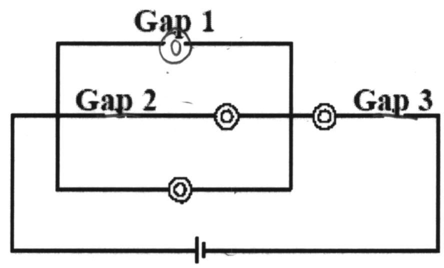
Physics 112 Sample Test 2

1.1) This circuit contains a battery and three light bulbs (indicated by concentric circles). There are also three gaps in the circuit. You want to close all three gaps, but you have the option of closing them either with other light bulbs or with small pieces of wire. Which gaps should be closed with bulbs if you want the circuit to emit the maximum amount of light? Assumptions: all bulbs are identical; gaps not closed with bulbs are instead closed with wires and vice versa.

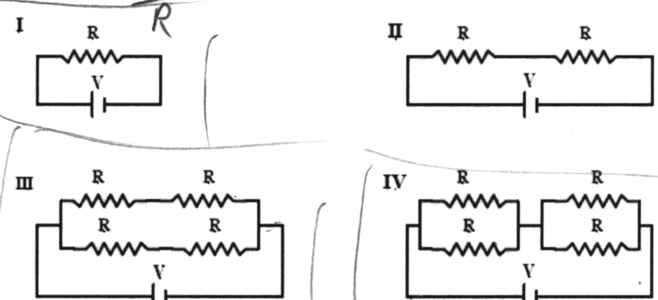


(a) Gap 1, only
(b) Gap 2, only
(c) Gap 3, only
(d) Gaps 1 & 2, only
(e) Gaps 1, 2, & 3

Gaps 2 and 3 will increase R_{eff} and decrease current flow.

1.2) The batteries and resistors in these circuits are identical. For which circuit will the total current flowing from the battery be V/R?

(a) I, only
(b) I and III, only
(c) I and IV, only
(d) I, III, and IV, only
(e) I, II, III, and IV



$\frac{V}{R}$ $\qquad$ $V/2R$

$R_{eff} = \left(\frac{1}{2R} + \frac{1}{2R}\right)^{-1} = R \Rightarrow I = V/R$

$R_{eff} = \frac{1}{2}R + \frac{1}{2}R = R \Rightarrow I = V/R$

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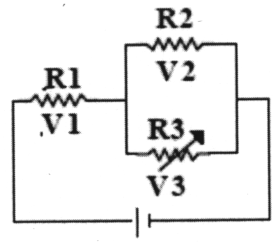
1.3) This circuit consists of a battery, two fixed resistors (R1 and R2), and an adjustable resistor (R3). The voltages across the three resistors are V1, V2, and V3. If the resistance R3 were to be increased, which of the voltages would also increase?

(a) V1, only
(b) V3, only
(c) V1 and V3, only
(d) V2 and V3, only
(e) V1, V2, and V3

as $R_3 \uparrow$, $R_{23} \uparrow$

as $R_{23} \uparrow$, $V_{23} \uparrow$

so, $V_1 \downarrow$

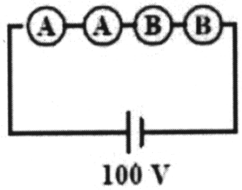


$V_{23} = V_2 = V_3$

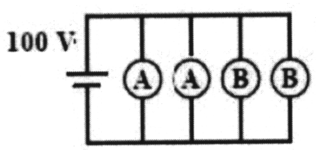
1.4) Type A light bulbs can be wired across a potential difference of up to 100 volts without burning out. Type B bulbs can only be wired across a potential difference of up to 50 volts without burning out. Which of the circuits here would emit the most amount of light without burning out a bulb? (Ignore changes in bulb properties with temperature.)

All voltages < 100V

all V = 100V



<A>



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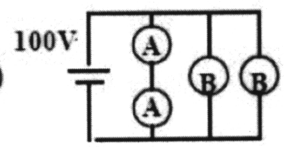
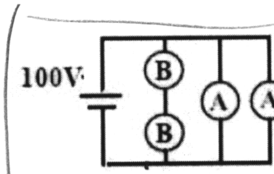


<C>

$V_B = 100V$
$V_A = 50V$



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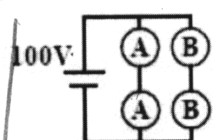
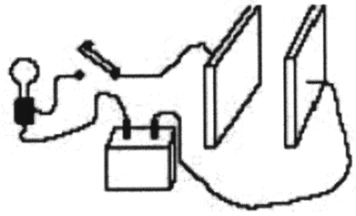


<E>

$V_A = 50V$
$V_B = 100V$

$V_B = 50V$
$V_A = 100V$

1.5) The picture here shows a circuit with a battery, a light bulb, a parallel plate capacitor, and a switch. If the battery were connected to the light bulb by itself, it would cause the bulb to emit a noticeable amount of light.



Which of the following best describes the intensity of the bulb's light if the switch were closed?

(a) The bulb would not light.
(b) The bulb would initially emit a bright light that subsequently gets dimmer.
(c) The buld would initially emit a dim light that subsequently gets bright.
(d) The bulb would emit a steady amount of light equally a bright as if just the bulb and battery were wired together.
(e) The bulb would emit a steady amount of light, but an amount that is substantially dimmer than if just the bulb and battery were wired together.

As capacitor charges, $\Delta V_c \uparrow$ from 0 so that $\Delta V_b \downarrow$ as capacitor charges.

1.6) After the switch has been closed for a very long time, a certain amount of charge will have built up on each plate. Which of the following changes to the apparatus would have caused the charge magnitude on each plate to be larger? (circle all that are correct)

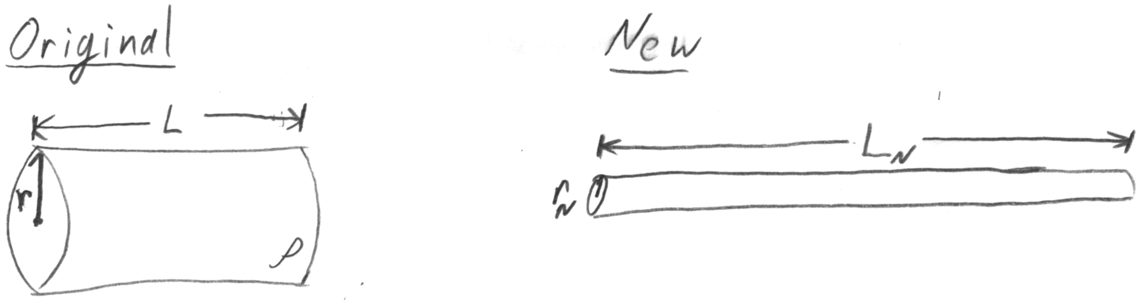
(a) Increasing the voltage of the battery
(b) Increasing the surface area of the metal plates
(c) Increasing the resistance of the light bulb → Causes capacitor to charge more slowly.

$Q = CV, \quad C = \frac{\epsilon_0 A}{d}$

$\Rightarrow Q = \frac{\epsilon_0 A}{d} V \qquad V\uparrow, Q\uparrow$

$A\uparrow, Q\uparrow$

2) A wire with resistance R is drawn out through a die so that its new length is three times its original length. Find the resistance of the new wire assuming that its resistivity did not change.



Given

$\rho_N = \rho$

$L_N = 3L$

R

Want

R_N

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} \qquad R_N = \frac{\rho_N L_N}{A_N} = \frac{\rho L_N}{\pi r_N^2}$$

$$\Rightarrow \frac{R_N}{R} = \frac{\rho L_N}{\pi r_N^2} \cdot \frac{\pi r^2}{\rho L} \Rightarrow \boxed{R_N = R\left[\frac{L_N}{L} \cdot \frac{r^2}{r_N^2}\right]}$$

*Now, we have to eliminate L and r

$$L_N = 3L \Rightarrow \boxed{\frac{L_N}{L} = 3}$$

*To get $\frac{r}{r_N}$, we use the fact that the volume of the wire didn't change.

$$V_N = V \Rightarrow \pi r_N^2 L_N = \pi r^2 L \Rightarrow \boxed{\frac{r^2}{r_N^2} = \frac{L_N}{L}}$$

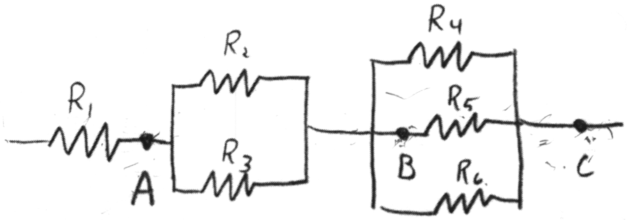
$$\Rightarrow \boxed{\frac{r^2}{r_N^2} = 3}$$

$$\Rightarrow R_N = R[3 \cdot 3] \Rightarrow \boxed{R_N = 9R}$$

3. A current of I_B flows through point *B*. All resistors have resistance R.

   a) What is the magnitude of the potential difference between points *A* and *C*, V_{AC}, in terms of I_B and *R*?

   b) What is the current in R_2, in terms of I_B and *R*?



Given

I_B, R

Want

ΔV_{AC}

I_2

a) Working from macro-scopic to micro-scopic:

$\Delta V_{AC} = I_A \cdot R_{eff}$, So we need to find R_{eff} and I_A

$R_{eff} = R_{23} + R_{456} \Rightarrow R_{eff} = \frac{1}{2}R + \frac{1}{3}R$

$\Rightarrow R_{eff} = \frac{5}{6}R$

$I_A = I_{23} = I_{456}$ so: $\Delta V_{456} = I_{456} \cdot R_{456} = I_A \cdot \frac{1}{3}R$

$\Rightarrow I_A = \frac{\Delta V_{456}}{\frac{1}{3}R}$ But, we need ΔV_{456}

$\Delta V_{456} = \Delta V_5 = I_B R_5 \Rightarrow \Delta V_{456} = I_B R \Rightarrow I_A = \frac{I_B R}{\frac{1}{3}R}$

$\Rightarrow I_A = 3I_B$

$\Rightarrow \Delta V_{AC} = 3I_B \cdot \frac{5}{6}R \Rightarrow \Delta V_{AC} = \frac{5}{2} I_B R$

b) $I_A = I_{23}$ and $\Delta V_{23} = I_{23} R_{23} \Rightarrow \Delta V_{23} = I_A \frac{1}{2}R \Rightarrow \Delta V_{23} = \frac{3}{2} I_B R$

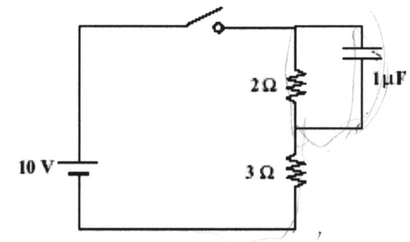
$I_3 = \frac{\Delta V_{23}}{R_3} \Rightarrow I_3 = \frac{3}{2} I_B$

Physics 112 Sample Test 2

4. Consider the circuit shown at the right.

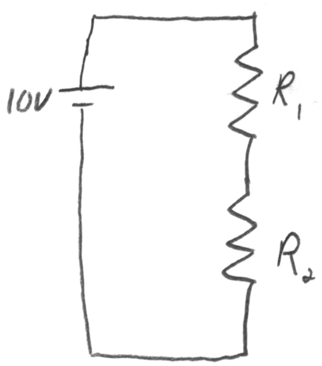
(a) If the switch is closed, and you wait sufficiently long that the currents in the circuit reach their steady-state values, what is (i) the rate at which energy leaves the battery and (ii) the charge on the capacitor? ("The rate at which energy leaves" is also called the power dissipation.)

(b) If the switch is then opened, how long does it take for the capacitor's charge to drop by 50%?



a) After a "long time," the capacitor is charged and no current flows through it.

Then, the equivalent circuit is:



i) $P = IV$, $V = V_b$, $I = \frac{V_b}{R_{eff}} = \frac{V_b}{R_1 + R_2}$

$$P = \frac{V_b}{R_1+R_2} \cdot V_b = \boxed{\frac{V_b^2}{R_1+R_2}}$$

$$\Rightarrow \boxed{P = \frac{(10V)^2}{5\Omega} = 20W}$$

ii) $Q = CV$, $V = V_1 = IR_1 = \frac{V_b}{R_1+R_2} \cdot R_1 = \frac{R_1}{R_1+R_2} V_b$

$$\Rightarrow \boxed{Q = \frac{R_1}{R_1+R_2} C V_b} \Rightarrow Q = \frac{2}{5}(1 \times 10^{-6} F)(10V)$$

$$\Rightarrow \boxed{Q = 4\mu C}$$

continued ↓

b) After the switch is opened, The equivalent circuit is:

Then, $V(t) = V_0 e^{-t/R_1C}$ and $V = \frac{Q}{C}$

$$\Rightarrow Q = Q_0 e^{-t/RC} \Rightarrow \frac{Q}{Q_0} = e^{-t/RC}$$

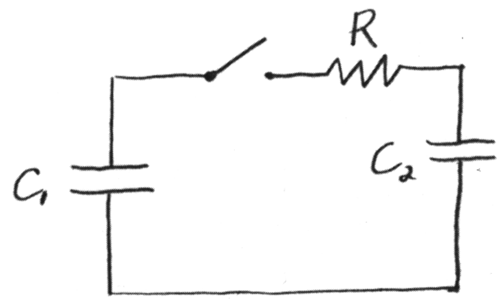
$$\Rightarrow \ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{R_1C} \Rightarrow t = R_1C\ln\left(\frac{Q_0}{Q}\right)$$

we want $\frac{Q}{Q_0} = \frac{1}{2} \Rightarrow \boxed{t = R_1C\ln(2)}$

$$t = (2\Omega)(1\times10^{-6}F)\ln(2)$$

$$\boxed{t = 1\mu s}$$

5. In the figure at the right, C_1 is initially charged to V_0 and C_2 is initially uncharged. The switch is then closed. Find the total energy dissipated in the resistor as the circuit comes to equilibrium. (HINT: Charge is conserved.)



Before the switch is closed, the total energy in the circuit is: $\boxed{U_I = \frac{1}{2} C_1 V_{10}^2}$

After the switch is closed, the charge on C_1 distributes across C_1 and C_2 until $\Delta V_1 = \Delta V_2$.

Because charge is conserved, $Q_T = Q_{1F} + Q_{2F}$

And $Q_T = Q_{1I} \Rightarrow \boxed{Q_{1I} = Q_{1F} + Q_{2F}}$

same potential

$$\Rightarrow C_1 V_0 = C_1 V_F + C_2 V_F$$

$$\Rightarrow \boxed{V_F = \frac{C_1}{C_1 + C_2} V_{10}}$$

Then, the total Energy in the circuit is:

$$U_F = \frac{1}{2} C_1 V_F^2 + \frac{1}{2} C_2 V_F^2 = \frac{1}{2}(C_1 + C_2) V_F^2$$

$$\Rightarrow \Delta U = U_F - U_I = \frac{1}{2}(C_1 + C_2) V_F^2 - \frac{1}{2} C_1 V_{10}^2$$

$$= \frac{1}{2}\cancel{(C_1 + C_2)} \frac{C_1^2}{(C_1 + C_2)^{\cancel{2}}} V_{10}^2 - \frac{1}{2} C_1 V_{10}^2 \Rightarrow$$

continued ↓

Sample Test 3, P5 continued

$$\Delta U = \frac{1}{2}\left[\frac{C_1^2}{(C_1+C_2)} - C_1\right]V_{10}^2$$

$$= \frac{1}{2}\left[\frac{C_1^2 - C_1(C_1+C_2)}{(C_1+C_2)}\right]V_{10}^2$$

$$= \frac{1}{2}\left[\frac{C_1^2 - C_1^2 - C_1C_2}{(C_1+C_2)}\right]V_{10}^2$$

$$\boxed{\Delta U = -\frac{1}{2}\frac{C_1C_2}{C_1+C_2}V_{10}^2}$$

which is the energy lost in the resistor.