Systems of Particles – Set 2

Billy and Sally are wearing ice skates standing on the ice at rest facing each other. They get into a terrible argument and Sally shoves Billy. Sally has a mass of 65 kg and Billy has a mass of 80 kg.

1) What is the acceleration of the center of mass before the push, during the push, and after the push? Explain why.

$\sum \vec{F}_{ext} = 0$, so $a_{cm} = 0$ Before, during, and after the push

During the push, Internal forces act on the system.

2) What is the velocity of the center of mass before the push, during the push, and after the push? Explain why.

$\sum \vec{F}_{ext} = 0$, so $\frac{d\vec{V}_{cm}}{dt} = 0$ and $\vec{V}_{cm} = $ constant.

Before the push, $\vec{V}_{cm} = 0$ and remains that way during and after.

3) If, after the after the push, Sally's velocity is 2 m/s, what is Billy's velocity?

$\vec{P}_I = \vec{P}_F$, $\vec{P}_I = 0$, $\vec{P}_F = m_B \cdot \vec{V}_B + m_s \vec{V}_s$

$0 = m_B \cdot V_B + m_s V_s \Rightarrow m_B V_B = m_s V_s \Rightarrow \boxed{V_B = \frac{-m_s}{m_B} V_s}$

$V_B = \frac{-65}{80} \cdot 2 = \boxed{-1.6 \text{ m/s}}$

4) Some time after the push, Billy is -10m from where the push occurred. Where is Sally at that time? **Do NOT use kinematics.**

They are initially at rest so: $\vec{V}_{cm} = 0$ which means $\vec{r}_{cm} = $ constant

$$\vec{r}_{cmI} = \vec{r}_{cmF} \Rightarrow \frac{1}{\cancel{M}}(m_B \vec{r}_{BI} + m_s \vec{r}_{SI}) = \frac{1}{\cancel{M}}(m_B \vec{r}_{BF} + m_s \vec{r}_{SF})$$

Let $\vec{r}_{BI} = \vec{r}_{SI} = 0$ Then:

$$0 = +m_B r_{BF} + m_s r_{SF} \Rightarrow \boxed{r_s = \frac{-m_B}{m_s} r_B} \quad \boxed{r_s = \frac{80}{65}(10) = 12 \text{ m}}$$

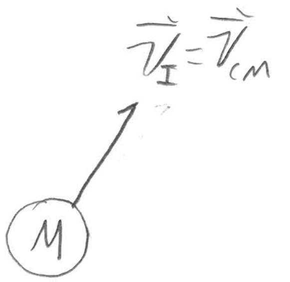
Systems of Particles – Set 2

A 4.0 kg puck is sliding along a frictionless surface when it explodes into two parts, one moving 30 m/s due north and the other at 5.0 m/s 30o north of east. What was the original velocity (x and y components) of the puck?

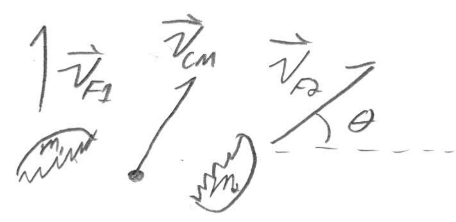
$\sum \vec{F}_{ext} = 0$ But $\vec{v}_{CMI} \neq 0$ and $\vec{P}_T \neq 0$

Two views of this problem:

Momentum is conserved or $\vec{v}_{cm}$ is invariant.



Initial



Final

conserve momentum: $(m_1 + m_2)\vec{v}_I = m_1\vec{v}_{1F} + m_2\vec{v}_{2F}$

Invariant v_{cm}:

$$\vec{v}_{CMI} = \vec{v}_{CMF}$$

$$\frac{1}{\cancel{M}}(m_1\vec{v}_{1I} + m_2\vec{v}_{2I}) = \frac{1}{\cancel{M}}(m_1\vec{v}_{1F} + m_2\vec{v}_{2F})$$

$$(m_1 + m_2)\vec{v}_I = m_1\vec{v}_{1F} + m_2\vec{v}_{2F}$$

Same thing...

Continued ↓

Systems of particles Set 2, P2

2D so break into x and y

x: $(m_1 + m_2) v_{Ix} = \cancel{m_1 v_{1Fx}}^{0} + m_2 v_{2Fx}$

$$\Rightarrow \boxed{v_{Ix} = \frac{m_2}{m_1 + m_2} v_{2Fx}} \Rightarrow v_{Ix} = \frac{\cancel{m}}{2\cancel{m}} v_{2Fx} = \tfrac{1}{2} v_{2Fx}$$

$v_{Ix} = \tfrac{1}{2} 5 \cos 30 = \boxed{2.1 \text{ m/s}}$

y: $\cancel{(m_1 + m_2)}^{4} v_{Iy} = \cancel{m_1}^{2} v_{1Fx} + \cancel{m_2}^{2} v_{2Fx}$

$$\boxed{v_{Iy} = \frac{\cancel{m}}{2\cancel{m}} (v_{1Fx} + v_{2F} \sin(\theta))}$$

$v_{Iy} = (30 + 5 \sin(30)) \tfrac{1}{2}$

$\cancel{\boxed{v_{Iy} = 32.5 \text{ m/s}}}$

$v_{Iy} = 16.25 \text{ m/s}$

Systems of Particles – Set 2

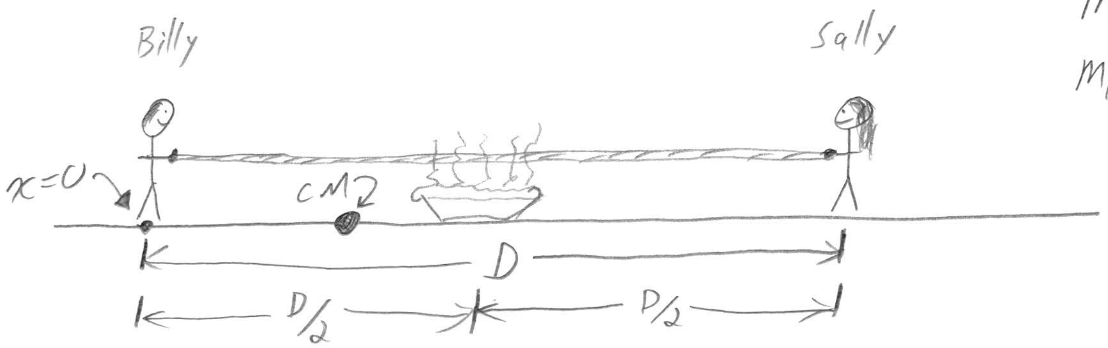
Billy and Sally are once again standing on the ice wearing ice skates (standing on a frictionless surface) initially at rest. They are holding the opposite ends of a rope that is stretched out between them. Placed exactly halfway between them is a delicious steaming hot apple pie and they both want it. They pull on the rope and begin moving towards the pie (and each other). Sally has a mass of 65kg and Billy has a mass of 80kg.

Who gets to the pie first? How far away from the pie is the loser when the winner gets there?

HINT: Question: How does the *position* of the center of mass change as they move?
Answer: It doesn't.

$M_S = 65\,kg$

$M_B = 80\,kg$



The Force exerted by the tension in the rope is internal, and $\sum F_{ext} = 0$.

So, $a_{cm} = 0$ and because they were initially at rest, $V_{cm} = 0$, and therefore $r_{cm} = \text{Const.}$

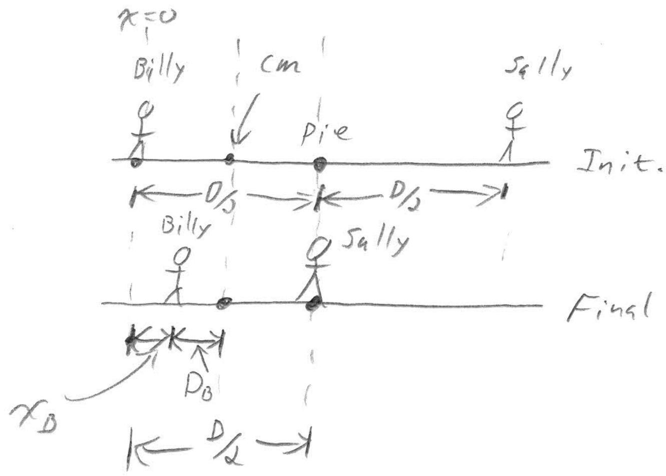
Billy is heavier, so the CM is closer to him. When they pull on the rope, they must meet at the CM.

Sally will get the pie first.

continued ↓

Systems of particles, set 2 P6 continued

Because x_{cm} is constant, we can set up



$$x_{cmI} = x_{cmF}$$

$$\frac{m_B \cancel{0}^{0} + m_S D}{\cancel{(m_B + m_S)}} = \frac{m_B x_B + m_S \frac{D}{2}}{\cancel{(m_B + m_S)}}$$

$$m_S D = m_B x_B + m_S \frac{D}{2}$$

$$m_B x_B = \frac{1}{2} m_S D$$

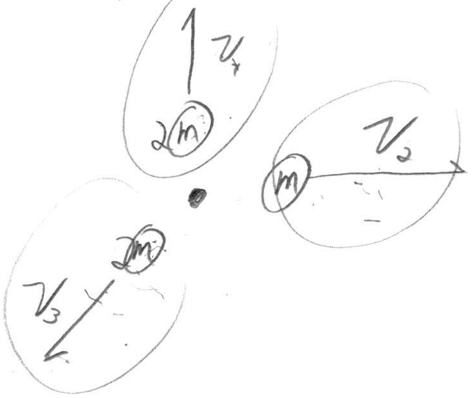
$$\boxed{x_B = \frac{1}{2} \frac{m_S}{m_B} D}$$

But, distance between Billy and the pie is:

$$D_B = D/2 - x_B = \frac{D}{2} - \frac{1}{2}\frac{m_S}{m_B} D = \boxed{\frac{D}{2}\left(1 - \frac{m_S}{m_B}\right)}$$

An object with a mass of $5m_o$ explodes at rest breaking into three pieces. One of the pieces with a mass of m_o travels in the x direction at 30.0 m/s. Another piece also with a mass of $2m_o$ travels in the y direction at 20.0 m/s. What is the magnitude and direction of the velocity of the last piece? What is the kinetic energy released in the explosion?

$\sum \vec{F}_{ext} = 0$, so $\vec{P}_I = \vec{P}_F = 0$



$P_{Ix} = P_{Fx}$

x: $0 = \cancel{2m v_{1x}}^{\,0} + \cancel{m} v_{2x} + \cancel{2m} v_{3x}$

$v_{3x} = -v_{2x} \cdot \frac{1}{2}$

$\boxed{v_{3x} = -15\ m/s}$

$P_{Iy} = P_{Fy}$

y: $0 = \cancel{2m} v_{1y} + \cancel{m v_{2y}}^{\,0} + \cancel{2m} v_{3y}$

$\boxed{v_{3y} = -v_{1y}}$

$\boxed{v_{3y} = -20\ m/s}$

Systems of Particles – Set 2

A spring loaded ball is dropped from 10m. After falling 2m, the spring sproings and the ball splits into two pieces, one with a mass m the other with a mass 2m. The spring acts only in the horizontal direction.

a) What is the velocity (both x and y components) of the center of mass as the pieces hit the ground?

$a_{cmx} = 0, \ a_{cmy} = g, \quad U_I = Mgh, \ U_F = 0, \ K_I = 0, \ K_F = \frac{1}{2}Mv^2$

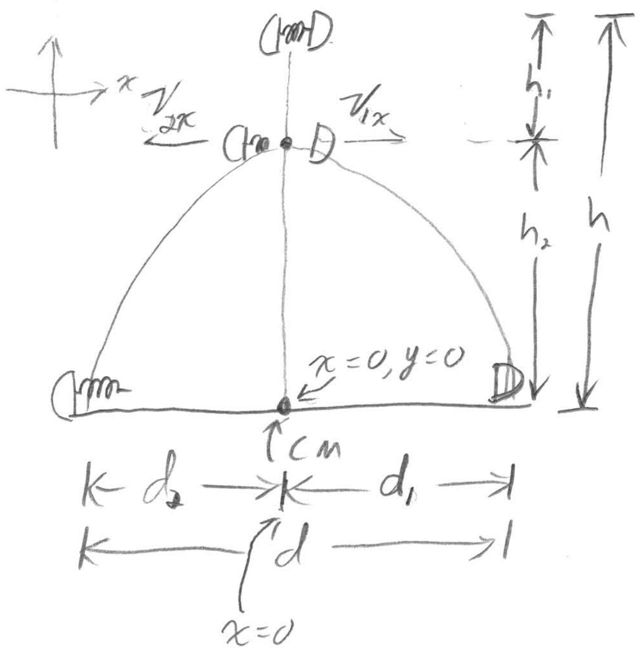
$$V_{cm} = \pm\sqrt{2gh} = ((2)(9.8)(10))^{1/2} = -14\ m/s$$

b) How long does it take the center of mass to get to the ground?

Kinematics: $V_{cm} = V_0 + a_{cmy}t \Rightarrow V_{cm} = 0 - gt \Rightarrow t = -\frac{V_{cm}}{g}$

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}}$$ and if $h = 10$, $t = 1.4s$

c) If the x component of the velocity of the lighter half is 4 m/s after the spring sproings, how far apart are the two halves when they hit the ground? (assume that the spring sproings VERY quickly)



$h_1 = 2$ meters

We can find the velocities of each piece at the moment of sproinging by conserving momentum in the x.

$P_{Ix} = P_{Fx}$

$0 = m_1V_{1x} + m_2V_{2x}$

$0 = m\,4\,m/s + 2m\,V_{2x}$

$\boxed{V_{2x} = -2\,m/s}$

continued. ↓

Systems of Particles Set 2, P3 continued

Now, I know V_{2x} and V_{1x} at the moment of sproing.
If I knew the time to get from $y=h_2$ to $y=0$,
I could find d:

$$d = d_1 + d_2 = \boxed{V_{1x} t_2 + V_{2x} t_2}$$, But I need t

I know how long to get from $y=h$ to $y=0$
I also can calculate how long to get from $y=h$ to $y=h_2$

In general (from part b): $t = \sqrt{\frac{2h}{g}}$ where h is the distance from rest.

so: $t_1 = \sqrt{\frac{2h_1}{g}} = \left(\frac{2 \cdot 2}{9.8}\right)^{1/2} = 0.64 s$

Now subtract: $t_2 = t - t_1 \Rightarrow \boxed{t_2 = 1.4 - 0.64 = 0.76 s}$

And finally, $d = (4 m/s + 2 m/s)(0.76 s) = \boxed{4.56 m}$

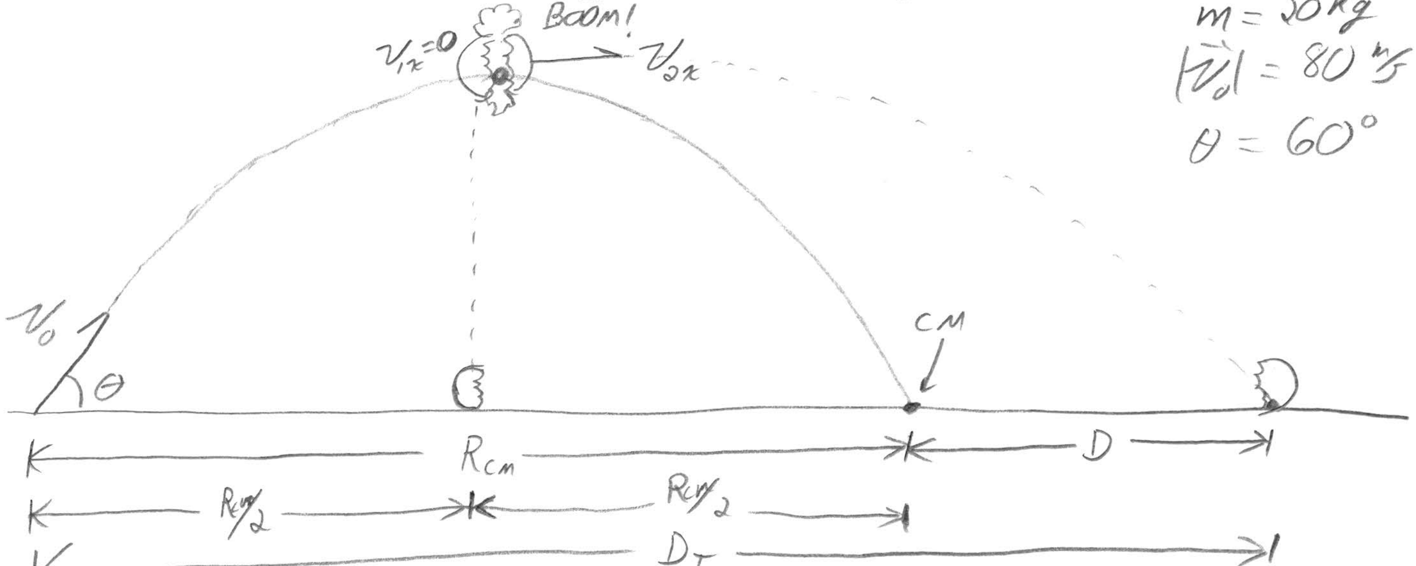
Systems of Particles – Set 2

A 20.0 kg projectile is fired at an angle of 60.0 degrees above the horizontal with a speed of 80.0 m/s. At the highest point of its trajectory, it explodes into two fragments with equal mass. The force of the explosion acts purely in the horizontal direction. One chunk falls vertically to the ground. Where does the second chunk land? How much energy was released in the explosion?

$m = 20\,kg$

$|\vec{v}_0| = 80\ m/s$

$\theta = 60°$



At the peak of the trajectory:

- $x_{cm} = R/2$, where R is the total distance that the center of mass will travel
- $v_{ycm} = 0$, All velocity is in the x.
- Since $a_x = 0$, $v_{xcm} = v_{0x}$ at the peak.
- Because m_1 falls vertically, $v_{1x} = 0$

* We can find v_{2x} at the peak by conserving momentum.

$$P_{Ix} = P_{Fx}$$

$$\cancel{m} v_{0x} = \cancel{\tfrac{m}{2} v_{1x}}^{\,0} + \tfrac{\cancel{m}}{2} v_{2x} \Rightarrow \boxed{v_{2x} = 2 v_{0x}}$$

continued

↓

Systems of Particles Set 2, P5 continued

Let's solve the trajectory problem for <u>R_{cm}</u>.

using kinematics:

$x_{cm} = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$

$R_{cm} = 0 + V_{0x}t + 0$

$R_{cm} = V_{0x}\frac{2V_{0y}}{g}$

$$\boxed{R_{cm} = \frac{2V_{0x}V_{0y}}{g}}$$

Distance to CM

$y_{cm} = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$

$0 = 0 + V_{0y}\cancel{t} - \frac{1}{2}gt^{\cancel{2}}$

$\Rightarrow V_{0y} = \frac{1}{2}gt$

$$\Rightarrow \boxed{t_T = \frac{2V_{0y}}{g}}$$

Total time of flight

Now: Because the explosion was in x <u>only</u>, Both particles fall from Rest in the y.

Therefore, Both pieces <u>and</u> the CM Hit the ground at the <u>same</u> time.

Particle 2 travels a horizontal distance <u>$\frac{R_{cm}}{2} + D$</u> in a time <u>$t_T/2$</u> at a V_{2x}.

$$x = V_x t$$

$$\frac{R_{cm}}{2} + D = V_{2x}\frac{t_T}{2} \Rightarrow \frac{R_{cm}}{2} + D = 2V_{0x}\frac{1}{\cancel{2}}\frac{\cancel{2}V_{0y}}{g}$$

$$\Rightarrow \frac{R_{cm}}{2} + D = \frac{2V_{0x}V_{0y}}{g}$$ → But this is just R_{cm}!

$$\Rightarrow D = R_{cm} - \frac{R_{cm}}{2} \Rightarrow \boxed{D = \frac{1}{2}R_{cm}}$$

continued ↓

Systems of Particles, P5 continued.

So! The total distance particle 2 travels is $\frac{3}{2} R_{cm}$.

$$D_T = \frac{3}{2} R_{cm} = \frac{3}{\cancel{2}} \frac{\cancel{2} V_{ox} V_{oy}}{g} = \frac{3 V_{ox} V_{oy}}{g}$$

$$D_T = \frac{3(V_0 \cos\theta)(V_0 \sin\theta)}{g} = \boxed{\frac{3V_0^2}{g} \sin\theta\cos\theta} =$$

$$D_T = \frac{3(80)^2}{9.8} \sin(60)\cos(60) = \boxed{848\,m}$$

Energy?

Just before the explosion, $K_I = \frac{1}{2} m V_{ox}^2$

Just after the explosion, $K_F = \cancel{\frac{1}{2}\frac{m}{2} V_{1x}^2}^{\,0} + \frac{1}{2}\frac{m}{2} V_{2x}^2$

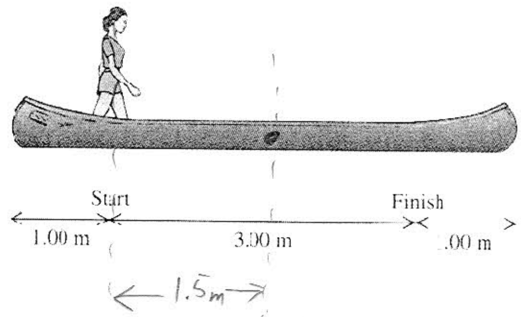
$$E = K_F - K_I = \frac{m}{4} V_{2x}^2 - \frac{m}{2} V_{ox}^2$$

$$= \frac{m}{4}(2V_{ox})^2 - \frac{m}{2} V_{ox}^2$$

$$= m V_{ox}^2 - \frac{1}{2} m V_{ox}^2 = \boxed{\frac{1}{2} m V_{ox}^2}$$

$$E = \frac{1}{2}(20)(80\cos\theta)^2 = \underline{1.6 \times 10^4 \, J}$$

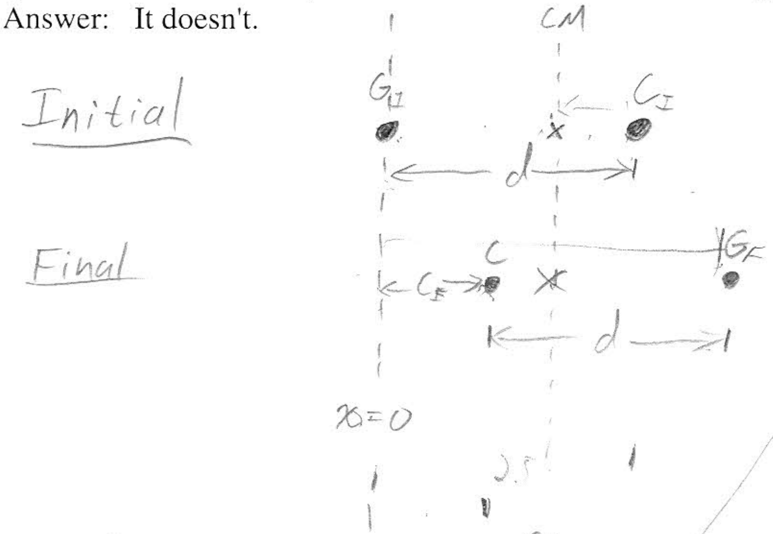
A 45 kg woman stands up in a 60 kg canoe of length 5.0 m. She walks from a point 1 m from one end to a point one meter from the other end. Ignoring resistance due to the water, how far does the canoe move?



HINT: Consider the canoe as a point mass at it's center of mass.

Question: How does the *position* of the center of mass change as they move?

Answer: It doesn't.



CM does not move

$C_I = d$, $C_F = ?$

$\boxed{G_F = C_F + d}$

$$x_{CMI} = x_{CMF}$$

$$\frac{\cancel{m_G G_I}^{\,0} + m_c C_I}{\cancel{m_G + m_c}} = \frac{m_G G_F + m_c C_F}{\cancel{m_G + m_c}}$$

$$m_c d = m_G(C_F + d) + m_c C_F$$

$$m_c d = m_G C_F + m_G d + m_c C_F$$

$$d(m_c - m_G) = (m_G + m_c) C_F$$

Continued ↓

Systems of Particles, Set 2 P 7 continued

$$C_F = \frac{m_c - m_G}{m_c + m_G} d$$

total canoe movement:

$$C_T = C_F - C_I = \frac{m_c - m_G}{m_c + m_G} d - d$$

$$= d\left[\frac{m_c - m_G}{m_c + m_G} - 1\right]$$

$$= d\left[\frac{\cancel{m_c} - m_G - \cancel{m_c} - m_G}{m_c + m_G}\right]$$

$$\boxed{C_T = -d\frac{2 m_G}{m_c + m_G}} = -(1.5)\frac{(2)(45)}{(45+60)} = \boxed{-1.3\ m}$$