Magnetics – Set 5

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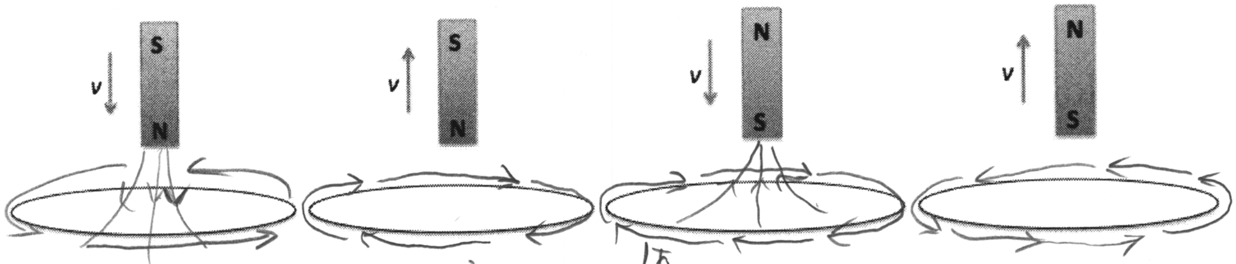
Name: ____________________

Problems Solved __/ 8

In which direction will the induced current flow for the following configurations:



opposite

$\frac{d\Phi}{dt}$ increasing up

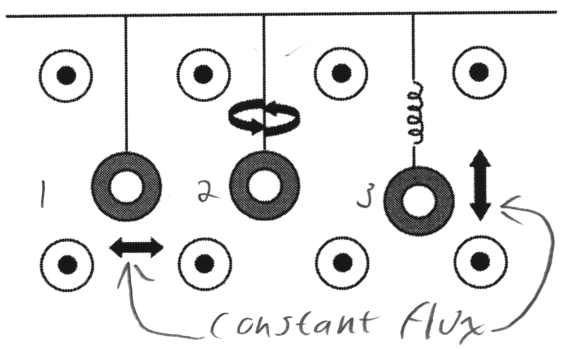
So coil acts opposite

$\frac{d\Phi}{dt}$ increasing down

So coil acts opposite

The three loops of wire shown in the sketch are all in a region of space with a uniform, constant magnetic field. Loop 1 swings back and forth as the bob on a pendulum; loop 2 rotates about a vertical axis; and loop 3 oscillates vertically on the end of a spring. Which loop or loops have a magnetic flux that changes with time?

a) Loop 1
b) Loop 2
c) Loop 3
d) Loops 1 and 3
e) All of the above



For an induced current to appear in a wire loop, which of the following must be true?

a) there must be a large magnetic flux through the loop
b) the loop's plane must be parallel to the magnetic field
c) the loop's plane must be perpendicular to the magnetic field
d) ~~the magnetic~~ flux through the loop must vary with time
e) None of the above

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

A magnet moved into a coil of wire will induce voltage in the coil. What is the effect of moving a magnet into a coil with more loops?

a) greater induced voltage
b) lower induced voltage
c) the same induced voltage
d) None of the above

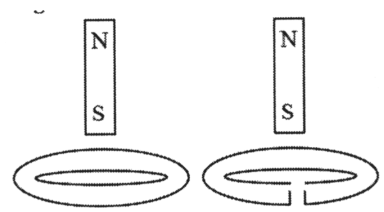
$\mathcal{E} = -N \frac{d\Phi_B}{dt}$

Which of the following will produce a greater induced voltage?

a) moving a magnet into a coil very slowly
b) moving a magnet into a coil very quickly
c) Neither of these actions produces an induced voltage

The magnets shown in the sketch are dropped from rest through the middle of conducting rings. Notice that the ring on the right has a small break in it, whereas the ring on the left forms a closed loop. As the magnets drop toward the rings the magnet on the left has an acceleration that is _______ that of the magnet on the right.

a) more than
b) less than
c) the same as



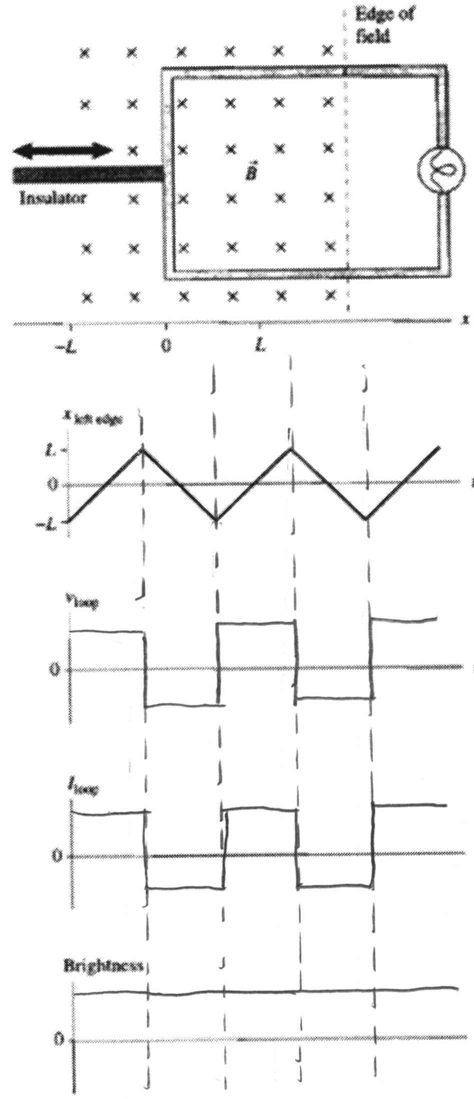
An insulating rod pushes a copper loop back and forth. The left edge of the loop, which is always in the magnetic field, oscillates between x=-L and x=+L, as shown in the top graph. The right edge of the loop, which includes a light bulb, is always outside the magnetic field.

a. Draw the velocity graph for the loop. Make sure it aligns with the position graph above it.

b. Draw a graph of the induced current in the loop as a funciotn of time. Let a clockwise current be positive and counterclockwise current be negative.

c. Draw a graph of the brightness of the lightbulb as a function of itme.

NOTE: There are no numbers on the vertical scale. The shape of each graph is the important result.



a) Velocity is the slope of position.

b) As the coil moves to the right, The downward point $\vec{B}$ flux is decreasing. Clockwise current gives a downward pointing $\vec{B}$ to counter the change.

As the coil moves to the left, Flux from downward $\vec{B}$ is increasing. CCW current gives an upward $\vec{B}$ to counter the change

c) The brightness of the bulb is independent of current direction. If the transitions are very fast, the bulb brightness will never change.

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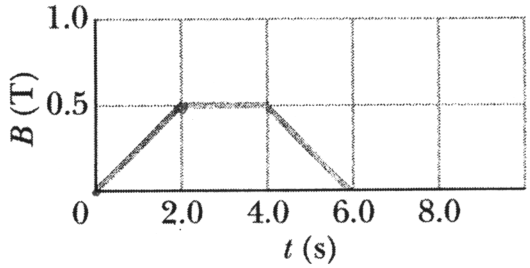
The magnetic field through a single loop of wire, 12 cm in radius and with 8.5 Ω resistance, changes with time as shown in the figure. Calculate the magnitude of the induced current in the following time intervals:

(a) $0 < t < 2.0$ s

(b) $2.0 < t < 4.0$ s

(c) $4.0 < t < 6.0$ s



a) $\Delta B = +0.5\,T$, $\Delta t = 2.0\,s$, $A = \pi(12\times10^{-2}\,m)^2 = 4.5\times10^{-2}\,m^2$

$$V = \frac{\Delta B \cdot A}{\Delta t} \qquad I = \frac{V}{R} \Rightarrow \boxed{I = \frac{\Delta B\, A}{R\,\Delta t}}$$

$$\Rightarrow I = \frac{(0.5T)(4.5\times10^{-2}\,m^2)}{(8.5\,\Omega)(2.0\,s)} = \boxed{1.3\times10^{-3}\text{ Amps}}$$

b) $\Delta B = 0 \Rightarrow V = 0 \Rightarrow \boxed{I = 0}$

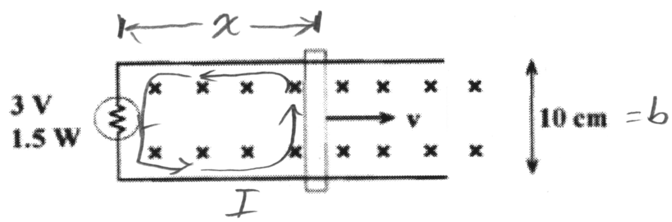
c) $\Delta B = -0.5\,T$, $\Delta t = 2.0\,s$

Same as part a but opposite direction.

So, same magnitude: $\boxed{I = 1.3\times10^{-3}\text{ Amps}}$

The circuit in the figure consists of a flashlight bulb, rated 3.0V/1.5W, and ideal wires with no resistance. The right wire of the circuit, which is 10 cm long, is pulled at constant speed *v* through a perpendicular magnetic field of strength 0.10 T.

a) What speed must the wire have to light the bulb to full brightness?
b) What force is needed to keep the wire moving?



a) $V = -\frac{d\Phi_B}{dt}$, $\Phi = BA \Rightarrow \Phi = Bxb$

B and b are constant. so: $\frac{d\Phi}{dt} = Bb\frac{dx}{dt}$

$$\Rightarrow \frac{d\Phi}{dt} = Bbv$$

So: $|V| = Bbv \Rightarrow \boxed{v = \frac{|V|}{Bb}} \Rightarrow v = \frac{(3V)}{(0.1T)(10\times10^{-2}cm)} = \boxed{300\ m/s}$

b) Bulb dissipates 1.5W at 3V and $P = VI$

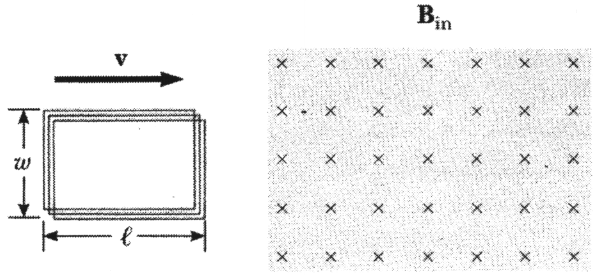
$$\Rightarrow I = \frac{P}{V}$$

and the force on a current carrying wire is:

$F = I(\vec{L} \times \vec{B})$, which is to the left, opposing the motion

So: $F = ILB \Rightarrow \boxed{F = \frac{PLB}{V}} \Rightarrow \boxed{F = 0.005\ N}$

A rectangular coil with resistance R has N turns (loops), each of length l and width w as shown in the figure. The coil moves into a uniform magnetic field B with constant velocity v. Find the magnitude and direction of the total magnetic force on the coil (a) as it enters the magnetic field (i.e. partially immersed), and (b) as it moves (totally immersed) within the magnetic field.



Given
R
N
ℓ
w
B
v

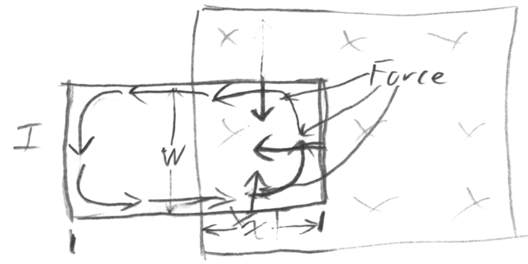
Want
F

a) $\Phi_B = BAN$, $A = wx$

$\Rightarrow \Phi_B = BNwx$

$\Rightarrow \frac{d\Phi_B}{dt} = BNw\frac{dx}{dt}$

$\Rightarrow \boxed{V = BNwv}$, $I = \frac{V}{R} \Rightarrow I = \frac{BNwv}{R}$



$F = I(\vec{L} \times \vec{B}) \Rightarrow F = \frac{BNwv}{R} wB$

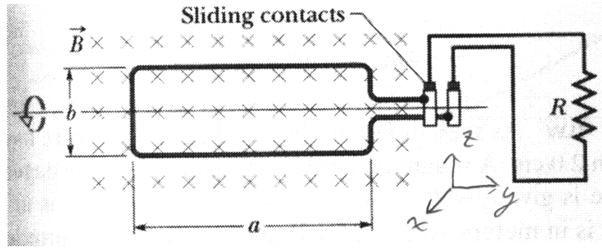
$\Rightarrow \boxed{F = \frac{B^2 w^2 N v}{R}}$

b) When completely immersed, $\frac{d\Phi_B}{dt} = 0$

So: $\boxed{F = 0}$

A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field $\vec{B}$, as indicated in the figure. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact.

Find the emf in induced in the coil as a function of time.



As the coil spins: $\Phi = N\vec{B}\cdot\vec{A}$

where $\vec{A} = A\hat{n}$

So: $\boxed{\Phi = NBA\cos\theta}$

But, θ is a function of time: $\theta = \omega t$

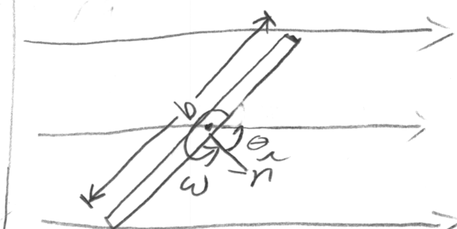
$\Rightarrow \Phi = NBA\cos(\omega t)$

$\Rightarrow V = -\dfrac{d\Phi}{dt}$

$= -NBA\dfrac{d}{dt}(\cos(\omega t))$

$\Rightarrow V = \omega NBA\sin(\omega t)$

$\Rightarrow \boxed{V = \omega abB\sin(\omega t)}$



Side View

Flux depends on cross section of loop exposed to $\vec{B}$

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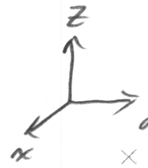

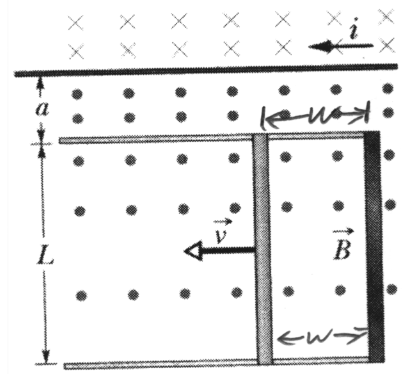

The figure shows a rod of length L that is forced to move at constant speed v along horizontal rails. The rod, rails , and connecting strip at the right form a conductin loop. The rod has resistance R and the rest of the loop has negligible resistance. A current I through the long straight wire at distance a from the lop sets up a (non-uniform) magnetic field through the loop.

a) Find the emf and the current in the loop.
b) Find the magnitude of the force on the rod required to keep it moving at a constant velocity.

a) $\mathcal{E} = -\frac{d\Phi_B}{dt}$, $\Phi_B = \int \vec{B}\cdot d\vec{A}$

$\vec{B} \parallel \vec{A}$
$\Rightarrow \vec{B}\cdot\vec{A} = BA$

In this case, $\vec{B}$ is <u>spatially</u> non-uniform, so we have to integrate to get the flux.

$\vec{B} = \frac{\mu_0 I}{4\pi r}\hat{\imath} \Rightarrow$ Field due to long wire, where r is distance from the wire.

$\Rightarrow \Phi_B = \int \vec{B}\cdot d\vec{A}$, $dA = w\,dl\,\hat{\imath}$

B is uniform in this dimension

Integrating in the direction of the non-uniform B

$\Rightarrow \Phi_B = \int_a^{a+L} \left(\frac{\mu_0 I}{4\pi l}\right) w\,dl$, $(\hat{\imath}\cdot\hat{\imath}) = 1$

$\Rightarrow \Phi_B = \frac{\mu_0 I w}{4\pi}\int_a^{a+L} \frac{dl}{l} \Rightarrow \boxed{\Phi_B = \frac{\mu_0 I w}{4\pi}\ln\left(\frac{a+L}{a}\right)}$

continued
↓

$V = -\frac{d\Phi_B}{dt}$, Now we take the time derivative.

$\Rightarrow V = -\frac{d}{dt}\left[\frac{\mu_0 I w}{4\pi}\ln\left(\frac{a+L}{a}\right)\right]$

The only time varying variable is w.

$\Rightarrow V = -\frac{\mu_0 I}{4\pi}\ln\left(\frac{a+L}{a}\right)\frac{dw}{dt}$

$\Rightarrow \boxed{V = -\frac{\mu_0 I}{4\pi}\ln\left(\frac{a+L}{a}\right)v}$

b) I is clockwise around the loop to oppose the increasing flux out of the page.

So: $I_1 d\vec{l} = I_1 dl\hat{k}$, $\vec{B} = \frac{\mu_0 I_2}{4\pi l}\hat{\imath}$

Then: $d\vec{F}_B = Id\vec{l}\times\vec{B}$

$= d\vec{F}_B = I_1 dl\frac{\mu_0 I_2}{4\pi l}(\hat{k}\times\hat{\imath})$

* careful! I_1 and I_2 are different.
  I_1 is induced current in the loop
  I_2 is the current in the long wire

continued ↓

Magnetics Set 5, P8 continued.

$$\Rightarrow \vec{F}_B = +\frac{\mu_0 I_1 I_2}{4\pi}\int_a^{a+L} \frac{dl}{l}\,\hat{j}$$

$$\Rightarrow \vec{F}_B = \frac{\mu_0 I_1 I_2}{4\pi}\ln\left(\frac{a+L}{a}\right)\hat{j}$$

But $I_1 = \frac{V}{R}$ where V is the induced EMF.

Resistance

$$\Rightarrow I_1 = \frac{\mu_0 I_2}{4\pi R}\ln\left(\frac{a+L}{a}\right)v$$

$$\Rightarrow \vec{F}_B = \left[\frac{\mu_0 I_2}{4\pi}\ln\left(\frac{a+L}{a}\right)\right]\left[\frac{\mu_0 I_2}{4\pi R}\ln\left(\frac{a+L}{a}\right)v\right]\hat{j}$$

$$\Rightarrow \boxed{\vec{F}_B = \left[\frac{\mu_0 I_2}{4\pi}\ln\left(\frac{a+L}{a}\right)\right]^2\frac{v}{R}\hat{j}}$$

and the force required to keep constant v is equal and opposite, so that $a = 0$

$$\vec{F}_B + \vec{F}_{pull} = ma \Rightarrow \vec{F}_B + \vec{F}_{pull} = 0 \Rightarrow \vec{F}_B = -\vec{F}_{pull}$$

$$\Rightarrow \boxed{\vec{F}_{pull} = -\left[\frac{\mu_0 I_2}{4\pi}\ln\left(\frac{a+L}{a}\right)\right]^2\frac{v}{R}\hat{j}}$$