. When they move:

a) the moment of inertia of the system stays constant.
b) the angular momentum of the system stays constant.
c) the angular velocity of the system stays constant.
d) the merry-go-round slows down.

No External Torque,

L = Constant

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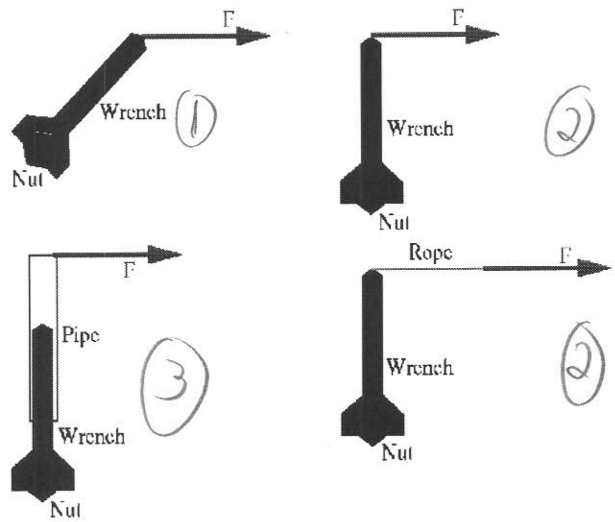
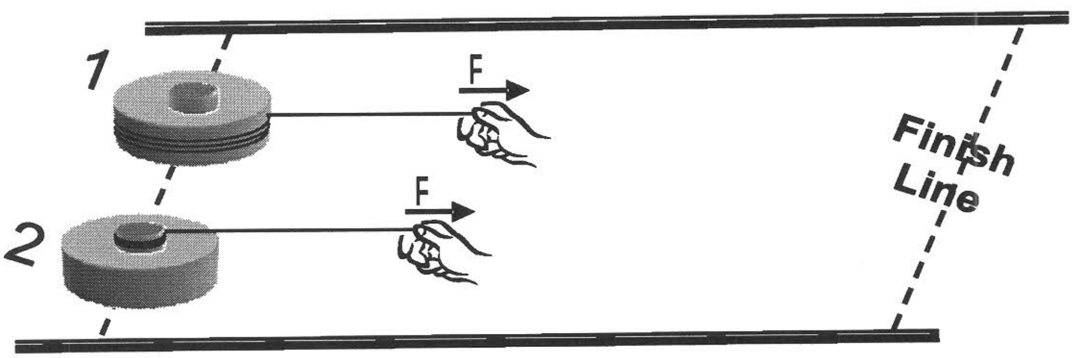
2.4) You are trying to turn a nut with a wrench. The same force is applied in each picture. Rank the pictures by torque, 1 = smallest. If any of the torques are the same, give them the same ranking.



2.5) Strings are wound around two identical pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force F. Both pucks start to move on a frictionless surface. Which puck arrives at the finish line first?

A) Puck 1
B) Puck 2
C) They arrive at the same time
D) There is not enough information to tell.

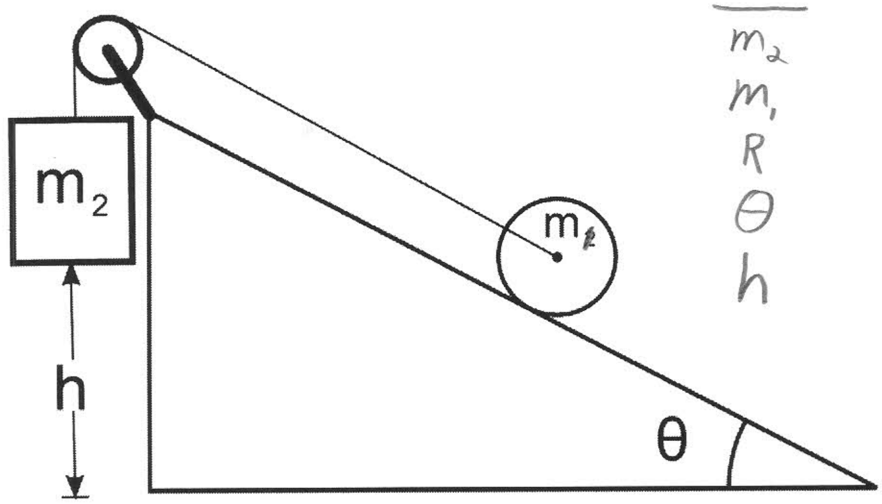


$F_T = ma$, $RF_T = I\alpha$

$\boxed{a} = \frac{F_T}{m}$ a is the same for both

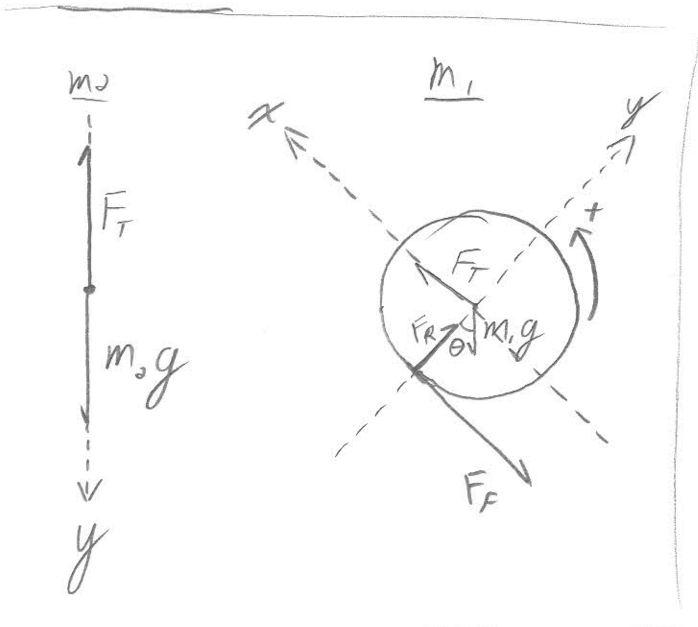
4) Mass m_2 is attached to a string that passes over a massless pulley. The other end of the string is attached to the central axle of a cylinder of mass m_1 and radius R. Assuming that $m_2 >> m_1$, the cylinder rolls without slipping up a slope that makes an angle θ with the horizontal.

Assuming that the system starts from rest, use **Torque and Kinematics**, to find an expression for the velocity, v, of m_2 after it falls a distance h.



Given	Want
m_2	v
m_1	
R	
θ	
h	

FBD



NSL

$m_2: \boxed{m_2 g - F_T = m_2 a}$ ①

$m_1: \boxed{x: F_T - F_F - m_1 g \sin\theta = m_1 a}$ ②

$\boxed{y: F_R - m_1 g \cos\theta = 0}$ ③

$\text{Rot}: \boxed{F_F R = I\alpha}$ ④

continued
↓

Extra Space

* Eliminate F_f between eq ② and ④:

From ④: $F_f = \frac{I}{R}\alpha$

into ② $\boxed{F_T - \frac{I}{R}\alpha - m_1 g \sin\theta = m_1 a}$ ⑤

* Solve ① for F_T: $F_T = m_2 g - m_2 a$

into ⑤ $\boxed{m_2 g - m_2 a - \frac{I}{R}\alpha - m_1 g \sin\theta = m_1 a}$ ⑥

* Fix α using $a = R\alpha$ and solve ⑥ for a

$$\Rightarrow m_2 g - m_2 a - \frac{I}{R^2} a - m_1 g \sin\theta = m_1 a$$

$$\Rightarrow (m_2 - m_1 \sin\theta) g = (m_1 + m_2 + I/R^2)\, a$$

$$\Rightarrow \boxed{a = \frac{m_2 - m_1 \sin\theta}{m_1 + m_2 + I/R^2}\, g}$$

Kinematics

$$y = \cancel{y_0}^{\,0} + \cancel{v_{0y}}^{\,0} + \tfrac{1}{2} a_y t^2 \qquad v = \cancel{v_0}^{\,0} + a_y t$$

$$h = \tfrac{1}{2} a_y t^2 \qquad t = \frac{v}{a_y}$$

$$\boxed{v = \sqrt{2ha}} \Rightarrow \boxed{v = \left[2h \frac{m_2 - m_1 \sin\theta}{m_1 + m_2 + I/R^2}\, g\right]^{1/2}}$$

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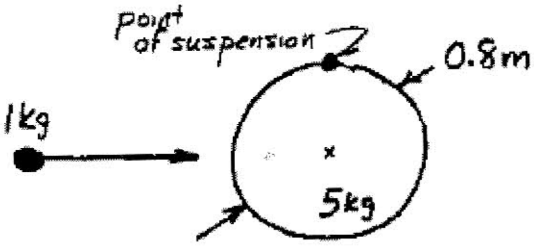
5) A uniform 5-kg disk has an 80-cm diameter and is suspended from its edge so that it can swing freely. A small, dense 1-kg blob of Silly Putty is thrown horizontally at the disk with a speed of 7 m/s, and it sticks to the middle of the disk's edge as shown.

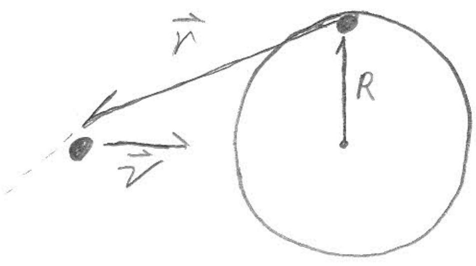


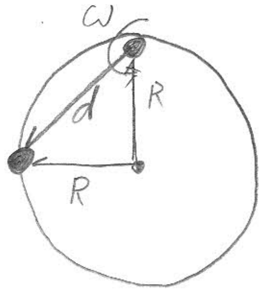
a. What is the angular speed about the point of suspension of the disk-blob combination immediately after the collision?
(For a disk, $I_{CM} = \frac{1}{2}MR^2$.)

Conservation of Angular Momentum

B4 - not rigid body

After - Rigid Body





$d = \sqrt{2}\, R$

$$L_I = L_F$$

$$m(\vec{r} \times \vec{v}) = I\omega$$

$$mrv\sin\theta = I\omega$$

But, $r\sin\theta = R$

so: $\boxed{mvR = I\omega}$

$$\Rightarrow \omega = \frac{mvR}{\left(\frac{3}{2}M + 2m\right)R^{\cancel{2}}}$$

$$\boxed{\omega = \frac{m}{\frac{3}{2}M + 2m}\,\frac{v}{R}}$$

Find I:

$$I = I_{Disk} + I_{clay}$$

$$I = \left[I_{cm,Disk} + MR^2\right] + md^2$$

$$I = \left[\tfrac{1}{2}MR^2 + MR^2\right] + m2R^2$$

$$I = \left(\tfrac{3}{2}M + 2m\right)R^2$$