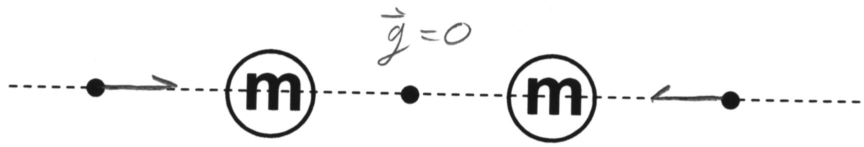
Electrostatics – Set 2

Name: ____________________

Problems Solved __/ 8

For each figure, draw the net gravitational <u>field</u> vector, $\vec{g}_{net}$, at each of the points marked with a dot (a location in empty space). Or, if appropriate, label the dot with "$\vec{g} = 0$."

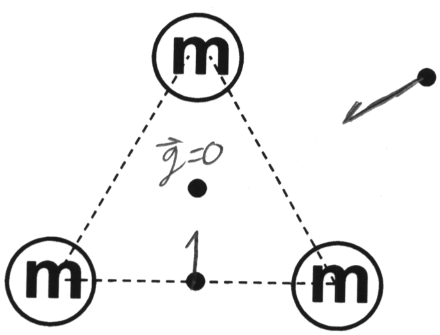
(a)



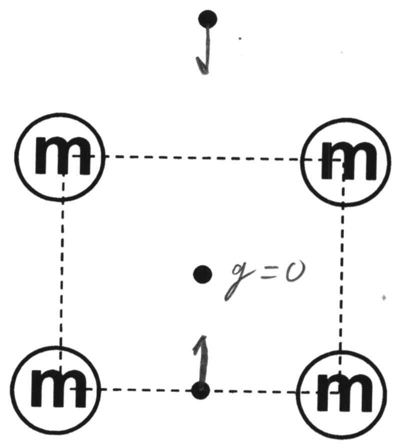
Is there a force acting on anything at the location of these dots? No!

What is located at the dots? The Field, or potential For a Force.

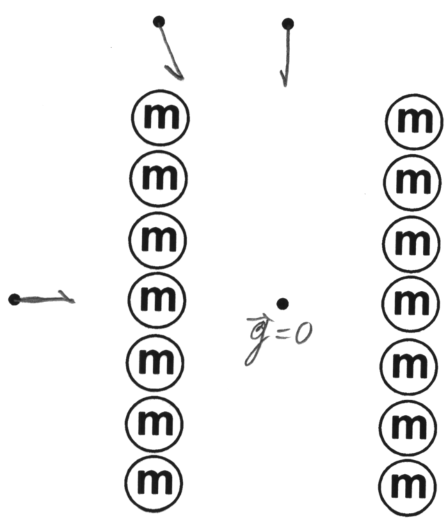
(b)



(c)



(d)

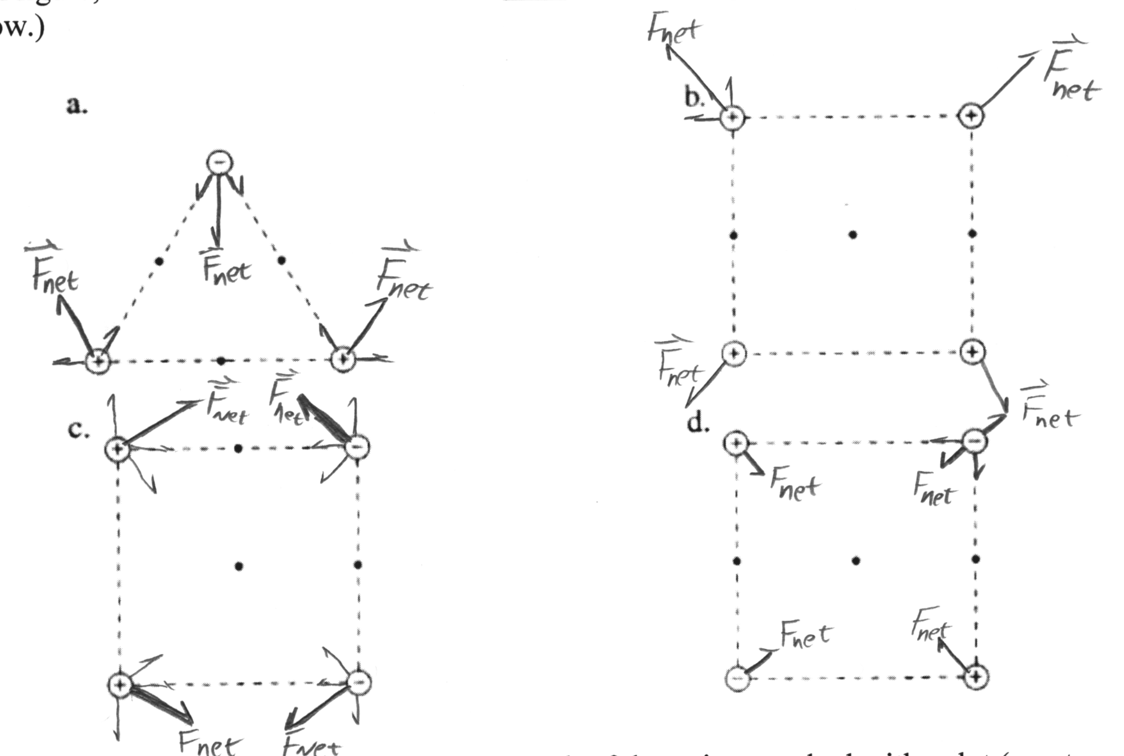


(i) If I place a mass at the location of the center dot in (d), what type of net force will be exerted on that mass?

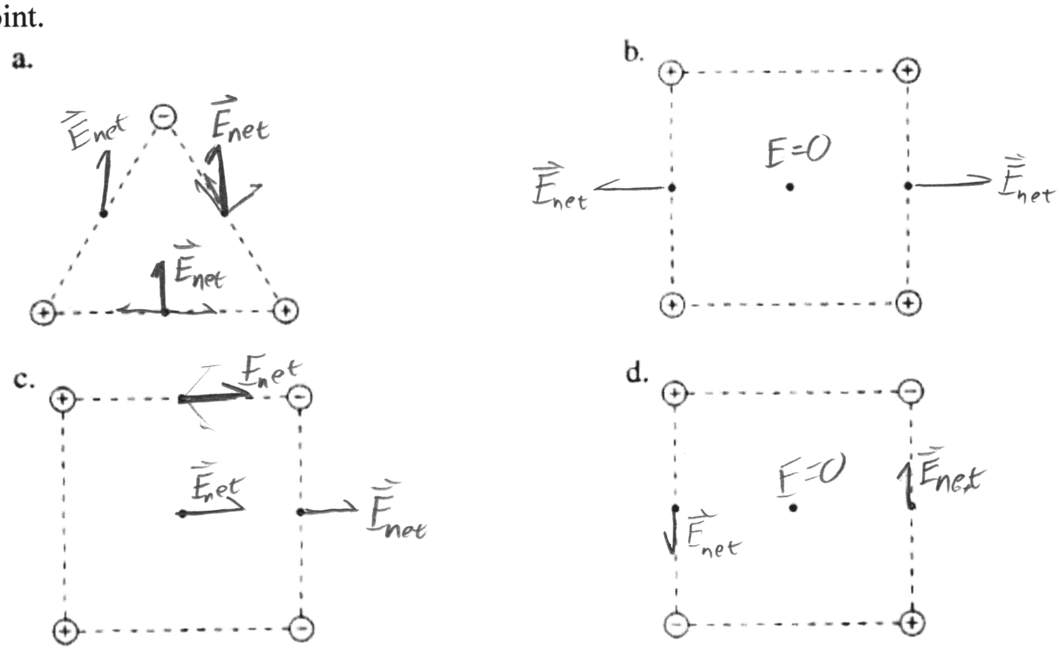
(ii) At the left-most dot?

(iii) How far away must I move the left-most dot in order to completely remove the net gravitational force acting on a mass at that dot?

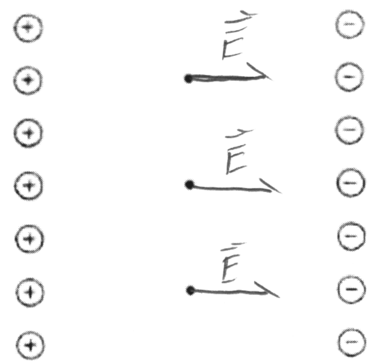
1. For each figure, draw and label the net electric <u>force</u> vector acting on each of the charges. (Ignore the dots for now.)



Now draw and label the net electric <u>field</u> vector at each of the points marked with a dot (empty space) or, if appropriate, label the dot E = 0. The lengths of your vectors should indicate the magnitude of the field at each point.

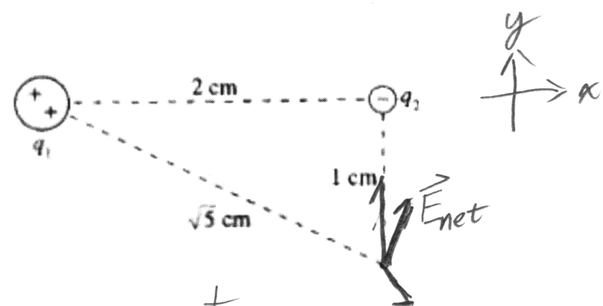


Draw the electric field vector at the three points marked with a dot. Hint:Use symmetry to determine if horizontal or vertical components cancel out.



At the position of the dot, draw the electric field due to each charge as well as the net electric field. Then, in the blanks, state whether the x- and y-components of the net field are positive or negative.

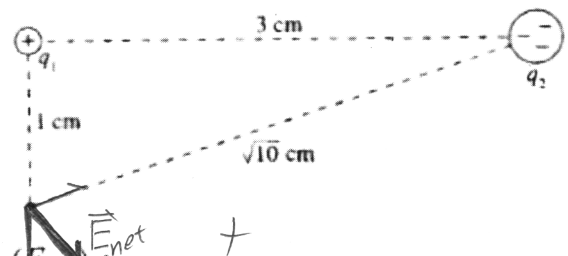
a.



$(E_{net})_x$: +

$(E_{net})_y$: +

b.

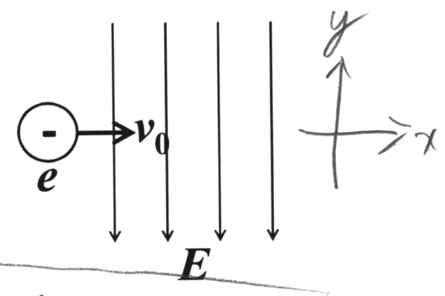


$(E_{net})_x$: +

$(E_{net})_y$: −

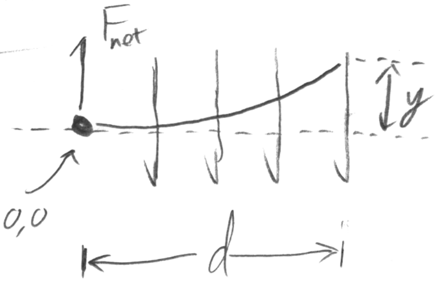
Electrosatics – Set 2

An electron with charge ~~q_e~~ and mass m_e is projected into a uniform electric field, $\vec{E} = -|\vec{E}|\hat{j}$, with an initial velocity $\vec{v}_0 = |\vec{v}_0|\hat{i}$ (perpendicular to the field).



(a) How far has the electron moved in the $\hat{j}$ direction after it has traveled a distance d in the $\hat{i}$ direction? Is the deflection in the positive or negative $\hat{j}$ direction?

Given	want
$\vec{E}$	$y\hat{j}$
$\vec{v}_0$	
q_e	
m_e	
d	



$$\boxed{\vec{F}_{net} = (-q_e)(-|\vec{E}|)\hat{j} = q_e|\vec{E}|\hat{j}}$$

$$\Rightarrow \boxed{\vec{a} = \frac{q_e|\vec{E}|}{m_e}\hat{j}}$$

x: $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$d = |\vec{v}_0|t \Rightarrow \boxed{t = \frac{d}{|\vec{v}_0|}}$

y: $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$\boxed{y = \frac{1}{2}a_y t^2}$

$$\boxed{y = \frac{1}{2}\frac{q_e|\vec{E}|}{m_e}\frac{d^2}{|\vec{v}_0|^2}}$$

(b) Write the final velocity in unit vector notation.

$v_x = v_{0x} + a_x t$

$\Rightarrow v_{xF} = |\vec{v}_0|$

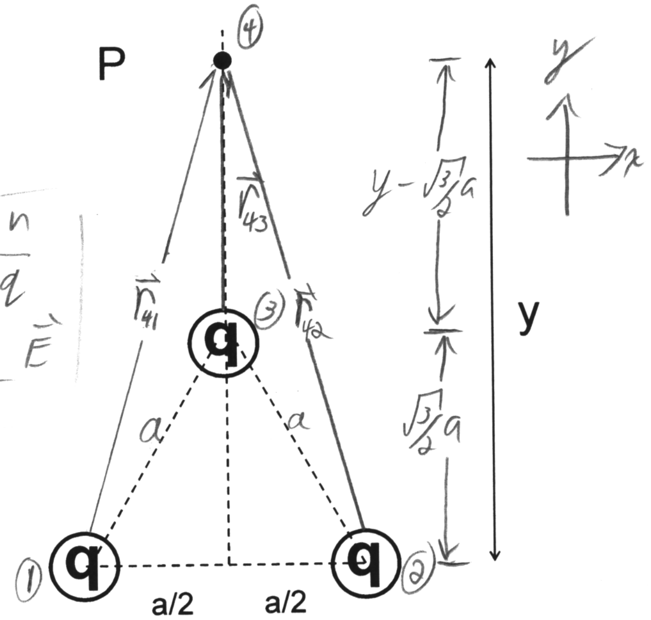
$v_y = v_{0y} + a_y t$

$\Rightarrow v_{yF} = \frac{q_e|\vec{E}|}{m_e}\frac{d}{|\vec{v}_0|}$

$$\boxed{\vec{v}_F = \left(|\vec{v}_0|\hat{i} + \frac{q_e|\vec{E}|d}{m_e|\vec{v}_0|}\hat{j}\right)}$$

~~(c) What are the magnitude and direction of its velocity vector at this time?~~

Three identical charges q form an equilateral triangle of side a.
(a) Find an expression for the electric field at an arbitrary point P that sits on the y-axis somewhere above the uppermost charge. Write your simplified answer in terms of k, q, y, and a. (Hint: First draw all individual E-fields at P and then label all important distances and angles.)



Given a, y, q
want: $\vec{E}$

$$\vec{E} = \vec{E}_{41} + \vec{E}_{42} + \vec{E}_{43}$$

$$= \frac{kq}{|\vec{r}_{41}|^2}\hat{r}_{41} + \frac{kq}{|\vec{r}_{42}|^2}\hat{r}_{42} + \frac{kq}{|\vec{r}_{43}|^2}\hat{r}_{43}$$

$$= \frac{kq}{|\vec{r}_{41}|^2}\cdot\frac{\frac{a}{2}\hat{\imath} + y\hat{\jmath}}{|\vec{r}_{41}|} + \frac{kq}{|\vec{r}_{42}|^2}\cdot\frac{-\frac{a}{2}\hat{\imath} + y\hat{\jmath}}{|\vec{r}_{42}|}$$

$$+ \frac{kq}{|\vec{r}_{43}|^2}\,\frac{0\hat{\imath} + (y - \frac{\sqrt{3}}{2}a)}{|\vec{r}_{43}|}$$

$\hat{\imath}$ component cancels!

$$= kq\left[\left(\frac{1}{|\vec{r}_{41}|^3}\left(\frac{a}{2} - \frac{a}{2}\right)\right)\hat{\imath}\right.$$

$$\left. + \left(\frac{2y}{(\frac{a^2}{4} + y^2)^{3/2}} + \frac{\cancel{(y - \frac{\sqrt{3}}{2}a)}}{(y - \frac{\sqrt{3}}{2}a)^{\cancel{3}\,2}}\right)\hat{\jmath}\right]$$

$$|\vec{r}_{41}| = |\vec{r}_{42}| = \left(\frac{a^2}{4} + y^2\right)^{1/2}$$

$$|\vec{r}_{43}| = y - \frac{\sqrt{3}}{2}a$$

$$\vec{r}_{41} = \frac{a}{2}\hat{\imath} + y\hat{\jmath}, \quad \vec{r}_{42} = -\frac{a}{2}\hat{\imath} + y\hat{\jmath}$$

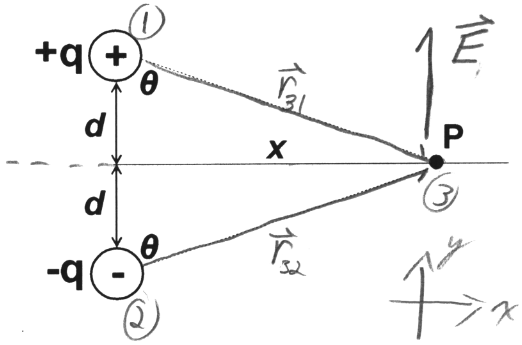
$$\boxed{\vec{E} = kq\left(\frac{2y}{(\frac{a^2}{4} + y^2)^{3/2}} + \frac{1}{(y - \frac{\sqrt{3}}{2}a)^2}\right)\hat{\jmath}}$$

(b) Show that your result reduces to the field of a point charge $3q$ for the condition $y >> a$ (i.e., when a effectively goes to 0).

as y grows, $\left(\frac{a^2}{4} + y^2\right)^{3/2} \to y^3$ and $\left(y - \frac{\sqrt{3}}{2}a\right)^2 \to y^2$

Then: $\vec{E} \to kq\left(\frac{2y}{y^3} + \frac{1}{y^2}\right)\hat{\jmath} = \boxed{\frac{k3q}{y^2}}$

* Same result as a charge of $3q$ at the origin.



The figure shows positive and negative charges of equal magnitude placed a distance $2d$ apart. This configuration is called an electric dipole, and it plays a very important role in chemistry.

(a) Derive an expression for the electric field due to these charges at point P, a distance x along the perpendicular bisector of the line joining the charges.

(b) If a charge Q is placed at point P, what net electric force vector acts on it?

(c) Show what happens to the electric field at P if $x >> d$. Explain your result.

Given
d, x, q
want
$\vec{E}$

$$|\vec{r}_{31}| = |\vec{r}_{32}| = (x^2 + d^2)^{1/2}$$
$$\vec{r}_{31} = (x\hat{\imath} + d\hat{\jmath})$$
$$\vec{r}_{32} = (x\hat{\imath} - d\hat{\jmath})$$

a) $$\vec{E} = \vec{E}_{31} + \vec{E}_{32}$$
$$= \frac{kq}{|\vec{r}_{31}|^2}\hat{r}_{31} + \frac{k(-q)}{|\vec{r}_{32}|^2}\hat{r}_{32}$$
$$= \frac{kq}{|\vec{r}_{31}|^2}\cdot\frac{\vec{r}_{31}}{|\vec{r}_{31}|} - \frac{kq}{|\vec{r}_{32}|^2}\cdot\frac{\vec{r}_{32}}{|\vec{r}_{32}|}$$
$$= \frac{kq}{|\vec{r}_{31}|^3}(x\hat{\imath} + d\hat{\jmath}) - \frac{kq}{|\vec{r}_{32}|^3}(x\hat{\imath} - d\hat{\jmath})$$
$$= \frac{kq}{(x^2+d^2)^{3/2}}\left[\cancel{(x - x)\hat{\imath}} + (d+d)\hat{\jmath}\right]$$

$$\Rightarrow \boxed{\vec{E} = \frac{2kqd}{(x^2+d^2)^{3/2}}\hat{\jmath}}$$

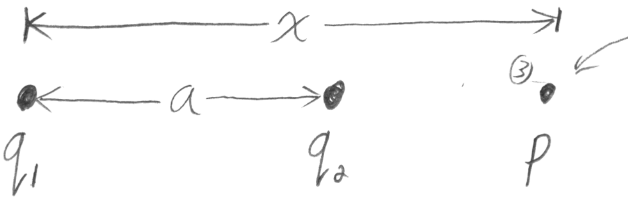
b) $$\vec{F} = Q\vec{E} \Rightarrow \boxed{\vec{F} = \frac{2kqQd}{(x^2+d^2)^{3/2}}\hat{\jmath}}$$

c) when $x >> d$, $(x^2+d^2)^{3/2} \rightarrow (x^2)^{3/2} = x^3$

Then: $\boxed{\vec{E} \rightarrow \frac{2kqd}{x^3}\hat{\jmath}}$, Field (and force) drop as $\frac{1}{x^3}$!

Wolfson 20.40

Given	Want
$q_1 = 3q$	x where $\lvert E \rvert = 0$
$q_2 = -2q$	
a	



P must be here to get opposing Fields

$$\vec{E}_{net} = \vec{E}_{31} + \vec{E}_{32}$$

$$= \frac{k(3q)}{|\vec{r}_{31}|^2}\hat{r}_{31} + \frac{k(-2q)}{|\vec{r}_{32}|^2}\hat{r}_{32}$$

$\vec{r}_{31} = x\hat{\imath}$

$\vec{r}_{32} = (x-a)\hat{\imath}$

$$= \frac{3kq}{|\vec{r}_{31}|^2}\frac{\vec{r}_{31}}{|\vec{r}_{31}|} - \frac{2kq}{|\vec{r}_{32}|^2}\frac{\vec{r}_{32}}{|\vec{r}_{32}|}$$

$$= kq\left[3\frac{\cancel{x}}{x^{\cancel{3}\,2}}\hat{\imath} - 2\frac{\cancel{(x-a)}}{(x-a)^{\cancel{3}\,2}}\hat{\imath}\right]$$

$$\vec{E}_{net} = kq\left(\frac{3}{x^2} - \frac{2}{(x-a)^2}\right)\hat{\imath}$$

$$\text{want: } |\vec{E}| = kq\left(\frac{3}{x^2} - \frac{2}{(x-a)^2}\right) = 0$$

$$\Rightarrow \frac{3}{x^2} = \frac{2}{(x-a)^2} \Rightarrow 3(x-a)^2 = 2x^2$$

$$\Rightarrow \sqrt{3}(x-a) = \sqrt{2}x$$

$$\Rightarrow x = \frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}a \Rightarrow \boxed{x = 5.45a}$$

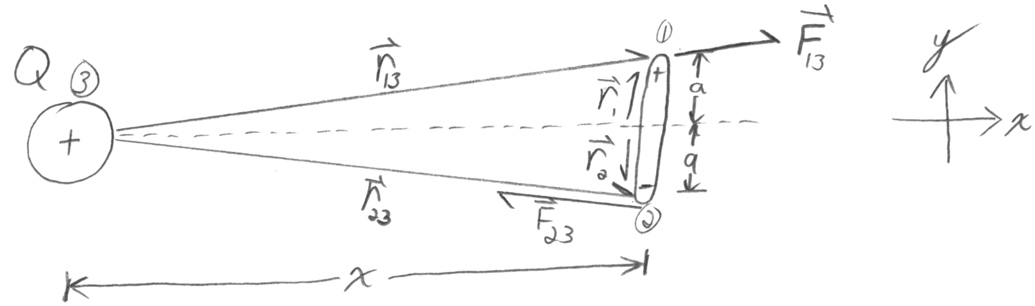
Wolfson 20.63

Given

Q, q, x, a

want

$\vec{T}_{net}, \vec{F}_{net}$ at $x \gg a$



$\vec{r}_1 = a\hat{j}\ ,\ \vec{r}_2 = -a\hat{j}$

$\vec{r}_{13} = (x\hat{i} + a\hat{j})$

$\vec{r}_{23} = (x\hat{i} - a\hat{j})$

a) Find net Torque

$$\vec{T}_{net} = \vec{T}_{13} + \vec{T}_{12}$$

$$= \vec{r}_1 \times \vec{F}_{13} + \vec{r}_2 \times \vec{F}_{23}$$

$$= (a\hat{j}) \times \frac{kQq}{|\vec{r}_{13}|^2}\hat{r}_{13} + (-a\hat{j}) \times \frac{-kQq}{|\vec{r}_{23}|^2}\hat{r}_{23}$$

$$= a\hat{j} \times \frac{kQq}{|\vec{r}_{13}|^3}(x\hat{i} + a\hat{j}) - a\hat{j} \times \frac{-kQq}{|\vec{r}_{23}|^3}(x\hat{i} - a\hat{j})$$

$$\boxed{* \quad \hat{j} \times \hat{i} = -\hat{k} \quad \text{and} \quad \hat{j} \times \hat{j} = 0}$$

$$= \frac{kQqax}{(x^2+a^2)^{3/2}}(\hat{j} \times \hat{i}) + \frac{kQqax}{(x^2+a^2)^{3/2}}(\hat{j} \times \hat{i})$$

$$\vec{T}_{net} = -\frac{kQqax}{(x^2+a^2)^{3/2}}\hat{k}$$ and when $x \gg a$:

$$\boxed{\vec{T}_{net} \cong \frac{-kQqa}{x^2}\hat{k}}$$

b) $\vec{F}_{net} = \vec{F}_{13} + \vec{F}_{12}$

$$= \frac{kQq}{|\vec{r}_{13}|^2}\hat{r}_{13} - \frac{kQq}{|\vec{r}_{23}|^2}\hat{r}_{23}$$

$$= \frac{kQq}{|\vec{r}_{13}|^3}(x\hat{\imath} + a\hat{\jmath}) - \frac{kQq}{|\vec{r}_{23}|^3}(x\hat{\imath} - a\hat{\jmath})$$

$$= \frac{kQq}{(x^2+a^2)^{3/2}}\left[(x-x)\hat{\imath} + (a+a)\hat{\jmath}\right]$$

$$\Rightarrow \boxed{\vec{F}_{net} = \frac{2kQqa}{(x^2+a^2)^{3/2}}\hat{\jmath}}$$

and as $x \gg a$, $(x^2+a^2)^{3/2} \rightarrow x^3$

Then: $\boxed{\vec{F}_{net} \rightarrow \frac{2kQqa}{x^3}\hat{\jmath}}$