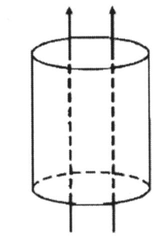
Electrostatics – Set 5

Name: ____________________

Problems Solved __/ 9

For each of the closed cylinders shown below, are the electric fluxes through the top, the side wall, and the bottom positive (+), negative (-), or zero (0)? Is the net flux +, -, or 0? (Arrows represent electric field lines.)

a.



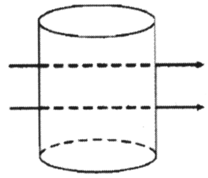
Φ_{top} = +

Φ_{wall} = 0

Φ_{bot} = −

Φ_{net} = 0

b.



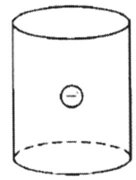
Φ_{top} = 0

Φ_{wall} = 0

Φ_{bot} = 0

Φ_{net} = 0

c.



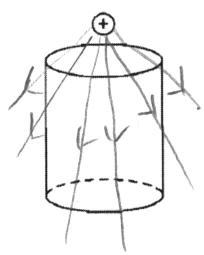
Φ_{top} = −

Φ_{wall} = −

Φ_{bot} = −

Φ_{net} = −

d.



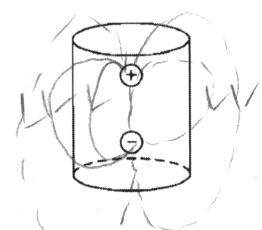
Φ_{top} = −

Φ_{wall} = +

Φ_{bot} = +

Φ_{net} = 0

e.



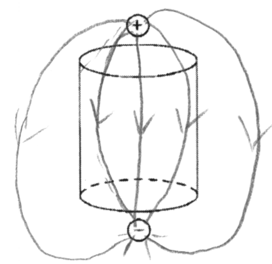
Φ_{top} = +

Φ_{wall} = 0

Φ_{bot} = −

Φ_{net} = 0

f.



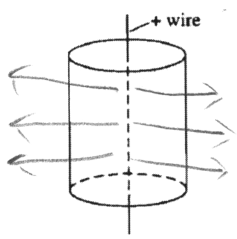
Φ_{top} = −

Φ_{wall} = 0

Φ_{bot} = +

Φ_{net} = 0

g.



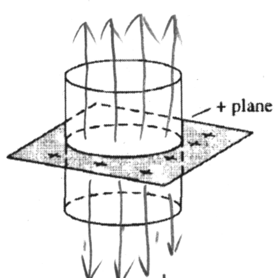
Φ_{top} = 0

Φ_{wall} = +

Φ_{bot} = 0

Φ_{net} = +

h.



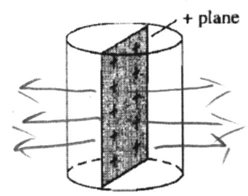
Φ_{top} = +

Φ_{wall} = 0

Φ_{bot} = +

Φ_{net} = +

i.



Φ_{top} = 0

Φ_{wall} = +

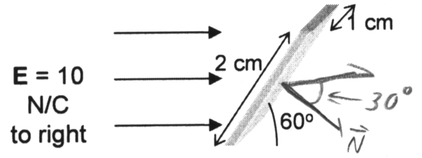
Φ_{bot} = 0

Φ_{net} = +

Calculate the net electric flux through the surfaces indicated.

$$\Phi = \int \vec{E}\cdot d\vec{A}$$

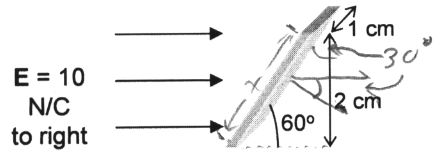
a)



$\vec{E}\cdot\vec{A} = 10\left(\frac{N}{C}\right)\cdot(2cm^2)\cos(30)$

$= 10\sqrt{3}\ N\cdot cm^2/C$

b)

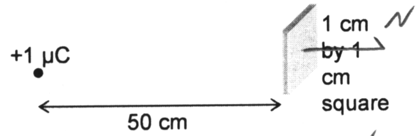


$\vec{E}\cdot\vec{A} = 10\left(\frac{N}{C}\right)\cdot(x\cos 30)(1cm)$

$= 20\left(\frac{N\cdot cm^2}{C}\right)$

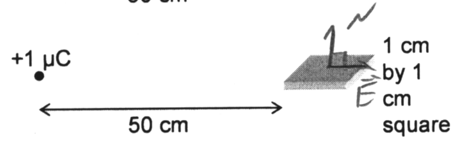
E is approx. uniform across square

c)



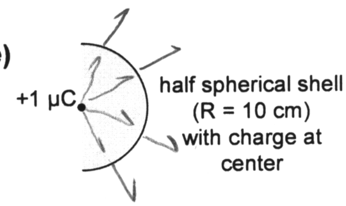
$\vec{E}\cdot\vec{A} = \frac{kq}{r^2}\cdot 1cm^2 = \frac{(9\times10^9)(1\times10^{-6}C)}{50^2} = 3.6\frac{N\cdot m^2}{C}$

d)



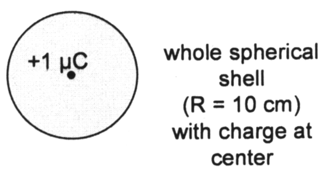
$\vec{E}\cdot\vec{A} = 0$

e)



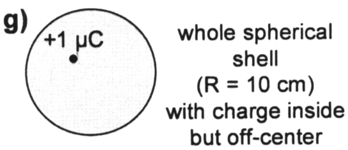
$\vec{E}\cdot\vec{A} = \frac{(9\times10^9)(1\times10^{-6})}{\cancel{10^2}}\cdot 2\pi(\cancel{10^2}) = 5.6\times10^4\ \frac{N\cdot m^2}{C}$

f)



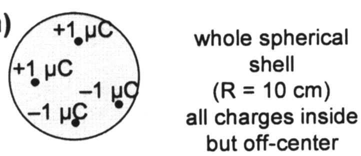
$\vec{E}\cdot\vec{A} = \frac{9\times10^9(1\times10^{-6})}{\cancel{10^2}}\cdot 4\pi(\cancel{10^2}) = 1.1\times10^5\ \frac{N\cdot m^2}{C}$

g)



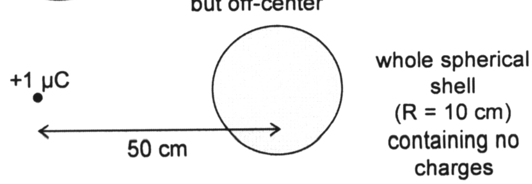
$\vec{E}\cdot\vec{A} = 1.1\times10^5\ \frac{N\cdot cm^2}{C}$

h)



$q_{enc} = 0 \Rightarrow \Phi = 0$

i)



$q_{enc} = 0 \Rightarrow \Phi = 0$

Consider this equation: $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

(a) Explain the equation in words.

The net flux through a surface is proportional to the total charge enclosed.

(b) Provide a name for each parameter and indicate its units (dimensions).

Φ_E = Electric flux $\left(\frac{N \cdot m^2}{C}\right)$

$\vec{E}$ = Electric field $\left(\frac{N}{C}\right)$

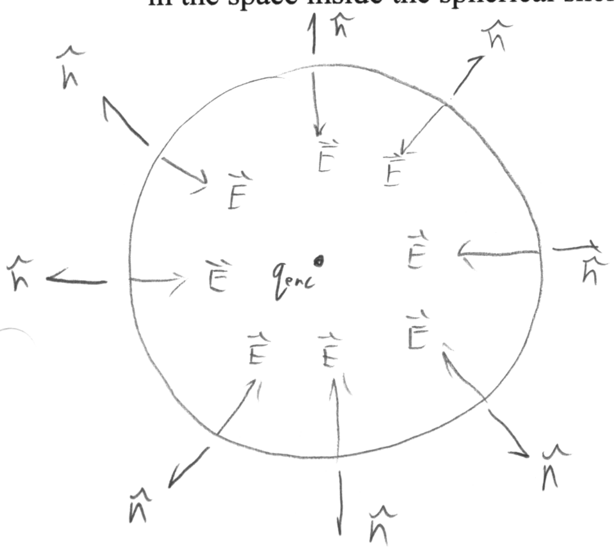
$\vec{A}$ = Directed area (m^2)

q_{in} = Charge enclosed by the surface

(c) What does the dot product do here?

Gives the component of the $\vec{E}$ field perpendicular to the surface, which effectively provides the cross setion presented to the field.

(d) A thin spherical shell of radius 0.750 m surrounds a collection of charged particles, but the sphere itself is not charged. The electric field everywhere at the location of the shell is measured to be 890 N/C and points radially toward the center of the sphere. Find the net charge contained in the space inside the spherical shell.

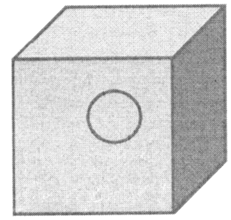


$\Phi_E = \frac{q_{enc}}{\epsilon_0} \Rightarrow q_{enc} = \epsilon_0 \Phi_E$

$\Phi_E = \vec{E} \cdot \vec{A} = -890 \, N/C \; 4\pi (0.75 m)^2 = -6.3 \times 10^3 \frac{N \cdot m^2}{C}$

$\Rightarrow q_{enc} = \left(8.8 \times 10^{-12} \frac{C^2}{N \cdot m^2}\right)\left(-6.3 \times 10^3 \frac{N \cdot m^2}{C}\right)$

$q_{enc} = -5.6 \times 10^{-8} C = \boxed{-56 \, nC}$



A charge of 170 μC is at the center of a cube of side length 80.0 cm.

(a) Find the total flux through the whole surface of the cube.
(b) Find the flux through each face of the cube.
(c) Speaking qualitatively, how would your answers above change if the charge were not at the cube's center?

a) $\Phi_E = \frac{q_{enc}}{\varepsilon_0} \Rightarrow \Phi_E = \frac{170 \times 10^{-6}\ C}{(8.8 \times 10^{-12}\ C^2/N\cdot m^2)}$

$$\boxed{\Phi_E = 1.9 \times 10^{7}\ \frac{N\cdot m^2}{C}}$$

b) There are six faces, so each face has 1/6th of the net flux.

$$\Phi_{E,\,1\,face} = \frac{1.97 \times 10^{7}\ \frac{N\cdot m^2}{C}}{6} = \boxed{3.2 \times 10^{6}\ \frac{N\cdot m^2}{C}}$$

c) Each face would have a different flux: Those closer to q would be higher, but those further away would be smaller by the same proportion. The net effect is that the net flux remains the same.

A solid sphere 2.0 cm in radius carries a uniform volume charge density. The electric field 1.0 cm from the sphere's center has a magnitude of 39,000 N/C and points outward from the sphere's center.

(a) What total charge is contained in the sphere?
(b) At what other distance does the electric field have a magnitude of 39,000 N/C?

a) Given

$R = 2.0\,cm$

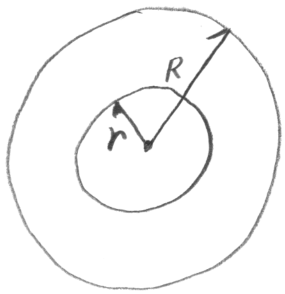
$\vec{E}_r = 3.9\times10^4\ N/C$

$r = 1.0\,cm$

$R = 2.0\,cm$

want

Q



Charge density:

$$\boxed{\rho = \frac{Q}{\frac{4}{3}\pi R^3}}$$

Gauss' Law

$$\Phi_E = \frac{q_{enc}}{\epsilon_0}, \quad \Phi_E = \vec{E}_r\cdot\vec{A}_r = E_r 4\pi r^2$$

$$q_{enc} = \rho V_r = \frac{Q}{\cancel{\frac{4}{3}\pi} R^3}\cdot\cancel{\frac{4}{3}\pi} r^3 = Q\frac{r^3}{R^3}$$

$$\Rightarrow E_r\cdot 4\pi \cancel{r^2} = \frac{Q}{\epsilon_0}\frac{r^{\cancel{3}}}{R^3}$$

$$\Rightarrow \boxed{Q = E_r 4\pi\epsilon_0 \frac{R^3}{r}} \Rightarrow \boxed{Q = (3.9\times10^4 \tfrac{N}{C})\,4\pi\,(8.8\times10^{-12})\frac{(2\times10^{-2})^3}{1\times10^{-2}} = 3.45\,nC}$$

b)

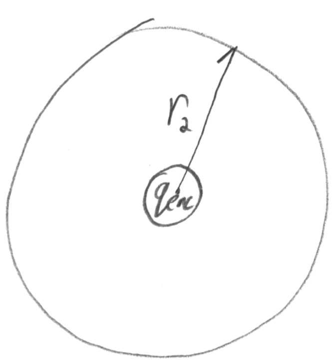
Given

Q_{enc}

E_{r_2}

want

r_2



$$\Phi_E = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E_r\cdot 4\pi r_2^2 = \frac{Q_{enc}}{\epsilon_0}$$

$$r_2^2 = \frac{1}{E_r}\cdot\frac{Q_{enc}}{4\pi\epsilon_0} = \frac{k\,Q_{enc}}{E_r}$$

$$\boxed{r = \left[\frac{k\,Q_{enc}}{E_r}\right]^{1/2}} \Rightarrow r = \left[\frac{9\times10^9\left(\frac{N\cdot m^2}{C^2}\right)3.45\times10^{-9}C}{3.9\times10^4\left(\frac{N}{C}\right)}\right.$$

$$\boxed{r = 0.028\,m}$$

Wolfson, Volume II, 2nd Edition, Problem 21.25

Gauss' Law: $\Phi_E = \frac{q_{enc}}{\epsilon_0}$

a) $q_{enc} = -q \Rightarrow \Phi_E = -\frac{q}{\epsilon_0}$

b) $q_{enc} = -2q \Rightarrow \Phi_E = -\frac{2q}{\epsilon_0}$

c) $q_{enc} = 0 \Rightarrow \Phi_E = 0$

d) $q_{enc} = 0 \Rightarrow \Phi_E = 0$

Wolfson, Volume II, 2nd Edition, Problem 21.29

Given

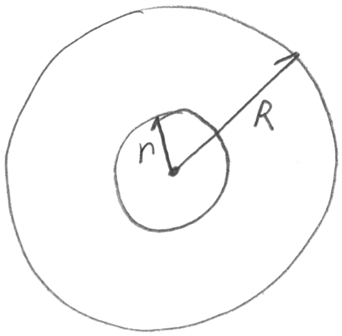
$R = 25\,cm$ $\quad r_a = 15\,cm$ $\quad r_c = 50\,cm$

$Q = 14\,\mu C$ $\quad r_b = 25\,cm$

want

E_r at each r

For a and b, $r < R$



$$\Phi_E = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E \cdot A = \frac{1}{\epsilon_0} Q \frac{\cancel{4/3}\cancel{\pi} r^3}{\cancel{4/3}\cancel{\pi} R^3}$$

$$\Rightarrow E \cdot 4\pi \cancel{r^2} = \frac{1}{\epsilon_0} Q \frac{r^{\cancel{3}}}{R^3}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} Q \frac{r}{R^3} \Rightarrow \boxed{E = \frac{kQ}{R^3} r}$$

a) $$\frac{kQ}{R^3} r_a = \frac{(9\times10^9 \frac{Nm^2}{C^2})(14\times10^{-6} C)}{(25\times10^{-2})^3}(15\times10^{-2}) = \boxed{1.2\times10^6 \frac{N}{C}}$$

b) $$r_b = R \Rightarrow E = \frac{kQ}{R^{\cancel{3}2}}\cancel{R} = \frac{kQ}{R^2} = \frac{(9\times10^9 \frac{Nm^2}{C^2})(14\times10^{-6} C)}{(25\times10^{-2})^2}$$

$$\boxed{= 2.0\times10^6 \,N/C}$$

c) $$\Phi_E = \frac{q_{enc}}{\epsilon_0} \Rightarrow E\cdot 4\pi r_c^2 = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E = \frac{kQ}{r_c^2} = 5.0\times10^5 \,N/C}$$

Wolfson, Volume II, 2nd Edition, Problem 21.46

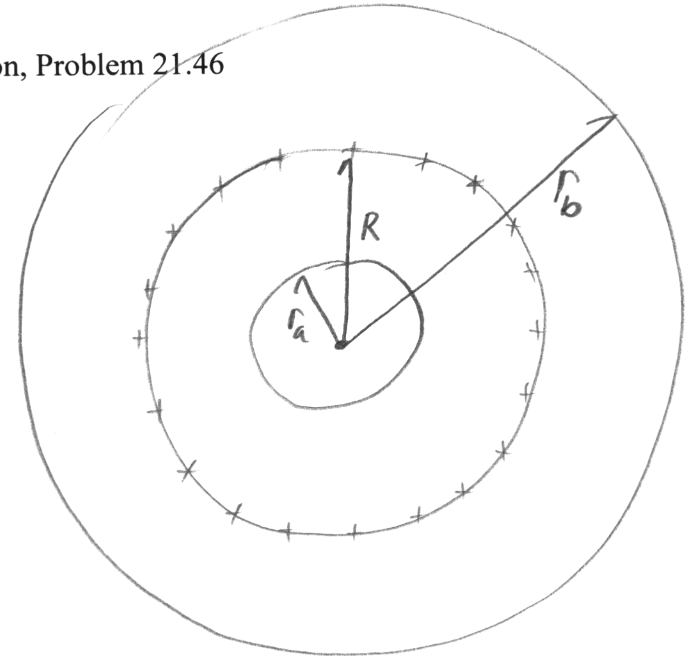
Given

$R = 70cm \quad r_a = 50cm$

$\vec{E}_R = 26 \text{ kN/C} \quad r_b = 190cm$

Want

E_a, E_b, Q_{net}



a) $\Phi_E = \frac{Q_{enc}}{\epsilon_0}$

But at r_a, $Q_{enc} = \emptyset \Rightarrow \Phi = 0 \Rightarrow \boxed{E_a = 0}$

b) $\Phi_{Eb} = \frac{Q_{enc}}{\epsilon_0}$, $Q_{enc} = \epsilon_0 \Phi_{ER} = \epsilon_0 \vec{E}_R \cdot \vec{A}_R = \epsilon_0 E_r 4\pi R^2$

$\Phi_{Eb} = \frac{\cancel{\epsilon_0} E_R 4\pi R^2}{\cancel{\epsilon_0}} \Rightarrow E_b A_b = E_R 4\pi R^2$

$\Rightarrow E_b \cdot \cancel{4\pi} r_b^2 = E_R \cancel{4\pi} R^2$

$\Rightarrow \boxed{E_b = E_R \frac{R^2}{r_b^2}} \Rightarrow E_b = 26 \times 10^3 \text{ N/C} \cdot \frac{(70cm)^2}{(190cm)^2}$

$\boxed{= 3.5 \times 10^3 \text{ N/C}}$

c) $Q_{enc} = \epsilon_0 \Phi_{ER} \Rightarrow \boxed{Q_{enc} = \epsilon_0 E_R 4\pi R^2}$

$Q_{enc} = (8.8 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(26 \times 10^3 \text{ N/C}) 4\pi (70 \times 10^{-2} m)^2$

$\boxed{= 1 \times 10^{-6} C}$

Wolfson, Volume II, 2[nd] Edition, Problem 21.48

Given

$q_1 = -2Q$

R

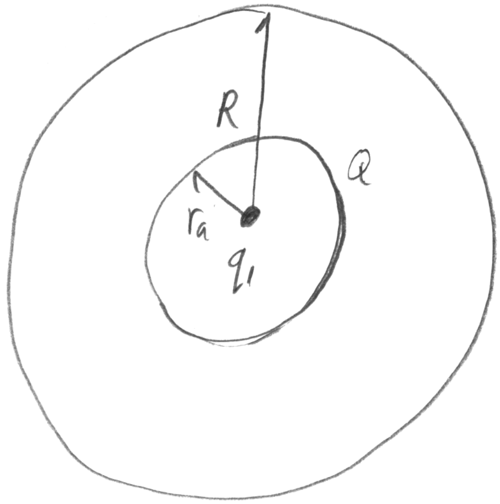
Q

$r_a = \frac{1}{2}R$

$r_b = 2R$

Want

E_a, E_b



a) $\Phi_E = \frac{q_{enc}}{\epsilon_0}$

at r_a, $q_{enc} = -2Q$

So: $E_a \cdot 4\pi r_a^2 = \frac{-2Q}{\epsilon_0} \Rightarrow E_a = \frac{-2kQ}{r_a^2} \Rightarrow \boxed{E_a = \frac{-8kQ}{R^2}}$

b) at r_b, $Q_{enc} = -2Q + Q \Rightarrow \boxed{Q_{enc} = -Q}$

So: $E_b \cdot 4\pi r_b^2 = \frac{-Q}{\epsilon_0} \Rightarrow \boxed{E_b = \frac{-kQ}{4R^2}}$

c) If the charge on the shell were doubled,

Answer (a) wouldn't change as Q_{enc} wouldn't change

Answer (b) would go to zero, however, since

$\underline{Q_{enc} = -2Q + 2Q = 0}$