Electrostatics – Set 1

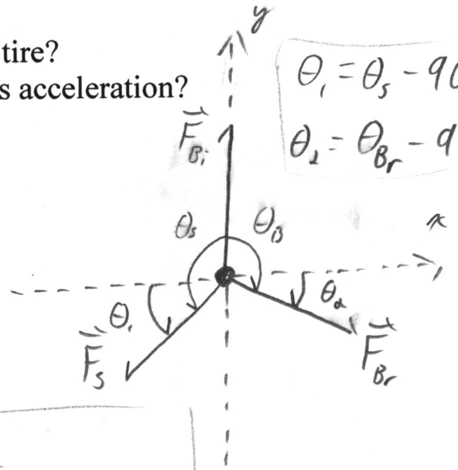
Name: ____________________

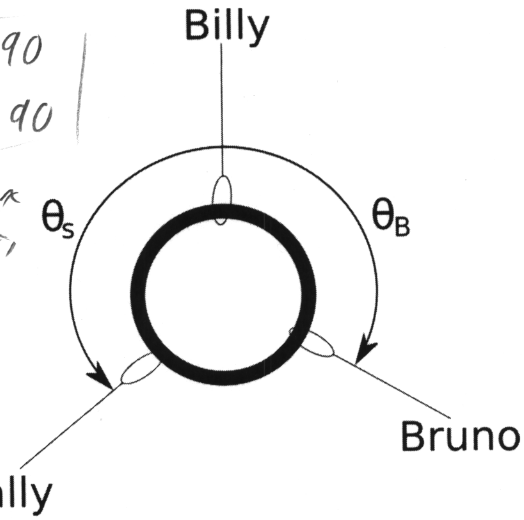
Problems Solved __/ 5

Billy, Sally, and Bruno are playing three way tug of war using ropes attached to a bicycle tire. Billy is pulling with 15 N of force, Sally with 20 N of force, and Bruno with 5 N of force. Sally's rope is 120^0 counterclockwise from Billy's rope. Bruno's rope is 110^0 clockwise from Billy's rope.

a) What is the net force on the tire?
b) In what direction is the tire's acceleration?

$\theta_1 = \theta_s - 90$
$\theta_2 = \theta_{Br} - 90$





Given
$|\vec F_{B_i}| = 15N$ $\quad \theta_s = 120^\circ$
$|\vec F_s| = 20N$ $\quad \theta_{Br} = 110^\circ$
$|\vec F_{Br}| = 5N$

Want
$\vec F_{net}, \theta_{net}$

Fun Facts
$\sin(\theta - 90) = -\cos\theta$
$\cos(\theta - 90) = \sin\theta$

a) $\vec F_{net} = \vec F_{B_i} + \vec F_s + \vec F_{Br}$

$$= |\vec F_{B_i}|\hat j - |\vec F_s|\cos\theta_1 \hat i - |\vec F_s|\sin\theta_1 \hat j + |\vec F_{Br}|\cos\theta_2 \hat i - |\vec F_{Br}|\sin\theta_2 \hat j$$

$$= |\vec F_{B_i}|\hat j - |\vec F_s|\sin\theta_s \hat i + |\vec F_s|\cos\theta_s \hat j + |\vec F_{Br}|\sin\theta_{Br} \hat i + |\vec F_{Br}|\cos\theta_{Br} \hat j$$

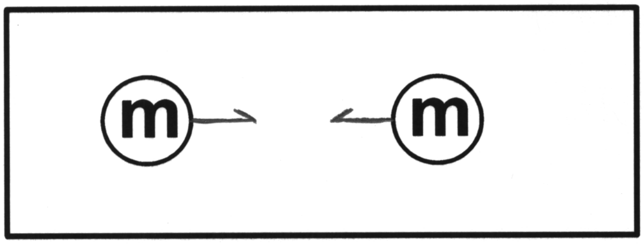
$$\boxed{\begin{aligned} &\Rightarrow \vec F_{net} = (-|\vec F_s|\sin\theta_s + |\vec F_{Br}|\sin\theta_{Br})\hat i + (|\vec F_{B_i}| + |\vec F_s|\cos\theta_s + |\vec F_{Br}|\cos\theta_{Br})\hat j \\ &\Rightarrow \vec F_{net} = -12N\hat i + 3.3N\hat j \end{aligned}}$$

b) $\vec F_{net} = m\vec a \Rightarrow \vec F_{net} \parallel \vec a$

$$\boxed{\text{So, } \theta_{net} = \tan^{-1}\left(\frac{F_{net,y}}{F_{net,x}}\right) = \tan^{-1}\left(\frac{3.3}{-12}\right) = 164^\circ}$$

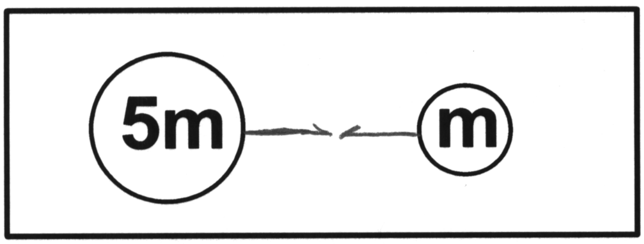
For each figure, draw the net gravitational <u>force</u> vector, $\vec{F}_{g,net}$, acting on each of the masses. The lengths of your vectors should indicate the relative magnitudes of the forces.

(a)



$\vec{F}_{12} = -\vec{F}_{21} \Rightarrow |\vec{F}_{12}| = |\vec{F}_{21}|$

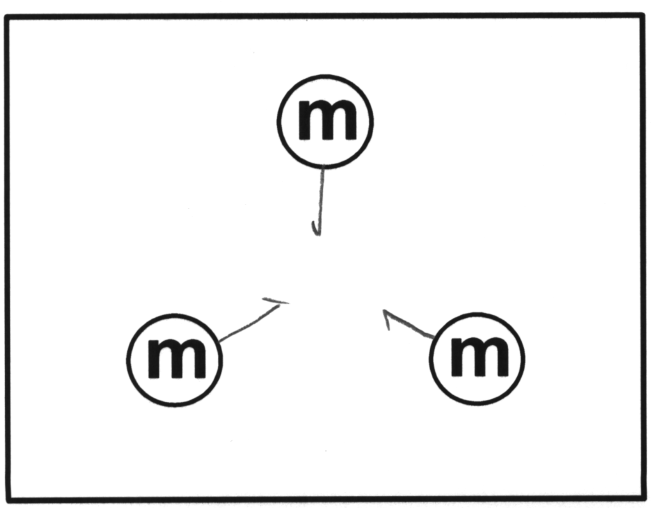
(b)



$\vec{F}_{12} = -\vec{F}_{21}$ still

but both are 5x larger than (a)

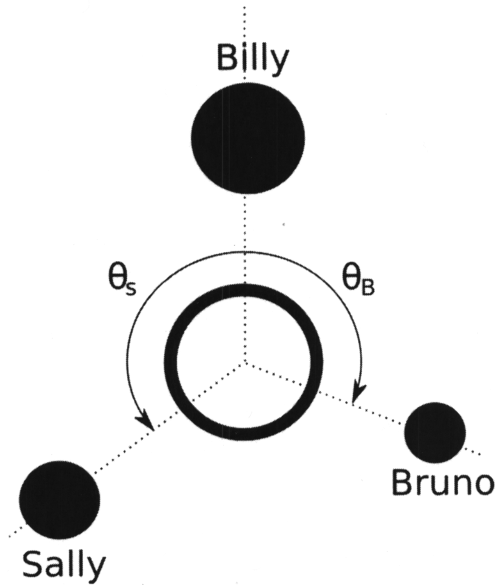
(c)



all mags are equal

Billy, Sally, and Bruno have suddenly become small planets and are attempting to pull the tire with without touching it, using only their gravitational force. Billy is 6×10^{3} m away from the tire and has a mass of 2×10^{6} kg. Sally is $3\times10^{\cancel{6}\,3}$ m away from the tire and has a mass of 1.5×10^{6} kg. Bruno is $1\times10^{\cancel{6}\,3}$ m away from the tire and has a mass of 5×10^{5} kg. The mass of the tire is 10 kg. $\theta_s = 120^\circ$ and $\theta_B = 110^\circ$

What is the net force on the tire?



Given

$d_{Bi} = 6\times10^{3}\,m$ $m_{Bi} = 2\times10^{6}\,kg$

$d_s = 3\times10^{3}\,m$ $m_s = 1.5\times10^{6}\,kg$

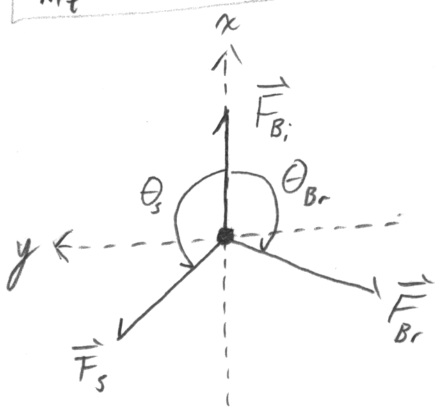
$d_{Br} = 1\times10^{3}\,m$ $m_{Br} = 5\times10^{5}\,kg$

$\theta_s = 120^\circ$ $\theta_{Br} = 110^\circ$

m_t

Want

$\vec{F}_{net}$



I've aligned my coord. system so that the angles are measured from the x-axis.

$\hat{\imath}\uparrow,\ \hat{\jmath}\leftarrow$

$$\vec{F}_{net} = \vec{F}_{Bi} + \vec{F}_s + \vec{F}_{Br}$$

$$= -\frac{G m_{Bi} m_t}{d_{Bi}^2}\hat{r}_{Bi,t} - \frac{G m_s m_t}{d_s^2}\hat{r}_{s,t} - \frac{G m_{Br} m_t}{d_{Br}^2}\hat{r}_{Br,t},$$

$$= -\frac{G m_{Bi} m_t}{d_{Bi}^2}(-\hat{\imath}) - \frac{G m_s m_t}{d_s^2}(\cos\theta_s\,\hat{\imath} - \sin\theta_s\,\hat{\jmath})$$

$$- \frac{G m_{Br} m_t}{d_{Br}^2}(\cos\theta_{Br}\,\hat{\imath} + \sin\theta_{Br}\,\hat{\jmath})$$

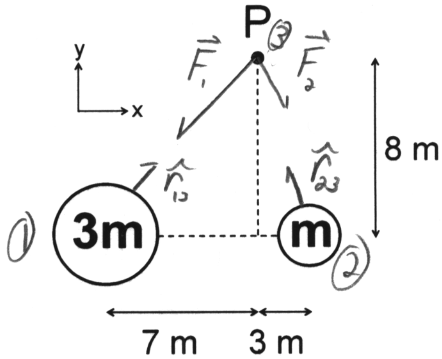
continued ↓

Electrostatics - Set 1, P3, continued

$$\boxed{\vec{F}_{net} = Gm_t\left[\left(\frac{m_{Bi}}{d_{Bi}^2} - \frac{m_s}{d_s^2}\cos\theta_s - \frac{m_{Br}}{d_{Br}^2}\cos\theta_{Br}\right)\hat{\imath} + \left(\frac{m_s}{d_s^2}\sin\theta_s - \frac{m_{Br}}{d_{Br}^2}\sin\theta_{Br}\right)\hat{\jmath}\right]}$$

$$\boxed{\vec{F}_{net} = 2.06\times10^{-10}\,N\,\hat{\imath} - 2.17\times10^{-10}\,N\,\hat{\jmath}}$$

Consider the figure. A mass of $2m$ is placed at point P.



a) Calculate the net gravitational force vector, $\vec{F}_{g,net}$, that will act on it. Write your answer in unit vector notation. (Let $m = 3.2\times10^4$ kg.)

$G = 6.67\times10^{-11}\ m^3/kg\cdot s^2$

b) Calculate the magnitude and direction of $\vec{F}_{g,net}$.

Given

$d_1 = 7m$

$d_2 = 3m$

$d_3 = 8m$

$m = 3.2\times10^4\,kg$

Want

$\vec{F}_{net}$

a) $\vec{F}_{net} = \vec{F}_1 \;\; \vec{F}_2$

$$= -\frac{G(2m)(3m)}{|\vec{r}_{13}|^2}\hat{r}_{13} + -\frac{G(2m)(m)}{|\vec{r}_{23}|^2}\hat{r}_{23}$$

$$\boxed{\text{In general, } \hat{r}_{ij} = \frac{\vec{r}_{ij}}{|\vec{r}_{ij}|}}$$

$$\Rightarrow \vec{F}_{net} = -\frac{G6m^2}{|\vec{r}_{13}|^3}\vec{r}_{13} + -\frac{G2m^2}{|\vec{r}_{23}|^3}\vec{r}_{23}$$

$$\boxed{\Rightarrow \vec{F}_{net} = -\frac{G6m^2}{[d_1^2+d_3^2]^{3/2}}(d_1\hat{\imath} + d_3\hat{\jmath}) - \frac{G2m^2}{[d_2^2+d_3^2]^{3/2}}(-d_2\hat{\imath} + d_3\hat{\jmath})}$$

$$\Rightarrow \vec{F}_{net} = -3.41\times10^{-4}(+7\hat{\imath} + 8\hat{\jmath}) - 2.19\times10^{-4}(+3m\hat{\imath} + 8m\hat{\jmath})$$

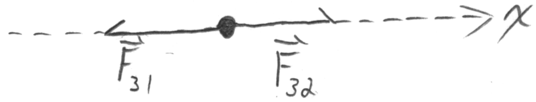
$$\boxed{\vec{F}_{net} = -1.73\times10^{-3}\hat{\imath} - 4.48\times10^{-3}\hat{\jmath}}$$

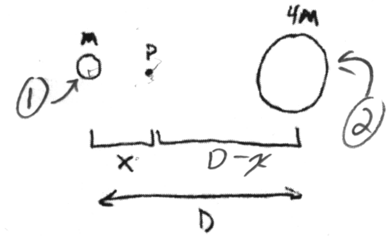
b) $\theta = \tan^{-1}\left(\frac{-4.48\times10^{-3}}{-1.73\times10^{-3}}\right) \Rightarrow \boxed{\theta = 249^\circ \text{ ccw from } +x\text{-axis}}$

Two masses M and $4M$ are separated by a distance D. Determine the location x of a point measured from M at which the net force on a *third* mass weighing m_3 would be zero.

Given	Want
$m_1 = M$	x
$m_2 = 4M$	
m_3	
D	

FBD for m_3





$$\vec{F}_{net,3} = \vec{F}_{31} + \vec{F}_{32}$$

$$= -\frac{Gm_1m_3}{|\vec{r}_{31}|^2}\hat{r}_{31} - \frac{Gm_2m_3}{|\vec{r}_{32}|^2}\hat{r}_{32}$$

$$= -\frac{Gm_1m_3}{x^2}\hat{x} - \frac{Gm_2m_3}{(D-x)^2}(-\hat{x}) \quad , \quad \text{want } |\vec{F}_{net}| = 0$$

$$\Rightarrow |\vec{F}_{net}| = 0 \Rightarrow \frac{Gm_2m_3}{(D-x)^2} - \frac{Gm_1m_3}{x^2} = 0$$

$$\Rightarrow \frac{\cancel{G}m_2\cancel{m_3}}{(D-x)^2} = \frac{\cancel{G}m_1\cancel{m_3}}{x^2} \Rightarrow m_2x^2 = m_1(D-x)^2$$

$$\Rightarrow 4\cancel{M}x^2 = \cancel{M}(D-x)^2$$

$$\Rightarrow 2x = D - x$$

$$\boxed{\Rightarrow x = \tfrac{1}{3}D}$$

Wolfson Chapter 8 Question 11

Given: $m_1 = M_E$

$|\vec{F}_{gp}| = 2|\vec{F}_{gE}|$

$M_E = 5.97 \times 10^{24}\,kg$

$R_E = 6.37 \times 10^{6}\,m$

Want: R_p

$$|\vec{F}_{gp}| = \frac{G m_A M_E}{R_p^2}, \quad |\vec{F}_{gE}| = \frac{G m_A M_E}{R_E^2}$$

$$\frac{\cancel{G m_A M_E}}{R_p^2} = 2\,\frac{\cancel{G m_A M_E}}{R_E^2}$$

$$\Rightarrow R_p^2 = \tfrac{1}{2} R_E^2$$

$$\Rightarrow \boxed{R_p = \frac{R_E}{\sqrt{2}}} \Rightarrow \boxed{R_p = 4.50 \times 10^{6}\,m}$$

Wolfson Chapter 8 Question 36

Given

$d_1 = 15\,m$

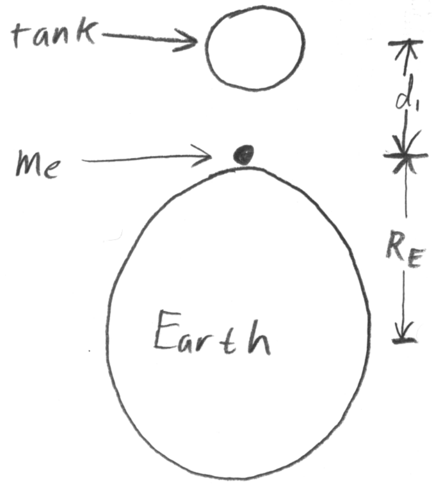
$R_E = 6.37\times10^6\,m$

$m_t = 4\times10^6\,kg$

$m_E = 5.97\times10^{24}\,kg$

Want

$$\frac{|\vec{F}_{mt}|}{|\vec{F}_{mE}|}$$



$$|\vec{F}_{mt}| = \frac{G m_m m_t}{d_1^2}$$

$$|\vec{F}_{mE}| = \frac{G m_m m_E}{R_E^2}$$

So: $$\frac{|\vec{F}_{mt}|}{|\vec{F}_{mE}|} = \frac{\cancel{G m_m} m_t}{d_1^2} \cdot \frac{R_E^2}{\cancel{G m_m} m_E} \Rightarrow \boxed{\frac{|\vec{F}_{mt}|}{|\vec{F}_{mE}|} = \frac{R_E^2}{d_1^2} \cdot \frac{m_t}{m_E}}$$

$$\boxed{\frac{|\vec{F}_{mt}|}{|\vec{F}_{mE}|} = 1.21\times10^{-7}}$$

a bit less than 1...