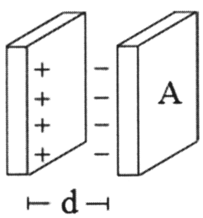
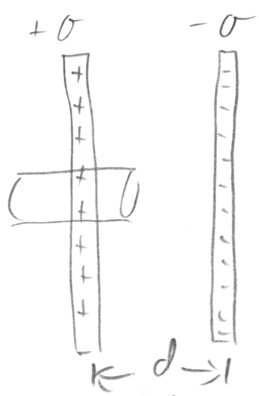
Name: ______________________ **Problems Solved __/ 7**

A capacitor is formed from two parallel plates. The plates are separated by a distance d and each have a surface area of A and hold a charge of + q and – q respectively. Calculate the missing values in the table.

The electric field should be in terms of σ and ε_o.
The potential across the gap should be in terms of q, A, d, and ε_o.
The capacitance should be in terms of A, d, and ε_o.

Be sure to show the details of your calculations and any assumptions you make.

System	Geometry	E-field in gap	ΔV across gap	$C = \frac{\lvert q \rvert}{\lvert \Delta V \rvert}$
Parallel Plate Capacitor		a) $E = \frac{\sigma}{\varepsilon_o}$	b) $\Delta V = -\frac{\sigma}{\varepsilon_o} d$	c) $C = \frac{\varepsilon_o A}{d}$



$$\Phi = \frac{Q_{enc}}{\varepsilon_o} \Rightarrow E 2\pi r^2 = \frac{\sigma \pi r^2}{\varepsilon_o} \Rightarrow E = \frac{\sigma}{2\varepsilon_o} \text{ 1 plate}$$

2 plates: $\boxed{E = \frac{\sigma}{\varepsilon_o}}$

$$\Delta V = -\int_0^d \vec{E} \cdot d\vec{s} = -\int_0^d \frac{\sigma}{\varepsilon_o} ds = -\frac{\sigma}{\varepsilon_o} d$$

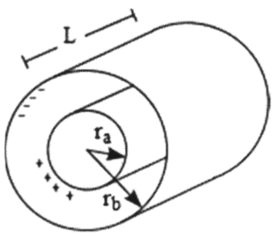
$$C = \frac{|q|}{|\Delta V|}, \quad q = \sigma A \Rightarrow C = \frac{\sigma A}{\sigma d} \varepsilon_o$$

$$\boxed{C = \frac{\varepsilon_o A}{d}}$$

A capacitor is formed from two concentric cylinders. The smaller cylinder has a radius of r_a and the larger cylinder has a radius of r_b. A positive charge of q is on the inner cylinder and a negative charge of q is on the outer cylinder. Calculate the missing values in the table.

The electric field should be in terms of q, π, ε_o, L and r.
The potential across the gap should be in terms of q, π, ε_o, L, r_a, and r_b.
The capacitance should be in terms of π, ε_o, L, r_a, and r_b.

Be sure to show the details of your calculations and any assumptions you make.

System	Geometry	E-field in gap	ΔV across gap	$C = \frac{\lvert q\rvert}{\lvert\Delta V\rvert}$
Cylindrical Capacitor		a) $\vec{E} = \frac{q}{2\pi\epsilon_o L r}\hat{r}$	b) $\Delta V = -\frac{q}{2\pi\epsilon_o L}\ln\left(\frac{r_b}{r_a}\right)$	c) $C = \frac{2\pi\epsilon_o L}{\ln\left(\frac{r_b}{r_a}\right)}$

$$\Phi = \frac{q_{enc}}{\epsilon_o} \Rightarrow E \cdot A = \frac{q}{\epsilon_o} \Rightarrow E \cdot 2\pi r L = \frac{q}{\epsilon_o}$$

$$\Rightarrow \vec{E} = \frac{q}{2\pi r L \epsilon_o}\hat{r}$$

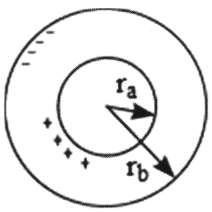
$$\Delta V = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{s} = -\int_{r_a}^{r_b} \frac{q}{2\pi r L \epsilon_o} dr = -\frac{q}{2\pi L \epsilon_o} \ln\left(\frac{r_b}{r_a}\right)$$

$$C = \frac{|q|}{|\Delta V|} = \frac{q}{q} 2\pi L \epsilon_o \ln\left(\frac{r_b}{r_a}\right)^{-1}$$

A capacitor is formed from two concentric spheres. The smaller sphere has a radius of r_a and the larger sphere has a radius of r_b. A positive charge of q is on the inner sphere and a negative charge of q is on the outer sphere. Calculate the missing values in the table. By sure to show the details of your calculations and any assumptions you make.

The electric field should be in terms of q, π, ε_o and r.
The potential across the gap should be in terms of q, π, ε_o, r_a, and r_b.
The capacitance should be in terms of π, ε_o, r_a, and r_b.

Be sure to show the details of your calculations and any assumptions you make.

System	Geometry	E-field in gap	ΔV across gap	$C = \frac{\lvert q \rvert}{\lvert \Delta V \rvert}$
Spherical Capacitor		a) $\vec{E} = \frac{q}{4\pi\varepsilon_o r^2}\hat{r}$	b) $\Delta V = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$	c) $C = 4\pi\varepsilon_o \frac{r_a r_b}{r_b - r_a}$

$$\Phi = \frac{Q_{enc}}{\varepsilon_o} \Rightarrow EA = \frac{q}{\varepsilon_o} \Rightarrow E 4\pi r^2 = \frac{q}{\varepsilon_o}$$

$$\Rightarrow \vec{E} = \frac{q}{4\pi\varepsilon_o r^2}\hat{r}$$

$$\Delta V = -\int_{r_a}^{r_b} \frac{q}{4\pi\varepsilon_o r^2} dr \Rightarrow \Delta V = -\frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = -\frac{q}{4\pi\varepsilon_o}\left(\frac{r_b - r_a}{r_a r_b}\right)$$

$$C = \frac{|q|}{|\Delta V|} = \frac{q}{q} 4\pi\varepsilon_o \left(\frac{r_a r_b}{r_b - r_a}\right)$$

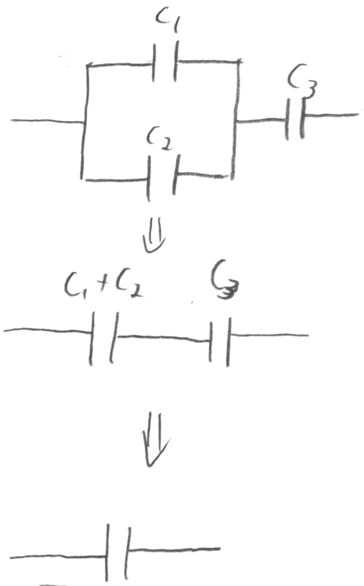
For the capacitor network shown in the figure:

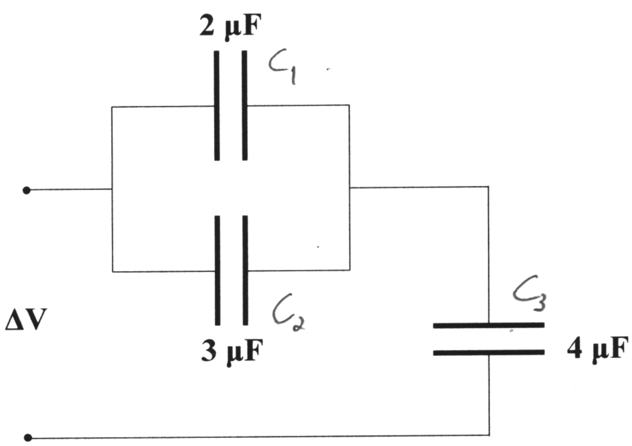
a) Find the equivalent capacitance of the network of three capacitors.

b) If ΔV = 10 V, find the charge on each capacitor and the voltage across each capacitor.

c) Find the total electric potential energy stored in this system.

a)





$$\frac{1}{C_1+C_2} + \frac{1}{C_3} = \frac{1}{C_T} \Rightarrow \frac{C_1+C_2+C_3}{C_3(C_1+C_2)} = \frac{1}{C_T} \Rightarrow C_T = \frac{C_3(C_1+C_2)}{C_1+C_2+C_3}$$

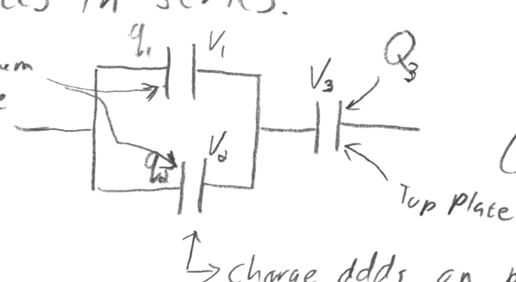
$$C_T = \frac{4(2+3)}{2+3+4} = 2.2\,\mu F$$

b) Must have same Q on plates in series.

$$V = V_1 + V_3$$

$$V_1 = V_2$$

* $C_T = \frac{Q_3}{V} \Rightarrow Q_3 = C_T V$



$$Q_3 = q_1 + q_2$$

* Given the charge on C_3 I can find the ΔV_3 there

$$C_3 = \frac{Q_3}{\Delta V_3} \Rightarrow \Delta V_3 = \frac{Q_3}{C_3} \Rightarrow \Delta V_3 = \frac{C_T}{C_3} V$$

continued

Energy Set 3, P5 - continued

* Now, C_1 and C_2 must have the same ΔV

$$\Delta V_1 = \Delta V_2 = \Delta V - \Delta V_3 = \Delta V - \frac{C_T}{C_3}\Delta V = \boxed{\Delta V\left(\frac{C_3 - C_T}{C_3}\right) = \Delta V_1 = \Delta V_2}$$

* And, Finally, Given the ΔV and C, we can find q_1 and q_2

$$q = CV \Rightarrow \quad q_1 = C_1 \Delta V_1 \Rightarrow \boxed{q_1 = C_1 \frac{C_3 - C_T}{C_3} \Delta V}$$

$$\boxed{q_2 = C_2 \frac{C_3 - C_T}{C_3} \Delta V}$$

* Plug in numbers at your leisure

c) Total Energy in the system:

$$\boxed{U = \tfrac{1}{2} C_T V^2}$$

Energy – Set 3 Page 5

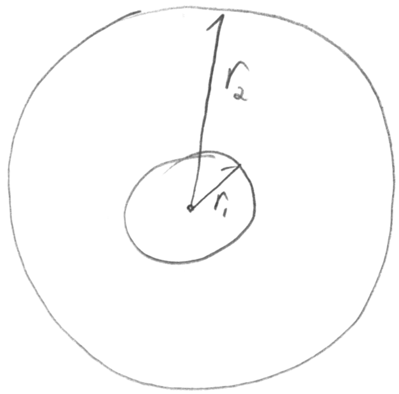
Consider a <u>spherical</u> capacitor whose concentric conductors have radii equal to 38.0 mm and 40.0 mm.

a) Calculate the system's capacitance by first finding the electric field and then the potential between the spheres. (Do so algebraically before plugging in numbers.)

b) Find the electric-field energy stored in this capacitor if the magnitude of charge on each sphere is 5.5 nC.

c) Consider a <u>parallel</u> <u>plate</u> capacitor with the same capacitance and same separation between plates as the spherical capacitor. What must be the area of each plate in the parallel plate capacitor for this condition to be met? How much electric-field energy is stored between the plates if the magnitude of charge on each plate is 5.5 nC?

a) $C = \frac{Q}{|\Delta V|}$ ⟹ Let's find ΔV



* Find E

$$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E 4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\Rightarrow \boxed{\vec{E} = \frac{kQ}{r^2}\hat{r}}$$

* Find ΔV

$$\Delta V = -\int \vec{E}\cdot d\vec{s} \Rightarrow \Delta V = -\int_{r_1}^{r_2} \frac{kQ}{r^2}dr \Rightarrow \Delta V = -kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) =$$

$$\Rightarrow |\Delta V| = kQ\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$$C = \frac{Q}{kQ}\frac{r_1 r_2}{r_2 - r_1} \Rightarrow \boxed{C = \frac{1}{k}\frac{r_1 r_2}{r_2 - r_1}}$$

$$\Rightarrow C = \frac{1}{9\times10^9}\frac{(38\times10^{-3}\,m)(40\times10^{-3}\,m)}{(40\times10^{-3}\,m) - (38\times10^{-3}\,m)} = \boxed{8.4\times10^{-11}\,F}$$

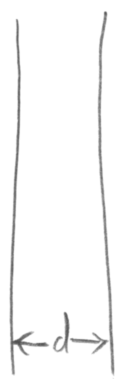
Energy Set 3, P5 - continued

b) $U = \frac{1}{2} C V^2$ and $C = \frac{Q}{|\Delta V|} \Rightarrow |\Delta V| = \frac{Q}{C}$

$$\Rightarrow U = \frac{1}{2} C \frac{Q^2}{C^2} \Rightarrow \boxed{U = \frac{1}{2} \frac{Q^2}{C}}$$

$$U = \frac{1}{2} \frac{(5.5 \times 10^{-9} C)^2}{(8.4 \times 10^{-11} F)} = \boxed{1.8 \times 10^{-7} J}$$

c)



parallel plate: $E = \frac{\sigma}{\epsilon_0}$

$$\Rightarrow \boxed{\Delta V = \frac{\sigma}{\epsilon_0} d}$$

$$C = \frac{Q}{\Delta V}, \quad Q = \sigma A$$

$$\Rightarrow C = \frac{\sigma A}{\sigma d} \epsilon_0 \Rightarrow \boxed{C = \frac{\epsilon_0 A}{d}}$$

assuming $d = r_2 - r_1$ from part a:

$$C_a = \frac{\epsilon_0 A}{r_2 - r_1} \Rightarrow \boxed{A = C_a (r_2 - r_1) \frac{1}{\epsilon_0}}$$

$$A = (8.4 \times 10^{-11})(40 \times 10^{-3} m - 38 \times 10^{-3} m) \frac{1}{8.8 \times 10^{-12}}$$

$$\boxed{A = 0.019 m^2 = 190 cm^2}$$

Energy – Set 3

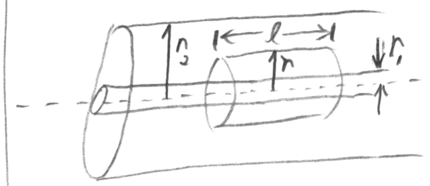
A long straight wire with a 1.0 mm radius has a uniform linear charge density of $\lambda = -5.0$⊠10^{-8} C/m. The wire is surrounded by a cylindrical conducting shell with an inner radius of 5.0 mm. If an electron at rest is released at the inner wire, what is its speed when it reaches the shell?

$m_{electron} = 9.11\text{x}10^{-31}$ kg
$e = 1.60\text{x}10^{-19}$ C
$k = 1/(4\pi\varepsilon_o) = 8.99\text{x}10^{9}$ Nm^2/C^2
$\varepsilon_o = 8.85\text{x}10^{-12}$ C^2/Nm^2

$r_1 = 1.0\,\text{mm}$
$\lambda = -5\times10^{-8}\,\text{C/m}$
$r_2 = 5.0\,\text{mm}$

(Show all steps.)

* Find E



$$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{\lambda \ell}{\epsilon_0} \Rightarrow E 2\pi r \ell = \frac{\lambda \ell}{\epsilon_0}$$

$$\Rightarrow \boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}}$$

* Find ΔV

$$\Delta V = -\int_{r_1}^{r_2} \vec{E}\cdot d\vec{s} = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi\epsilon_0 r}dr = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)$$ → inner to outer

* Find v

$$-\Delta U = \Delta K \qquad \Delta U = q\Delta V \Rightarrow \Delta U = \frac{-q\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)$$

$$\Rightarrow -\left(\frac{-q\lambda}{2\pi\epsilon_0}\right)\ln\left(\frac{r_2}{r_1}\right) = \frac{1}{2}mv_f^2 \Rightarrow \boxed{v_f = \left[\frac{q\lambda}{\pi\epsilon_0 m}\ln\left(\frac{r_2}{r_1}\right)\right]^{1/2}}$$

$$\boxed{v_f = 2.2\times10^{7}\,\text{m/s}}$$

Wolfson, Volume II, 2nd Edition, 23.18

The electrical energy stored in a capacitor is $U_E = \frac{Q^2}{2C} = \frac{1}{2}Q(\Delta V) = \frac{1}{2}C(\Delta V)^2$.

Given
$A = (25\text{cm})^2$
$d = 5.0\text{mm}$
$Q = 1.1\mu C$

a) For a parallel plate capacitor, $E = \frac{\sigma}{\epsilon_0}$ (I'll just "remember" it this time!)

And, $Q = \sigma A \Rightarrow \sigma = \frac{Q}{A}$

So: $E = \frac{Q}{\epsilon_0 A} \Rightarrow E = \frac{1.1\times10^{-6}C}{(8.8\times10^{-12}\,C^2/N\cdot m^2)(25\times10^{-2}m)^2} = 2\times10^{6}\,N/C$

b) $\Delta V = Ed = (2\times10^{6}\,\frac{N}{C})(5.0\times10^{-3}m) = 1\times10^{4}V$

c) $U = \frac{1}{2}CV^2$, $C = \frac{Q}{V}$

$\Rightarrow U = \frac{1}{2}\frac{Q}{V}V^2 \Rightarrow U = \frac{1}{2}Q\Delta V$

$\Rightarrow U = \frac{1}{2}(1.1\times10^{-6}C)(1\times10^{4}V) = 5\times10^{-3}J$

5. Wolfson, Volume II, 2^{nd} Edition, 23.48

The electrical energy stored in a capacitor is $U_E = \frac{Q^2}{2C} = \frac{1}{2}Q(\Delta V) = \frac{1}{2}C(\Delta V)^2$.

Recall that average power = (change in energy)/(change in time).

Given

$U = 5.0\,J$

$t = 1.0\,ms$

$\Delta V = 200\,V$

a) $P = \frac{\Delta U}{\Delta t} = \frac{5.0\,J}{1\times10^{-3}\,s} = 5000$ Watts

b) $U = \frac{1}{2}C\Delta V^2 \Rightarrow C = \frac{2U}{\Delta V^2}$

$\Rightarrow C = \frac{2 \cdot 5}{(200)^2} = 2.5\times10^{-4}\,F$

c) $\bar{P} = \frac{\Delta U}{\Delta t} = \frac{5.0\,J}{10\,s} = 0.5$ Watts