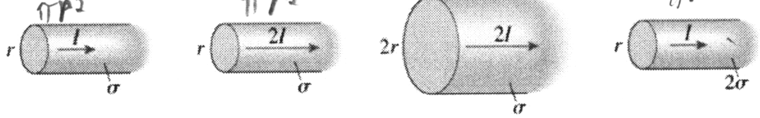
Circuits – Set 1
Name: ____________________

Problems Solved __/ 8

1. Rank in order, from largest to smallest, the current densities J_a to J_d in these four wires, which carry currents ranging from I to 2I.
   a. $J_b = J_d > J_a > J_c$
   b. $J_b > J_a > J_c > J_d$
   c. $\boxed{J_b > J_a = J_d > J_c}$
   d. $J_c > J_b > J_a > J_d$
   e. $J_c > J_b > J_a = J_d$

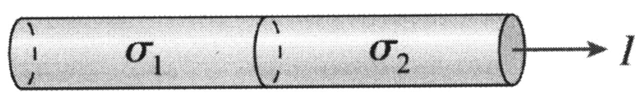
$\frac{I}{\pi r^2}$ $\frac{2I}{\pi r^2}$ $\frac{2I}{4\pi r^2} = \frac{1}{2}\frac{I}{\pi r^2}$ $\frac{I}{\pi r^2}$



(a) (b) (c) (d)

$$J = \frac{I}{A}$$

2. A wire carrying a current I has two equal length segments that have equal diameters. If the conductivities of the material in the two segments have a ratio **σ_1:σ_2**=2:1, what is the ratio E_1:E_2 of the electric field strengths in the two segments of the wire.
   a. E_1:E_2=4:1
   b. E_1:E_2=2:1
   c. E_1:E_2=1:1
   d. $\boxed{E_1:E_2=1:2}$
   e. E_1:E_2=1:4



$$J = \sigma E \Rightarrow \frac{J}{J} = \frac{\sigma_1 E_1}{\sigma_2 E_2} \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{E_2}{E_1}$$

3. What is the ratio of the change in potential ΔV_1:ΔV_2?
   a. ΔV_1: ΔV_2=4:1
   b. ΔV_1: ΔV_2=2:1
   c. ~~ΔV_1: ΔV_2=1:1~~
   d. $\boxed{\Delta V_1: \Delta V_2=1:2}$
   e. ΔV_1: ΔV_2=1:4

$$\Delta V = E\ell \Rightarrow \frac{\Delta V_1}{\Delta V_2} = \frac{E_1 \ell}{E_2 \ell} = \frac{\sigma_2}{\sigma_1}$$

4. What is the ratio of the resistances R_1:R_2?
   a. R_1: R_2=4:1
   b. R_1: R_2 =2:1
   c. R_1: R_2=1:1
   d. $\boxed{R_1: R_{22}=1:2}$
   e. R_1: R_2=1:4

$$R = \frac{V}{I} \Rightarrow \frac{R_1}{R_2} = \frac{V_1}{I_1} \cdot \frac{I_2}{V_2}$$

5. What is the ratio of the power dissipated in each segment P_1:P_2?
   a. P_1: P_2=4:1
   b. P_1: P_2 =2:1
   c. P_1: P_2=1:1
   d. P_1: P_2=1:2
   e. P_1: P_2=1:4

$P = VI \Rightarrow \frac{P_1}{P_2} = \frac{V_1 \cancel{I}}{V_2 \cancel{I}}$

6. Two light bulbs operate on the same potential difference. Bulb A has four times the power output of bulb B. Which bulb has the greater current?
   a. Bulb A
   b. Bulb B
   c. Neither - they both have the same current

$P = \Delta V I$ $\quad \frac{P_A}{P_B} = \frac{\Delta V I_A}{\Delta V I_B}$

$P_A = 4P_B$ $\quad \frac{4\cancel{P_B}}{\cancel{P_B}} = \frac{I_A}{I_B} \Rightarrow I_A = 4I_B$

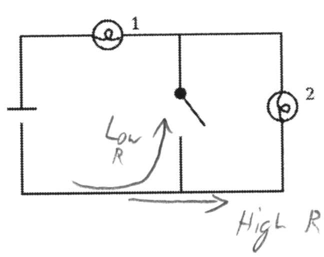
$I_A > I_B$

7. If a large resistor and a small resistor are connected in parallel, the equivalent resistance will be closer in value to that of the:
   a. large resistor
   b. small resistor
   c. Neither; it will be exactly between the two values
   d. None of the above

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$

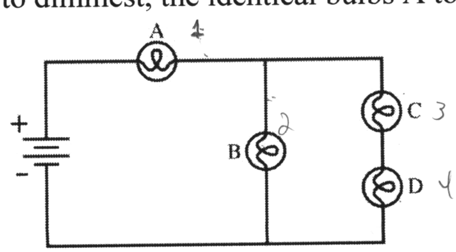
$R_T = \frac{R_1 R_2}{R_1 + R_2}$ as $R_2 \to \infty$ $R_T \to R_1$

8. Refer to the diagram. When the switch is closed the total power dissipated in the circuit will:
   a. increase
   b. decrease
   c. remain the same
   d. None of the above



Ø Low R, high I

9. Rank in order, from brightest to dimmest, the identical bulbs A to D.
   a. A = B = C = D
   b. A > B > C = D
   c. A > C > B > D
   d. A > C = D > B
   e. C = D > B > A



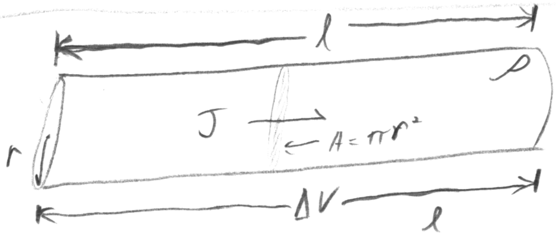
3=4

There is a potential difference of 2.5 V between opposite ends of a 6.0 m long iron wire.
Note: there is a table with resistivities in Wolfson.

a) Assuming a uniform electric field in the wire, what is the current density?
b) If the wire diameter is 1.0 mm, what is the total current?

Given
$\rho = 9.71\times10^{-8}\ \Omega\cdot m$
$\Delta V = 2.5\ V$
$l = 6.0\ m$
$r = 1.0\ mm$

a)



$J = \frac{1}{\rho}E, \quad |\Delta V| = +\int_0^l \vec{E}\cdot d\vec{s} = El \Rightarrow E = \frac{\Delta V}{l}$

$\boxed{J = \frac{1}{\rho}\frac{\Delta V}{l}} \Rightarrow J = \frac{1}{(9.71\times10^{-8}(\Omega\cdot m))}\cdot\frac{2.5(V)}{6.0\ m}$

$\Rightarrow J = 4.30\times10^{6}\ \frac{A}{V}\cdot\frac{1}{m}\cdot\frac{V}{m} = \boxed{4.3\times10^{6}\ \frac{A}{m^2}}$

b) $I = J\cdot A$

$\Rightarrow \boxed{I = J\cdot\pi r^2}$

$I = (4.3\times10^{6}\ \frac{A}{m^2})\pi(1\times10^{-3})^2$

$\boxed{I = 13.5\ A}$

The maximum safe current in 12-gauge (1.2 mm diameter) copper wire is 20 A.
a) What is the maximum current density?
b) What is the maximum electric field?

Given
$\rho = 1.68\times10^{-8}\ \Omega\cdot m$
$r = 1.2\ mm$
$I_{max} = 20\ A$

a) $I_{max} = J_{max}\cdot A \Rightarrow \boxed{J_{max} = \frac{I_{max}}{A}} \Rightarrow J_{max} = \frac{20\ A}{\pi(1.2\times10^{-3} m)^2} = \boxed{4.42\times10^{6}\ \frac{A}{m^2}}$

b) $J = \frac{1}{\rho}E \Rightarrow \boxed{E_{max} = \rho J_{max}} \Rightarrow (1.68\times10^{-8}\ \Omega\cdot m)(4.42\times10^{6}\ \frac{A}{m^2}) = E_{max}$

$\Rightarrow \boxed{E_{max} = 7.4\times10^{-2}\ \frac{V}{m}}$

Engineers call for a power line with a resistance per unit length of 50 mΩ/km.

a) What wire diameter is required if the line is made of copper?
b) What wire diameter is required if the line is made of aluminum?
c) If the costs of copper and aluminum wire are \$4.65/kg and \$2.30/kg, respectively, which material is more economical? The densities of copper and aluminum are 8.9 g/cm^3 and 2.7 g/cm^3, respectively.

Given

$\rho_c = 1.68\times10^{-8}\,\Omega\cdot m$

$\rho_a = 2.65\times10^{-8}\,\Omega\cdot m$

$\lambda = 50\ m\Omega/km$

Want

d_c, d_a

a) $R = \rho\frac{l}{A} \Rightarrow \lambda = \frac{R}{l} = \frac{\rho}{A}$

$\Rightarrow \lambda = \frac{\rho}{\pi r^2} \Rightarrow r\left[\frac{\rho}{\pi\lambda}\right]^{1/2} \Rightarrow \boxed{d = 2\left[\frac{\rho}{\pi\lambda}\right]^{1/2}}$

$d_c = 2\left[\frac{\rho_c}{\pi\lambda}\right]^{1/2} =$

$$\boxed{d_c = 2\left[\frac{1.68\times10^{-8}\,\Omega\cdot m}{\pi\cdot50\times10^{-6}\,\Omega/m}\right]^{1/2} = 0.02\,m = 20\,mm}$$

$$\boxed{d_a = 0.026\,m = 26\,mm}$$

c) Re-using ρ to mean mass density...

$\rho_c = 8.9\,g/cm^3$, $\rho_a = 2.7\,g/cm^3$, $C_c = 4.65\,\$/kg$, $C_a = 2.30\,\$/kg$

Now, use λ to mean linear mass density:

cross-section

$\lambda = \rho A \Rightarrow \lambda = \rho\pi r^2$

Now we can calculate cost per unit length

$\lambda' = \lambda\cdot C \Rightarrow \lambda' = \rho\pi r^2 C$

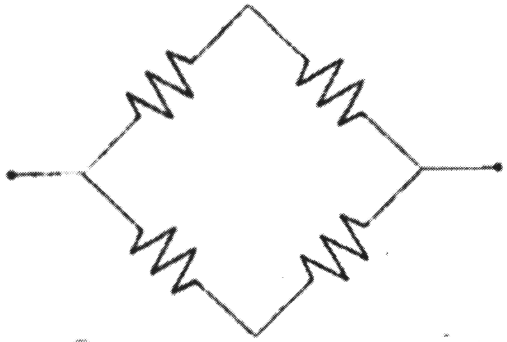
continued ↓

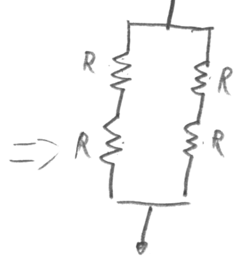
copper: $\lambda'_c = \rho_c \pi \left(\tfrac{1}{2} d_c\right)^2 C_c$

$$= \left(8.9\times10^{-2}\ \mathrm{kg/m^3}\right)\pi\left(\tfrac{1}{2}\cdot 0.02\ \mathrm{m}\right)^2 \cdot 4.65\ \$/\mathrm{kg}$$

$$\boxed{\begin{aligned} \lambda'_c &= 1.3\times10^{-4}\ \$/\mathrm{m} \\ \lambda'_a &= 1.08\times10^{-4}\ \$/\mathrm{m} \end{aligned}}$$

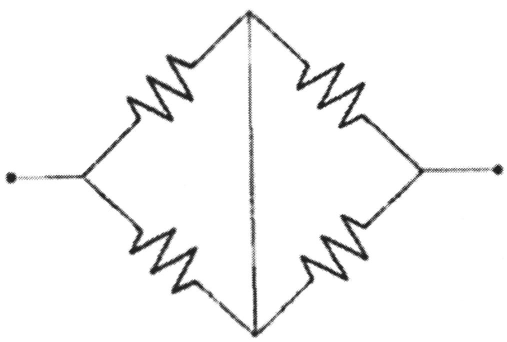
Find the equivalent resistance of the following systems. All resistors have a resistance R.





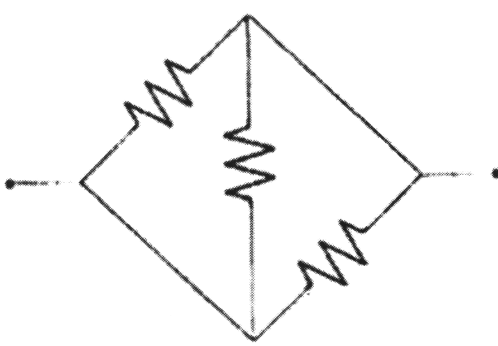
$$\frac{1}{R_T} = \frac{1}{R+R} + \frac{1}{R+R} = \frac{1}{2R} + \frac{1}{2R} = \frac{\cancel{2}}{\cancel{2}R} = \frac{1}{R}$$

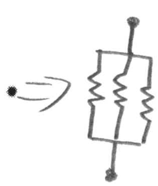
$$\boxed{\Rightarrow R_T = R}$$



$$R_T = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \left(\frac{2}{R}\right)^{-1} + \left(\frac{2}{R}\right)^{-1} = \frac{1}{2}R + \frac{1}{2}R$$

$$\boxed{\Rightarrow R_T = R}$$





$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

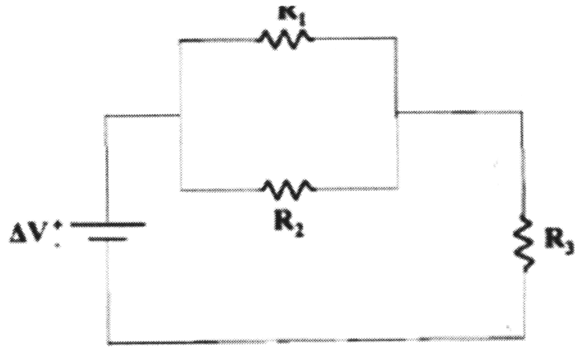
$$\Rightarrow \boxed{R_T = \tfrac{1}{3}R}$$

Circuits – Set 1

In this circuit, $R_1 = 1.0\ k\Omega$, $R_2 = 3.0\ k\Omega$ $R_3 = 2.0\ k\Omega$, and $\Delta V = 30\ V$

Find the:

a) Equivalent Resistance
b) Total Current delivered by the battery
c) Voltage across each resistor
d) Power dissipated by each resistor
e) Total power dissipated in the circuit.



a) $R_T = R_3 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$

$= R_3 + \left(\frac{R_1 + R_2}{R_1 R_2}\right)^{-1} = \boxed{R_3 + \frac{R_1 R_2}{R_1 + R_2} = R_T}$

$R_T = 2.0\,k\Omega + \frac{1.0\,k\Omega \cdot 3.0\,k\Omega}{4.0\,k\Omega} \Rightarrow \boxed{R_T = 2.75\,k\Omega}$

b) $V = IR \Rightarrow V_b = I_{net} \cdot R_T \Rightarrow \boxed{I_{net} = \frac{V_b}{R_T}} =$

$\Rightarrow \boxed{I_{net} = \frac{30\,V}{2.75 \times 10^3\,\Omega} = 0.011\,A}$

c) $I_{net} = I_3$, $V_3 = I_3 R_3 \Rightarrow V_3 = I_{net} R_3 \Rightarrow V_3 = 0.011\,A \cdot 2.0 \times 10^3\,\Omega$

$\Rightarrow \boxed{V_3 = 22\,V}$

$\boxed{V_1 = V_2 = V_b - V_3 = 30\,V - 22\,V = 8\,V}$

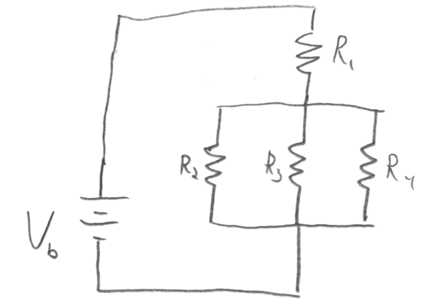
d) $P = VI \Rightarrow P_3 = V_3 I_3 \Rightarrow \boxed{P_3 = (22\,V)(0.011\,A)} = \boxed{0.24\,W}$

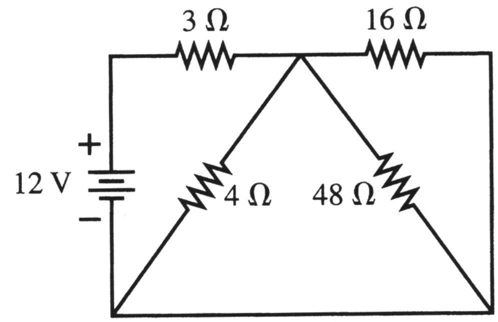
$P = VI$, $I = \frac{V}{R} \Rightarrow \boxed{P = \frac{V^2}{R}}$, $\boxed{P_1 = \frac{(8V)^2}{1 \times 10^3} = 0.06\,W}$

$\boxed{P_2 = \frac{(8V)^2}{3 \times 10^3} = 0.02\,W}$

e) $\boxed{P_T = P_1 + P_2 + P_3 = 0.32\,W}$

For the circuit shown in the figure, find the current through and the potential difference across each resistor.





Start by Finding R_{eff}

R_{eff} $R_1 + R_{234}$, $\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{3} \Rightarrow \boxed{R_{234} = 3}$

$\boxed{R_{eff} = 6}$

Now I can find the current through R_1

$V = IR \Rightarrow I = \frac{V}{R} \Rightarrow \boxed{I_{net} = \frac{V_b}{R_{eff}}}$

With the current, I can find the Voltage.

$I_1 = I_{net} \Rightarrow \boxed{V_1 = I_{net} R_1}$

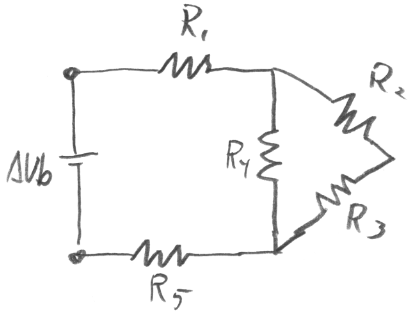
$\boxed{I_1 = 2\text{ A},\ V_1 = 6\text{ V}}$

Same Votage across R_2, R_3, and R_4

$V_{234} = V_b - V_1 \Rightarrow \boxed{V_2 = V_3 = V_4 = 6\text{ V}}$

Then, $I = \frac{V_{234}}{R} \Rightarrow \boxed{I_2 = 1.5\text{ A},\ I_3 = .125\text{ A},\ I_4 = .375\text{ A}}$

Wolfson, 25.37

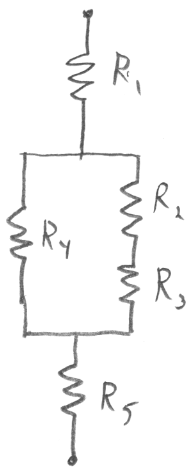


Want I_4

Given:

$R = 1\,k\Omega$ for all resistors.

$V_b = 6V$



$$R_{234} = \left(\frac{1}{R_2 + R_3} + \frac{1}{R_4}\right)^{-1} = \left(\frac{1}{2} + 1\right)^{-1} = \tfrac{2}{3}\,k\Omega$$

$$R_{eff} = R_1 + R_{234} + R_5 = \frac{8}{3}\,k\Omega$$

$$I_{tot} = \frac{V_b}{R_{eff}} = 6 \cdot \tfrac{3}{8} = \frac{9}{4}\,mA$$

$$V_{234} = I_{tot} \cdot R_{234} = \frac{9}{4} \cdot \frac{2}{3} = \tfrac{3}{2}V$$

$$\Rightarrow \boxed{I_4 = \frac{V_{234}}{R_4} = \tfrac{3}{2}\,mA}$$

Wolfson 25.42

Current in parallel circuits adds:

$I_T = \sum_{i=1}^{n} I_i$ and if each bulb draws the same current

$I_T = nI \Rightarrow \boxed{n = \frac{I_T}{I}}$, $\quad P = IV \Rightarrow I = \frac{P}{V}$

$\Rightarrow \boxed{n = \frac{I_T}{P}V}$ and we want $I_T = I_{max}$

$$\boxed{n = \frac{20\,A}{100\,W} \cdot 120\,V = 24 \text{ bulbs}}$$