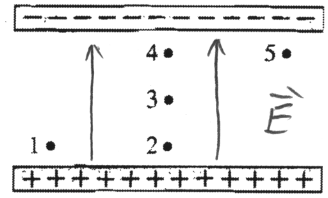
Name: ______________________ **Problems Solved** __/ 9

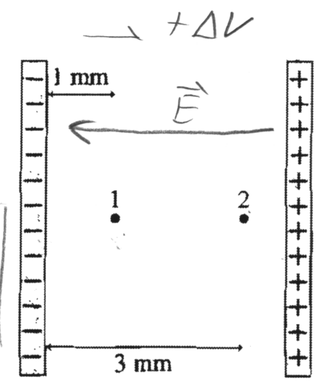
Rank in order from largest to smallest the electric potentials V_1 to V_5 in the picture. Does it matter what reference point you use?



V decreases from + plate to − plate

$V_4 = V_5 < V_3 < V_1 = V_2$

The figure to the right shows two points inside a capacitor.



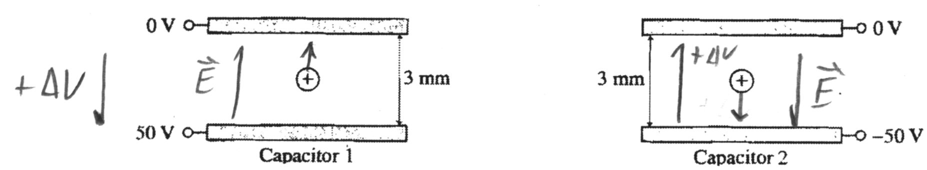
(a) What is the ratio of the electric potential differences $\frac{\Delta V_2}{\Delta V_1}$ with respect to the negative plate?

$$\Delta V = \frac{q\sigma}{\epsilon_0}\Delta x \Rightarrow \frac{\Delta V_2}{\Delta V_1} = \frac{\cancel{q\sigma}}{\cancel{\epsilon_0}}\Delta x_2 \frac{\cancel{\epsilon_0}}{\cancel{q\sigma}}\frac{1}{\Delta x_1} \Rightarrow \boxed{\frac{\Delta V_2}{\Delta V_1} = \frac{3}{1}}$$

(b) What is the ratio, $\frac{E_2}{E_1}$, of the electric field strength at these two points?

$\frac{E_2}{E_1} = 1$ because E is constant.

The figure shows two capacitors (sets of charged parallel plates), each with a 3 mm separation. A proton is released from rest in the center of each capacitor.



(a) Draw an arrow on each proton to show the direction it moves.

(b) Which proton reaches a capacitor plate first? Or are they simultaneous? Explain.

They reach the plate simultaneously.

Same ΔV, same $\vec{E}$, same force.

But opposite directions

A capacitor with plates separated by a distance d is charged to a potential difference ΔV_c. Then the two plates are pulled apart to a new separation of distance 2d. (Assume that the plates are very large compared to the separation distances.)

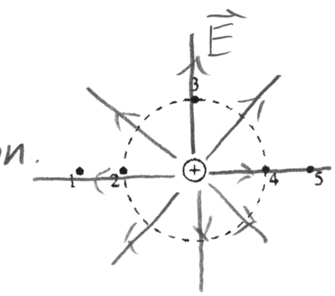
(a) Does the electric field strength E change as the separation increase? If so, by what factor? If not, why not?

$\vec{E}$ does not change. $|\vec{E}| = \frac{\sigma}{\epsilon_0}$ for a capacitor.

(b) Does the potential difference ΔV_c change as the separation increases? If so, by what factor? If not, why not?

Yes! $\Delta V = \frac{q\sigma}{\epsilon_0} 2d$, Increasing by a factor of 2

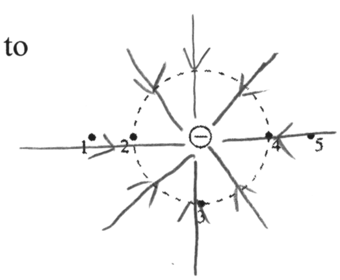
Rank the electric potentials V_1 to V_5 in order from largest to smallest.



ΔV decreases along $\vec{E}$ in the $\hat{r}$ direction.

$V_2 = V_3 = V_4 > V_1 = V_5$

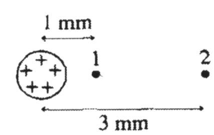
Rank in order, from most positive to most negative, the electric potentials V_1 to V_5 at the points shown.



ΔV decreases along $\vec{E}$ in the $-\hat{r}$ dir.

$V_1 = V_5 > V_2 = V_3 = V_4$

The figure shows two points near a positive point charge.



(a) What is the ratio of the potential differences $\frac{\Delta V_1}{\Delta V_2}$ with respect to infinity .

$\Delta V = \frac{kQ}{r}$ → with respect to Infinity

$\Rightarrow \frac{\Delta V_1}{\Delta V_2} = \frac{r_2}{r_1} \Rightarrow \frac{\Delta V_1}{\Delta V_2} = \frac{3}{1}$

(b) What is the ratio of the electric field strengths $\frac{E_1}{E_2}$ at these two points?

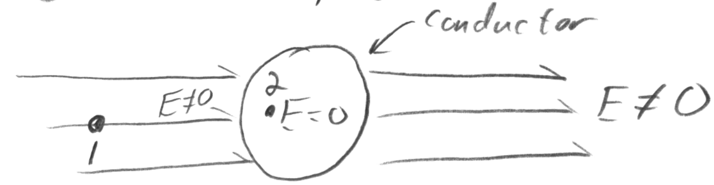
$E = \frac{kQ}{r^2} \Rightarrow \frac{E_1}{E_2} = \frac{r_2^2}{r_1^2} \Rightarrow \frac{E_1}{E_2} = 9$

Suppose that E = 0 V/m throughout some region of space. Can you conclude that V = 0 V in this region? Explain.

No!

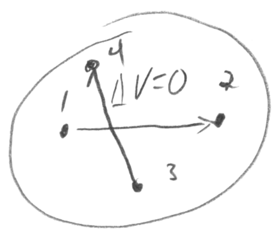
V only makes sense with respect to a reference

$\Delta V_{12} \neq 0$



Suppose that V = 0 V throughout some region of space. Can you conclude that E = 0 V/m in this region? Explain.

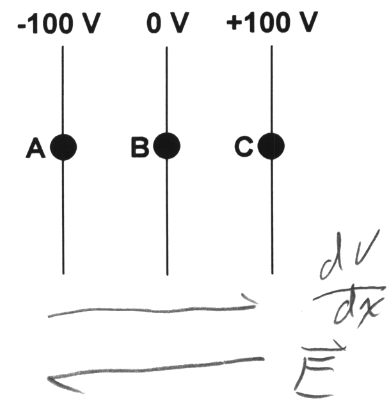
Yes.



If for every path in the space, $\Delta V = 0$, Then $\Delta V = \int \vec{E} \cdot d\vec{s} = 0, \Rightarrow \boxed{E = 0}$

A proton is released from rest at a point B, where the potential is 0 V. Afterward, the proton

(a) Remains at rest at B.
(b) Moves toward A with steady speed.
(c) Moves toward A with an increasing speed.
(d) Moves toward C with a steady speed.
(e) Moves toward C with an increasing speed.

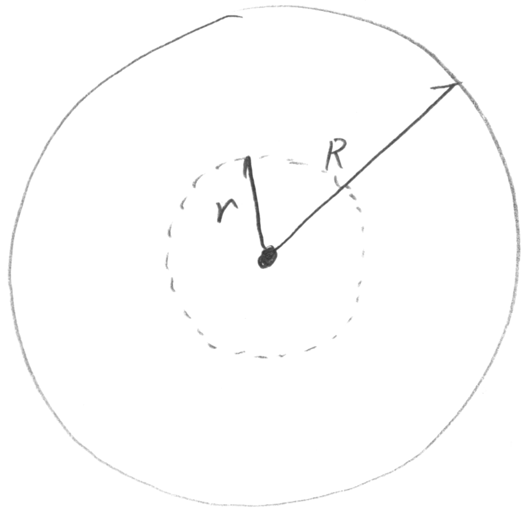


What is the answer if the proton is replaced by an electron?

The proton will gain Energy as it moves to lower potentials

A solid spherical insulator of radius R has a total charge Q distributed uniformly throughout its volume. Find the electric potential at the sphere's center with respect to infinity using $\Delta V = -\int \vec{E}\cdot d\vec{r}$.

Technique: Use **Gauss's Law** to find the electric field both outside and inside. Then, find the potential at the center by adding two the integrals, from r=0 to r=R and from r=R to $r = \infty$.



Find $\vec{E}$

$r < R$: $\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{\rho V}{\epsilon_0}$

$$A = 4\pi r^2,\quad \rho = \frac{Q}{\frac{4}{3}\pi R^3},\quad V = \frac{4}{3}\pi r^3$$

$$\Rightarrow E \cdot 4\pi \cancel{r^2} = \frac{1}{\epsilon_0}\frac{Q}{\cancel{\tfrac{4}{3}\pi} R^3}\cdot \cancel{\tfrac{4}{3}\pi} r^{\cancel{3}}$$

$$\Rightarrow \boxed{\vec{E}_{in} = \frac{Q}{4\pi\epsilon_0 R^3} r\,\hat{r}}$$

Entire sphere

$r > R$: $\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{Q}{\epsilon_0}$

$$\Rightarrow E\,4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \boxed{\vec{E}_{out} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}}$$

Find ΔV

$$\Delta V = -\int \vec{E}\cdot d\vec{r}$$

$$= -\int_0^R \vec{E}_{in}\cdot d\vec{r} - \int_R^\infty \vec{E}_{out}\cdot d\vec{r}$$

$$= -\int_0^R \frac{Q\,r}{4\pi\epsilon_0 R^3}\,dr - \int_R^\infty \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

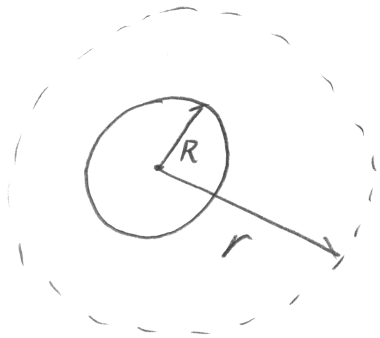
$$= -\frac{Q}{4\pi\epsilon_0}\left[\int_0^R \frac{r}{R^3}dr + \int_R^\infty \frac{1}{r^2}dr\right] = -\frac{Q}{4\pi\epsilon_0}\left[\left(\frac{1}{2}\frac{r^2}{R^3}\right)\Big|_0^R + \left(-\frac{1}{r}\right)\Big|_R^\infty\right]$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[\frac{1}{2R} + \frac{1}{R}\right] \rightarrow \text{From } 0 \text{ to } \infty$$

$\Delta V_{12} = -\Delta V_{21}$, so From ∞ to 0, $\boxed{\Delta V_{\infty 0} = \frac{3Q}{8\pi\epsilon_0}\frac{1}{R}}$

A solid spherical insulator of radius R has a total charge Q distributed uniformly throughout its volume.

Find the velocity of a particle of charge $-q$ and mass m released from rest at infinity as it reaches the sphere's surface.



What is $\vec{E}$ outside the sphere?

$$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E\,4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi r^2 \epsilon_0}\hat{r}}$$

What's ΔV for the trip?

* Integrate trip up (positive direction) then change the sign! (or be sad)

$$\Delta V = -\int_R^\infty \vec{E}\cdot d\vec{s} = -\int_R^\infty \frac{Q}{4\pi r^2 \epsilon_0}\,dr = -\frac{Q}{4\pi\epsilon_0}\int_R^\infty \frac{1}{r^2}\,dr = -\frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right|_R^\infty$$

$$\Rightarrow \Delta V = -\frac{Q}{4\pi\epsilon_0 R} \quad \text{for trip up}$$

$$\Rightarrow \boxed{\Delta V = \frac{Q}{4\pi\epsilon_0 R} \quad \text{for trip down}}$$

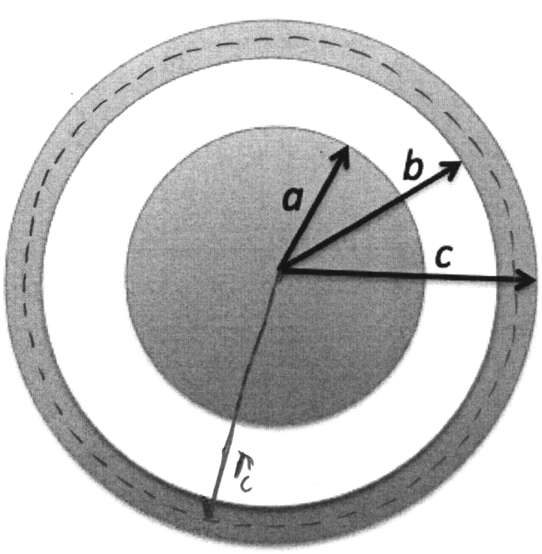
Use Work Energy Theorem to find V

$$-\Delta U + W_{NCF} = \Delta K, \quad W_{NCF} = 0, \quad \Delta U = q\Delta V$$

$$\Delta U = \frac{-qQ}{4\pi\epsilon_0 R} \Rightarrow -\left(\frac{-qQ}{4\pi\epsilon_0 R}\right) = \frac{1}{2}mV_F^2 - \cancel{\frac{1}{2}mV_I^2}^{\,0}$$

$$\Rightarrow \boxed{V = \left[\frac{qQ}{2\pi\epsilon_0 m R}\right]^{1/2}}$$

A solid *conducting* sphere with net charge $+Q$ and radius of a is surrounded by a concentric *insulating* spherical shell with an inner radius of b and an outer radius of c. The shell has a net charge of $-Q$ uniformly distributed throughout its volume.



a) Find the potential difference from the center to point a.
b) Find the potential difference from point a to point b.
c) Find the potential difference from point b to point c. (Just set up the integral, don't solve it)
d) Find the potential difference from point c to infinity.

a) Inside the conductor, $\vec{E}=0$

$$\text{so: } \boxed{\Delta V = -\int_0^a \vec{E}\cdot d\vec{s} = 0}$$

b) Find $\vec{E}$ in this region: $\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}}$

$$\text{So: } \Delta V = -\int_a^b \frac{Q}{4\pi\epsilon_0 r^2}\,dr = \frac{-Q}{4\pi\epsilon_0}\int_a^b \frac{1}{r^2}\,dr = -\frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Bigg|_a^b$$

$$\Rightarrow \boxed{\Delta V = -\frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a^2} - \frac{1}{b^2}\right]}$$

c) Find $\vec{E}$ in this region:

$$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{1}{\epsilon_0}\left[Q + \rho V_r\right], \quad \rho = \frac{-Q}{\frac{4}{3}\pi(c^3-b^3)}$$

$$\Rightarrow E\cdot 4\pi r^2 = \frac{1}{\epsilon_0}\left[Q - Q\frac{\cancel{\frac{4}{3}\pi}(r^3-b^3)}{\cancel{\frac{4}{3}\pi}(c^3-b^3)}\right] \qquad V_r = \tfrac{4}{3}\pi(r^3-b^3)$$

$$\Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\left[1 - \frac{r^3-b^3}{c^3-b^3}\right]\hat{r} \Rightarrow \Delta V = -\int_b^c \vec{E}\cdot d\vec{s}$$

$$\Rightarrow \boxed{\Delta V = -\frac{Q}{4\pi\epsilon_0}\int_b^c \frac{1}{r^2}\left[1 - \frac{r^3-b^3}{c^3-b^3}\right]dr}$$

hmmm... okay.

continued ↓

Energy Set 2 - P6 continued

d) Find $\vec{E}$ outside:

$$\Phi = \frac{Q_{enc}}{\epsilon_0}, \quad Q_{enc} = Q - Q = 0$$

so: $$\boxed{\Delta V = -\int_c^\infty \vec{E} \cdot d\vec{s} = 0}$$