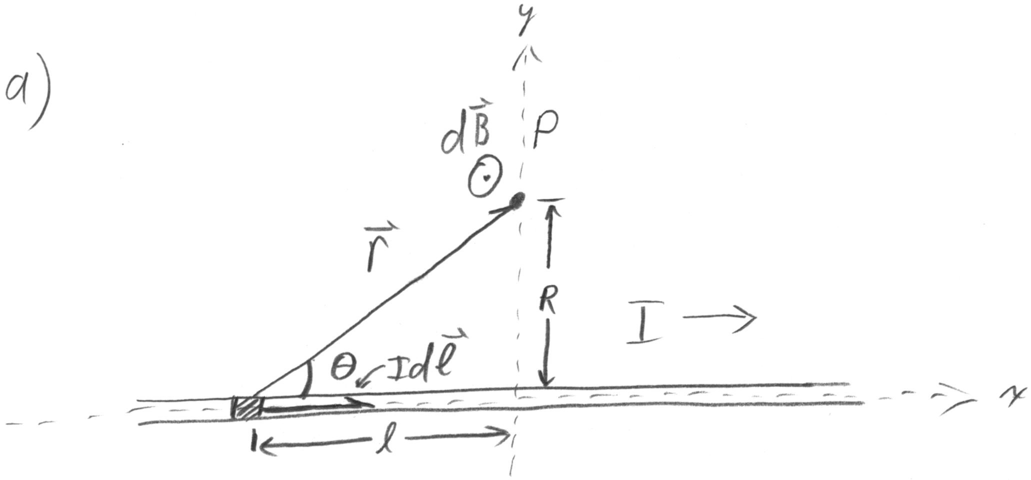
a) Find the magnetic field a distance R away from an infinitely long straight wire.
b) Find the magnetic field a perpendicular distance R away from the end of a semi-infinite wire,
c) Find the magnetic field a parallel distance R away from the end of a semi-infinite wire,

a)



$$d\vec{l} = dl\,\hat{\imath}$$
$$\vec{r} = l\hat{\imath} + R\hat{\jmath}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2},$$

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl\,\hat{\imath}\times(l\hat{\imath}+R\hat{\jmath})}{(l^2+R^2)^{3/2}}, \quad \hat{\imath}\times\hat{\imath}=0, \ \hat{\imath}\times\hat{\jmath}=\hat{k}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{IR\,\hat{k}}{(l^2+R^2)^{3/2}}dl \Rightarrow \vec{B} = \frac{\mu_0 RI}{4\pi}\int_{-\infty}^{\infty}\frac{dl\,\hat{k}}{(l^2+R^2)^{3/2}}$$

Let: $l = R\tan\theta \Rightarrow dl = R\sec^2\theta\, d\theta$

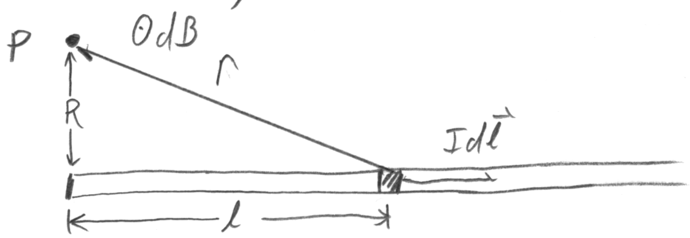
$$\Rightarrow \vec{B} = \frac{\mu_0 RI}{4\pi}\int_{-\pi/2}^{\pi/2}\frac{\cancel{R\sec^2\theta}\,\hat{k}}{R^{\cancel{3}}\sec^{\cancel{3}}\theta}d\theta \Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi R}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta\,\hat{k}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{2\pi R}\hat{k}}$$

continued ↓

Magnetics Set 3, P1 continued

b)

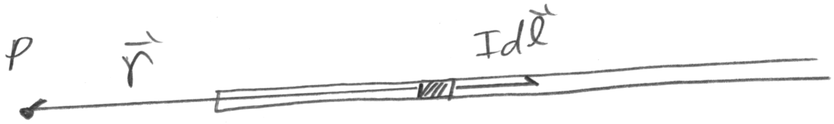


This is essentially the same integral as part a with different limits.

$$\vec{B} = \frac{\mu_0 R I}{4\pi} \int_0^\infty \frac{dl}{(l^2+R^2)^{3/2}} \hat{k}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 R I}{4\pi} \hat{k}}$$

c)

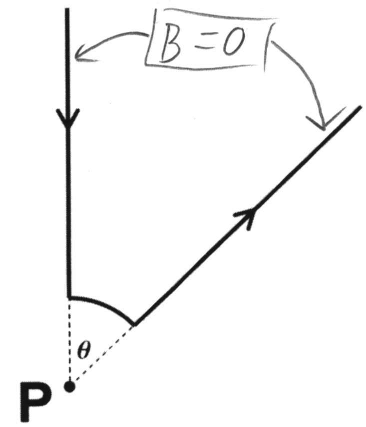


Here, $I d\vec{l} \times \hat{r} = 0$ since $dl \parallel \hat{r}$

so: $\underline{\vec{B} = 0}$

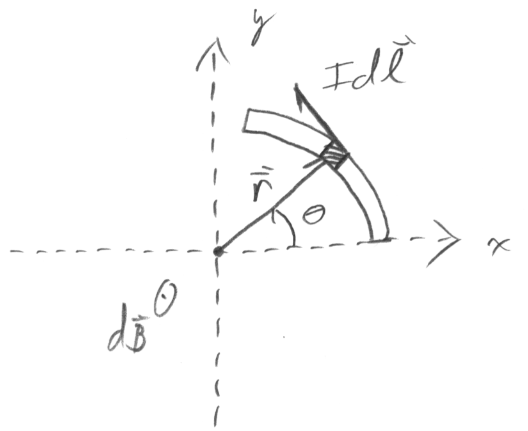
Magnetics – Set 3

A current path shaped as shown produces a magnetic field. The semi-circular arc subtends an angle of θ and has a radius r. The wire carried current I.



Derive an expression for the magnitude field vector at the center of the arc (point P).

I like to redraw this one:



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2}$$

$$d\vec{l} \times \hat{r} = |d\vec{l}||\hat{r}| \sin(90°)\hat{k}$$

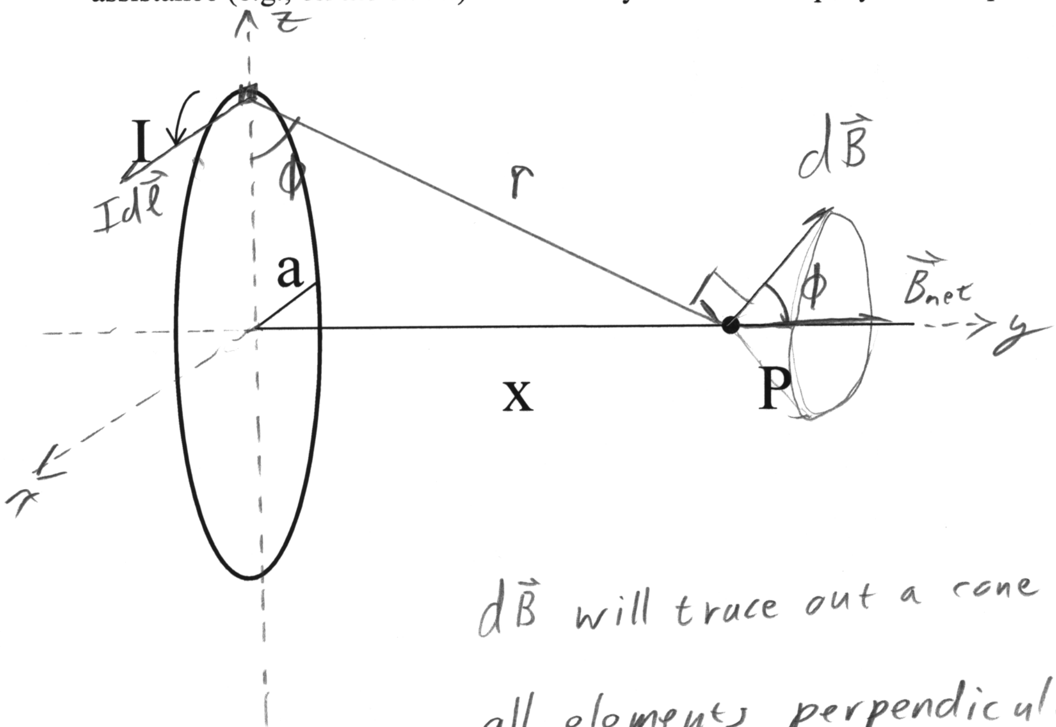
$$= dl\,\hat{k}$$

$$\Rightarrow d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{dl}{r^2}\hat{k}, \quad dl = r\,d\theta$$

$$\Rightarrow d\vec{B} = \frac{\mu_0 I}{4\pi r^{\cancel{2}}}\cancel{r}\,d\theta\,\hat{k} \Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi r}\int_0^\theta d\theta\,\hat{k}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I \theta}{4\pi r}\hat{k}}$$

Use the Biot-Savart law to derive a formula for the magnetic field a distance x along the axis of a vertical current loop with radius a and current I (the loop is perpendicular to the page). You may use Example 26.3 in your book as a guide, but be sure that you can eventually solve this problem with no assistance (e.g., on the exam). How does your result simplify for the special case of $x = 0$?



$d\vec{B}$ will trace out a cone around the y-axis

all elements perpendicular to $\hat{j}$ will

cancel, in pairs."

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,dl}{r^2}\cos\phi\,\hat{j}, \quad \cos\phi = \frac{a}{r} = \frac{a}{(x^2+a^2)^{1/2}}$$

$$dl = a\,d\theta$$

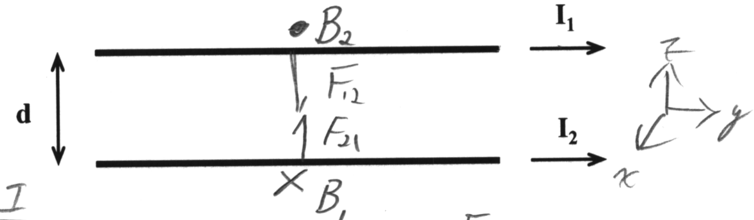
$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi}\frac{a\,d\theta}{r^2}\,\frac{a}{(x^2+a^2)^{1/2}}\hat{j} = \boxed{\frac{\mu_0 I}{r^2 4\pi}\,\frac{a^2}{(x^2+a^2)^{1/2}}\,2\pi\,\hat{j}}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2r^2}\,\frac{a^2}{(x^2+a^2)^{1/2}}} \quad \text{as } x \to 0, \quad \boxed{\vec{B} \to \frac{\mu_0 I a}{2r^2}}$$

We know that current-carrying wires produce magnetic fields, and we know that magnetic fields exert forces on current-carrying wires. Thus, two parallel current-carrying wires can either attract or repel each other, depending on their current directions.

(a) Consider two wires of length L (which is very long), separated by a distance d, that carry currents in the <u>same</u> direction. Do the wires attract or repel each other? What is the magnitude and direction of the force $\vec{F}_{21}$ exerted on wire 2 by wire 1? What is the force $\vec{F}_{12}$ exerted on wire 1 by wire 2?

Hint: Consider the B-field produced by wire 1 at the location of wire 2. Use this result to calculate the force on wire 2.



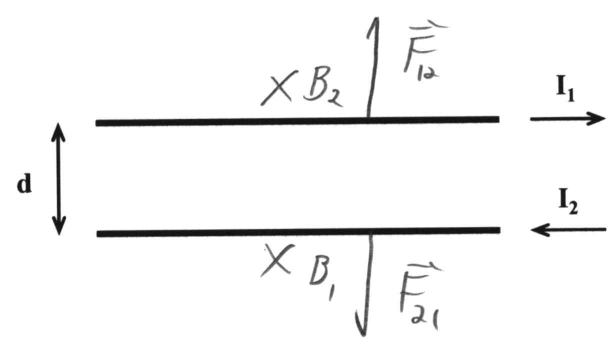
$F =$

For a "very long" wire, $|\vec{B}| = \frac{\mu_0 I}{2\pi R}$

and, Force on a wire, $\vec{F} = I(\vec{L} \times \vec{B})$

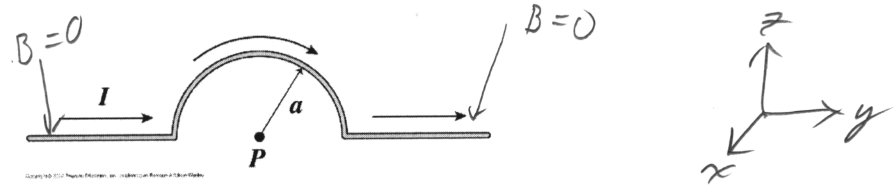
$$\boxed{\vec{F}_{12} = -ILB \cdot \frac{\mu_0 I}{2\pi d}\hat{k}}$$

(b) Now repeat the problem for two wires carrying currents in <u>opposite</u> directions.



$$\vec{F}_{12} = ILB\frac{\mu_0 I}{2\pi d}\hat{k}$$

Part of a long wire is bent into a semicircle of radius a, as in the figure below. A current I flows in the direction shown. Use the Biot-Savart law to find the magnetic field at the center of the semicircle (point P).



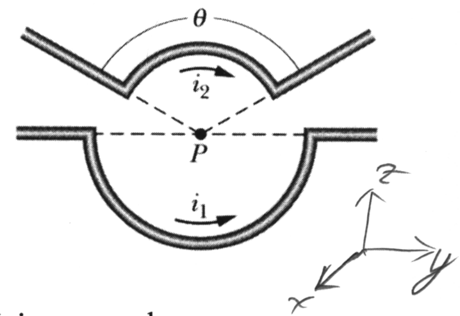
Earlier, we derived:

$$|\vec{B}| = \frac{\mu_0 I \theta}{4\pi r} \quad \text{for an arc.}$$

By the rhr, the direction is $-\hat{i}$

So: $$\vec{B} = -\frac{\mu_0 I \cancel{\pi}}{4 \cancel{\pi} r} \Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{4r}}$$

The figure shows two current segments. The lower segment carries a current of I_1 and includes a semicircular arc with radius r_1 subtending an angle of 180 degrees a center point P. The upper segment carries current $I_2 = 2I_1$ and includes a circular arc with radius r_2=0.5r_1 and subtends an angle of 120 degrees with the same center point P.



a) What is the magnitude and direction of the net magnetic field at point P for the indicated current directions?

b) What are the magnitude and direction of the net magnetic field at point P if I_1 is reversed.

$\theta_1 = 120° = 120° \frac{\pi}{180°}$ rad $= \frac{2}{3}\pi$

a) upper loop:

$$\vec{B}_u = -\frac{\mu_0 I \frac{2}{3}\pi}{4\pi r_2}\hat{\imath} = \frac{-\mu_0 I \frac{2}{3}\pi}{4\pi \cdot \frac{1}{2} r_1}$$

$$\vec{B}_L = \frac{\mu_0 I \pi}{4\pi r_1}\hat{\imath}$$

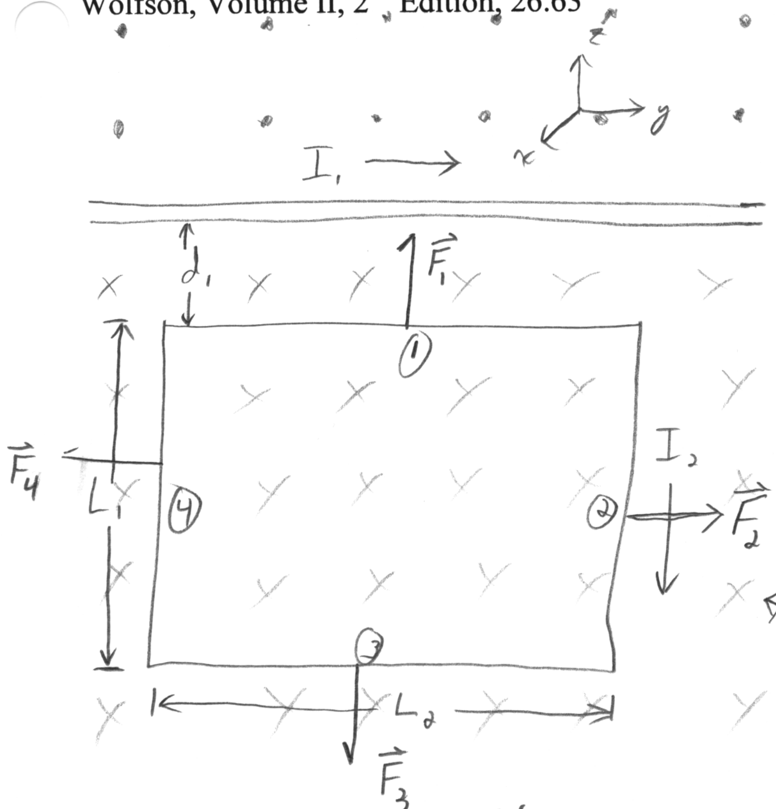
$$\vec{B}_{net} = \left[\frac{\mu_0 I \pi}{4\pi r_1} - \frac{\mu_0 I \frac{2}{3}\pi}{4\pi \frac{1}{2} r_1}\right]\hat{\imath}$$

$$= \frac{\mu_0 I}{4 r_1}\left[1 - \frac{1}{3}\right]\hat{\imath} \Rightarrow \boxed{\vec{B}_{net} = \frac{2}{3}\frac{\mu_0 I}{4 r_1}\hat{\imath}}$$

b) if I_1 is reversed, the sign of B_L flips to −

$$\Rightarrow \vec{B}_{net} = \frac{\mu_0 I}{4 r_1}\left[-1 - \frac{1}{3}\right] \Rightarrow \boxed{\vec{B}_{net} = -\frac{4}{3}\frac{\mu_0 I}{4 r_1}}$$

Wolfson, Volume II, 2nd Edition, 26.63



RHR gives field direction

<u>Given</u>

$I_1 = 20A$

$d = 2.0cm$

$L_1 = 5.0cm$

$L_2 = 10cm$

$I_2 = 500mA$

<u>Want</u>

$\vec{F}_{Bnet}$

From a previous problem, The field due to a long straight wire is:

$$\boxed{|\vec{B}| = \frac{\mu_0 I}{2\pi R}}$$

where R is the perpendicular distance to the wire.

We also know that the force on a piece of a current carrying wire in a magnetic field is:

$$\boxed{d\vec{F}_B = I\left(d\vec{l} \times \vec{B}\right)}$$

Now we can sum up the 4 sides of the loop

continued

↓

Magnetics Set 3 - P7 continued

Sides ① and ③, the field is uniform along the segment:

So: $\vec{F}_B = I_2(\vec{L} \times \vec{B})$, $\vec{L}_1 = L_2\hat{j}$, $\vec{L}_3 = -L_2\hat{j}$

$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi d}\hat{i}$, $\vec{B}_3 = \frac{-\mu_0 I}{2\pi(d+L_1)}$

$$\vec{F}_1 = I_2\left(L_2\hat{j} \times \left(-\frac{\mu_0 I_1}{2\pi d}\right)\hat{i}\right)$$

$$\Rightarrow \boxed{\vec{F}_1 = I_2 L_2 \frac{\mu_0 I_1}{2\pi d}\hat{k}}$$

$$\vec{F}_3 = I_2\left(-L_2\hat{j} \times -\left(\frac{\mu_0 I_1}{2\pi(d+L_1)}\right)\hat{i}\right)$$

$$\Rightarrow \boxed{\vec{F}_3 = -\frac{\mu_0 I_1 I_2 L_2}{2\pi(d+L_1)}\hat{k}}$$

Although we could integrate to get $\vec{F}_2$ and $\vec{F}_4$, it is clear that they cancel by symmetry.

So: $$\vec{F}_{net} = \vec{F}_1 + \vec{F}_3 = \left(\frac{\mu_0 L_2 I_1 I_2}{2\pi d} - \frac{\mu_0 L_2 I_1 I_2}{2\pi(d+L_1)}\right)\hat{k}$$

$$\Rightarrow \vec{F}_{net} = \frac{\mu_0 L_2 I_1 I_2}{2\pi}\left(\frac{1}{d} - \frac{1}{d+L_1}\right)\hat{k}$$

$$\Rightarrow \boxed{\vec{F}_{net} = \frac{\mu_0 L_2 I_1 I_2}{2\pi} \cdot \frac{L_1}{d(d+L_1)}\hat{k}}$$