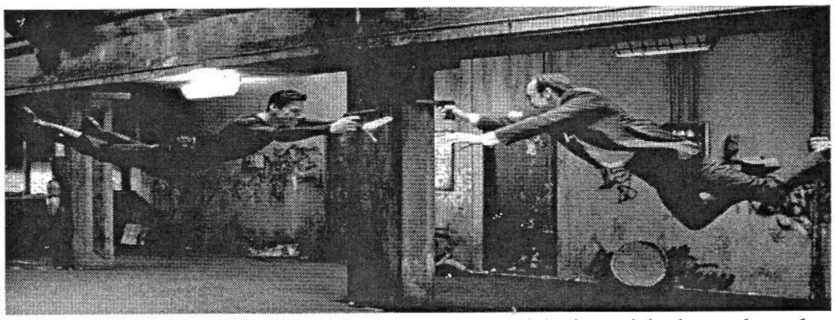
Systems of Particles – Set 4



Neo and Agent Smith are flying towards each other. They collide in mid air and grab onto each other (they stick together).

a) Assume that momentum is conserved in the Matrix and find an expression relating their initial velocities to their final velocity.

m_N v_N v_S m_S $(m_N + m_S)$ v_F

$$m_N v_N - m_S v_S = (m_N + m_S) v_F$$

$$\Rightarrow \boxed{v_F = \frac{m_N v_N - m_S v_S}{m_N + m_S}}$$

b) Let M_N = 70 kg, V_{NI} = 50 m/s, M_S = 100 kg, and V_{SI} = 35 m/s. Put these numbers into your expression and solve for their final velocity.

$$v_F = \frac{(70\,kg)(50\,m/s) - (100\,kg)(35\,m/s)}{(70\,kg + 100\,kg)} = \underline{0}$$

c) Calculate the pre-collision and post-collision kinetic energy of the system. Does this system conserve kinetic energy through the collision?

Pre

$K_I = K_N + K_S$

$K_I = \frac{1}{2} m_N v_N^2 + \frac{1}{2} m_S v_S^2$

= a positive number

Post

$K_F = \frac{1}{2}(m_N + m_S) v_F^2$

$K_F = 0$

K is not conserved

Systems of Particles – Set 3

A 4000 kg railroad car collides and sticks to a chain of three other 4000 kg cars initially sitting at rest on a rough track. The four cars travel together down the rough track for 1.5 m before they stop. Assuming $\mu_k = 0.10$, what is the velocity of the first car at impact?

Answer these important questions before "solving" this problem:

Does the train car conserve momentum throughout the entire problem? Why not?

No. Friction is an external force. $\Rightarrow \Sigma F_{ext} \neq 0$

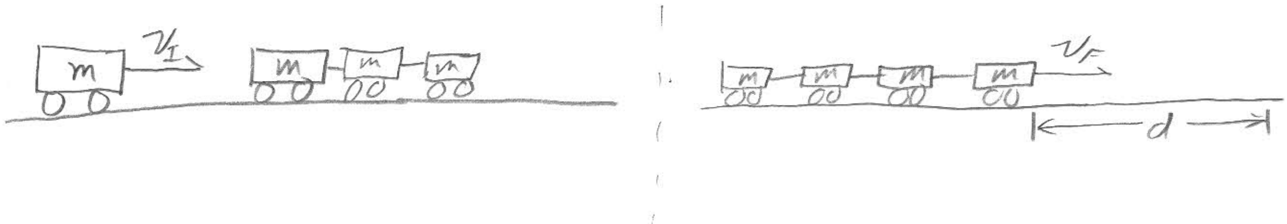
Is there a sub-problem where conservation of momentum can be applied? What is it?

Yes! The inelastic collision.

What other physics principal are you going to use to solve the problem?

Conservation of Energy.

Draw a picture (or pictures) showing the action.



Do the math and solve the problem.

(1) Collide

$\vec{P}_I = \vec{P}_F$

$mV_I = 4mV_F$

$V_I = 4V_F$

(2) Conserve Energy

$K_I = \frac{1}{2}4mV_F^2 \quad K_F = 0, \quad W_F = -\mu_k 4mgd$

$\frac{1}{2}4mV_F^2 = \mu_k 4mgd$

$V_F = (2\mu_k gd)^{1/2}$

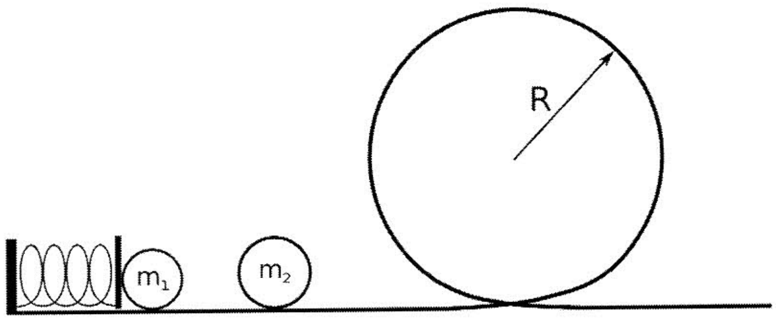
$$\boxed{V_I = (32\mu_k gd)^{1/2}}$$

$$V_I = (32(0.1)(9.8)(1.5))^{1/2} = \boxed{6.9\ m}$$

Sample Test 4
Phys 111 Spring 2011

3) In the system below, a ball of mass m_1 is placed against a spring with spring constant k that has been compressed a distance d. It is released from rest and collides with a second ball of mass m_2 which then goes around the loop the loop of radius R.

Find an expression for the minimum spring compression d in terms of m_1, m_2, k, R, and g such that m_2 makes it around the loop.



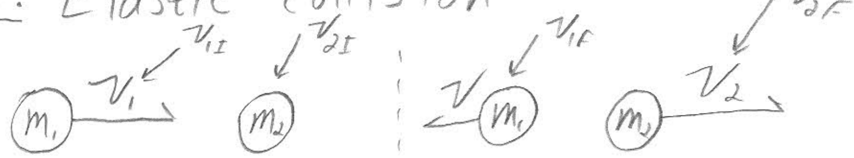
Stage 1: Spring releases

$U_I = \frac{1}{2}kd^2$ $\qquad$ $U_F = 0$

$K_I = 0$ $\qquad$ $K_F = \frac{1}{2}m_1 v_1^2$

$$\cancel{\tfrac{1}{2}} kd^2 = \cancel{\tfrac{1}{2}} m_1 v_1^2 \Rightarrow \boxed{d = \sqrt{\frac{m_1}{k}}\, v_1} \quad (1)$$

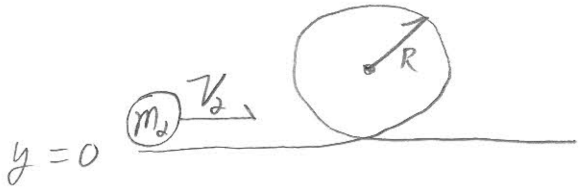
Stage 2: Elastic collision



$$v_{2F} = \cancel{\frac{m_2 - m_1}{m_1 + m_2} v_{2I}}^{\,0} + \frac{2m_1}{m_1 + m_2} v_{1I}$$

$$\Rightarrow v_2 = \frac{2m_1}{m_1 + m_2} v_1 \Rightarrow \boxed{v_1 = \frac{m_1 + m_2}{2m_1} v_2} \quad (2)$$

Stage 3: Loop the Loop



$U_I = 0$

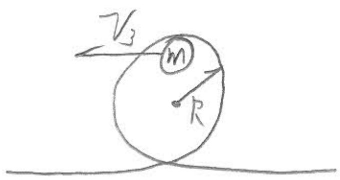
$K_I = \frac{1}{2} m_2 v_2^2$

$U_F = m_2 g (2R)$

$K_F = \frac{1}{2} m_2 v_3^2$

$$\frac{1}{2} m_2 v_2^2 = 2 m_2 g R + \frac{1}{2} m_2 v_3^2$$

But, what is v_3 so that m_2 loops?



FBD

NSC

$$\sum F = ma$$

uniform circular motion

$$\Rightarrow m_2 g + \cancel{N}^{\,0} = m_2 \frac{v_3^2}{R}$$

minimum speed ... $N \to 0$

$$\Rightarrow \cancel{m_2} g = \cancel{m_2} \frac{v_3^2}{R}$$

$$\Rightarrow \boxed{v_3 = \sqrt{gR}}$$

So: $\frac{1}{2} \cancel{m_2} v_2^2 = 2 \cancel{m_2} g R + \frac{1}{2} \cancel{m_2} g R$

$$\Rightarrow \boxed{v_2 = \sqrt{5gR}}$$ 3

continued ↓

Sample Test 4, P3 continued

Put it all together

From ①: $d = \sqrt{\frac{m_1}{k}}\, v_1$

plug in ②: $d = \sqrt{\frac{m_1}{k}}\, \frac{m_1 + m_2}{2m_1}\, v_2$

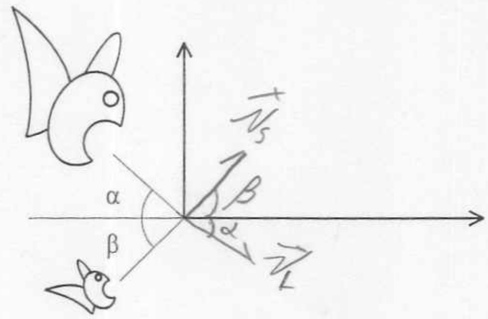
plug in ③: $d = \sqrt{\frac{m_1}{k}}\, \frac{m_1 + m_2}{2m_1} \sqrt{5gR}$

$$\Rightarrow \boxed{d = \sqrt{\frac{5gR}{k}} \cdot \frac{m_1 + m_2}{2\sqrt{m_1}}}$$

1. A large fish will soon make a dish of a smaller fish. What is the velocity of the large fish and his dinner immediately after he eats? Give both the magnitude and direction of the final velocity with respect to the x-axis.

$m_{\text{large fish}} = 4.0$ kg
$v_{o\,\text{large fish}} = 1.0$ m/s
$\alpha_{\text{large fish}} = 25.0°$

$m_{\text{small fish}} = 0.20$ kg
$v_{o\,\text{small fish}} = 5.0$ m/s
$\beta_{\text{small fish}} = 50.0°$



Conserve momentum in both axis

$$x: m_L v_L \cos\alpha + m_s v_s \cos\beta = (m_L+m_s) v_F \cos\theta$$

$$y: -m_L v_L \sin\alpha + m_s v_s \sin\beta = (m_L+m_s) v_F \sin\theta$$

Divide y by x to eliminate v_F

$$\frac{-m_L v_L \sin\alpha + m_s v_s \sin\beta}{m_L v_L \cos\alpha + m_s v_s \cos\beta} = \frac{\cancel{(m_L+m_s)}\cancel{v_F}\sin\theta}{\cancel{(m_L+m_s)}\cancel{v_F}\cos\theta}$$

$$\tan\theta = \frac{-m_L v_L \sin\alpha + m_s v_s \sin\beta}{m_L v_L \cos\alpha + m_s v_s \cos\beta} \Rightarrow$$

$$\theta = \tan^{-1}\left[\frac{-(4.0)(1.0)\sin(25) + (0.2)(5)\sin(50)}{(4.0)(1.0)\cos(25) + (0.2)(5)\cos(50)}\right] = \boxed{-12°}$$

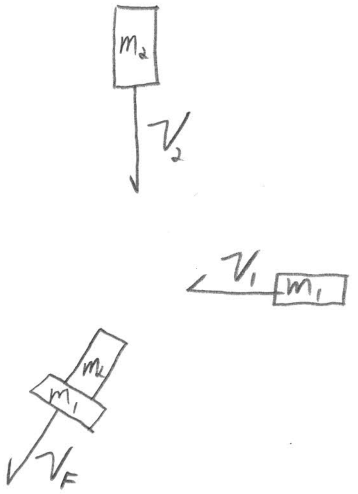
Plug back into x (or y) to get v_F

$$v_F = \frac{m_L v_L \cos\alpha + m_s v_s \cos\beta}{(m_L+m_s)\cos\theta} = \frac{(4)(1)\cos 25 + (0.2)(5)\cos(50)}{(4+0.2)\cos(-12)} = \boxed{1.0\ \text{m/s}}$$

You are driving West along Summit Ave, lawfully doing the speed limit (50 km/hr) in your new car which (as you've read in the owners manual) has a mass of 1500 kg. Sleepy McSnoozer is driving South along Cleveland in his 1965 Ford pickup truck loaded with bags of cement. His truck (plus cement) weighs 2300 kg. Sleepy runs the red light and smashes into your car. The cars fuse together and skid to a stop.

Certain that Sleepy was speeding, you measure the skid mark and find that the length of the skid is L = 18 m. You look up the rubber/asphalt coefficient of friction and find that it is $\mu_k = 0.6$.

What was Sleepy's velocity? Was he speeding? The speed limit is 50 km/hr.



$\mu_K = 0.6$

$L = 18$

$m_1 = 1500\,kg$

$m_2 = 2300\,kg$

$$v_1 = 50\,\frac{km}{hr} \cdot 1\times10^3\,\frac{m}{km} \cdot \frac{1}{3600} \cdot \frac{hr}{s} = 13.9\,m/s$$

Two parts, collision and skid. Conserve momentum for collision, Conserve energy to do skid.

Collision

$P_I = P_F$

$x: \ m_1 v_1 = (m_1 + m_2) v_{Fx}$

$y: \ m_2 v_2 = (m_1 + m_2) v_{Fy}$

$$\boxed{(1)\ v_{Fx} = \frac{m_1}{(m_1+m_2)} v_1, \quad (2)\ v_{Fy} = \frac{m_2}{(m_1+m_2)} v_2}$$

Skid

$U_I = 0 \qquad U_F = 0$

$K_I = \frac{1}{2}(m_1+m_2)|\vec{v}_F|^2 \qquad K_F = 0$

$W_f = -\mu_K (m_1 + m_2) g L$

$\frac{1}{2}\cancel{(m_1+m_2)}|\vec{v}_F|^2 = 2\mu_K \cancel{(m_1+m_2)} g L$

$$\boxed{|\vec{v}_F|^2 = 2\mu_K g L \quad (3)}$$

continued ↓

Systems of particles Set 3, P3 continued

Now: $|\vec{V}_F|^2$ is related to V_{Fx} and V_{Fy} by pythagoras.

$$④\ |\vec{V}_F|^2 = V_{Fx}^2 + V_{Fy}^2$$

Plugging ④ → ③:

$$⑤\ V_{Fx}^2 + V_{Fy}^2 = 2\mu_K g L$$

and plugging ① and ② → ⑤

$$\frac{m_1^2}{(m_1+m_2)^2} V_1^2 + \frac{m_2^2}{(m_1+m_2)^2} V_2^2 = 2\mu_K g L$$

and solve for V_1:

$$\frac{m_1^2 V_1^2 + m_2^2 V_2^2}{(m_1+m_2)^2} = 2\mu_K g L$$

$$m_1^2 V_1^2 + m_2 V_2^2 = 2\mu_K g L (m_1+m_2)^2$$

$$\Rightarrow \boxed{V_2 = \left[2\mu_K g L (m_1+m_2)^2 - m_1^2 V_1^2\right]^{1/2} \frac{1}{m_2}}$$

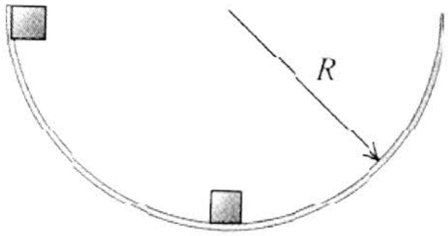
$$V_2 = \left[(2)(0.6)(9.8)(18)(1500+2300)^2 - (1500\cdot 13.9)^2\right]^{1/2} \frac{1}{2300}$$

$$V_2 = 22.3\,\tfrac{m}{s} \cdot 1\times10^{-3}\,\tfrac{km}{m} \cdot 3600\,\tfrac{s}{hr} = 80\,\tfrac{km}{hr}$$

Speeder!

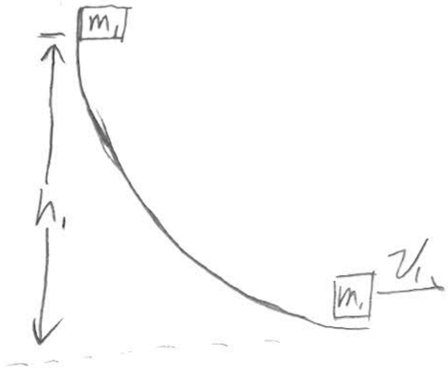
0

Two masses, m_1 and m_2, are released from rest in a frictionless hemispherical bowl of radius R from the positions shown in the figure. The upper mass collides with and sticks to the lower mass and the two slide up the other side together.



Derive an expression for their final height of the combined masses.

① Conserve Energy



$U_I = m_1 g h_1 \qquad U_F = 0$

$K_I = 0 \qquad K_F = \frac{1}{2} m_1 v_1^2$

$\cancel{m_1} g h_1 = \frac{1}{2} \cancel{m_1} v_1^2$

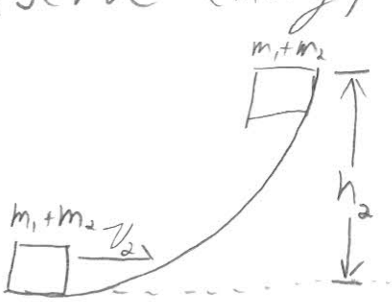
$$\boxed{v_1 = \sqrt{2 g h_1}} \quad (1)$$

② Conserve momentum

$m_1 v_1 + m_2 (0) = (m_1 + m_2) v_2$

$$\boxed{v_2 = \frac{m_1}{(m_1 + m_2)} v_1} \quad (2)$$

③ Conserve energy



$U_I = 0 \qquad U_F = (m_1 + m_2) g h_2$

$K_I = \frac{1}{2}(m_1 + m_2) v_2^2 \qquad K_F = 0$

$$\Rightarrow \boxed{h_2 = \frac{v_2^2}{2g}} \quad (3)$$

Systems of Particles Set 3, P6 continued.

From 3: $h_2 = \dfrac{v_2^2}{2g}$

Plug in 2: $h_2 = \dfrac{1}{2g}\left[\dfrac{m_1}{m_1+m_2}\right]^2 v_1^2$

plug in 1: $h_2 = \dfrac{1}{\cancel{2g}}\left[\dfrac{m_1}{m_1+m_2}\right]^2 \cancel{2g}h_1$

$$\boxed{h_2 = \left[\dfrac{m_1}{m_1+m_2}\right]^2 h_1}$$

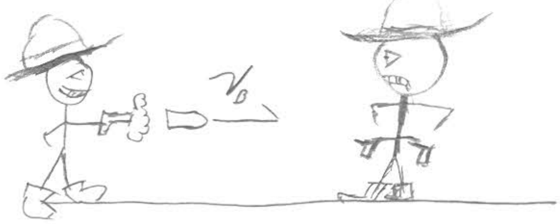
Sample Test 4
Phys 111 Spring 2011

4. In many classic westerns, gunfighters fly backwards several meters after being shot, often crashing through windows or saloon doors. Assume that a typical bullet weights 2 g and that a typical cowboy weights 80 kg.

a) If the bullet leaves the gun at 200 m/s, what is the velocity of the cowboy/bullet system after the impact?

b) What velocity does the bullet need for the cowboy to slide 3 meters across the floor after being shot (assuming $U_k = 0.5$)?

a)



Given

$m_B = 2\times10^{-3}$ kg

$M_c = 80$ kg

$V_{BI} = 200$ m/s

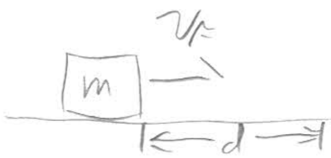
$V_F = ?$

$$P_i = m_B V_{BI} + \cancel{m_c V_{CI}}^{\,0} \qquad P_F = (m_B + m_c) V_F$$

$$\Rightarrow \boxed{V_F = \frac{m_B}{(m_B + m_c)} V_{BI}}$$

$$V_F = \frac{2\times10^{-3}}{80.002} \cdot 200 = \boxed{0.4 \text{ m/s}}$$

b) Slide to a stop in a distance d



$K_I = \frac{1}{2}(m_c + m_B) V_F^2 \qquad W_F = -\mu_k (m_c + m_B) g d$

$K_F = U_I = U_F = 0$

$$\Rightarrow \frac{1}{2}\cancel{(m_c + m_B)} V_F^2 = \mu_k \cancel{(m_c + m_B)} g d$$

$$V_F = (2\mu_k g d)^{1/2}$$

continued

Sample Test 4, P4 continued

$$\Rightarrow \frac{m_B}{(m_c + m_B)} v_{BI} = (2\mu_k g d)^{1/2}$$

$$\Rightarrow \boxed{v_{BI} = \frac{m_c + m_B}{m_B} (2\mu_k g d)^{1/2}}$$

$$v_{BI} = \frac{80.002}{2\times 10^{-3}} \left((2)(0.5)(9.8)(3)\right)^{1/2}$$

$$= \boxed{7.4 \times 10^{3} \text{ m/s}} = 16{,}000 \text{ miles/hour}$$

For comparison, the M16 muzzle velocity is approximately 1,000 m/s or 2,200 miles/hour

So... Flying backwards is bogus...