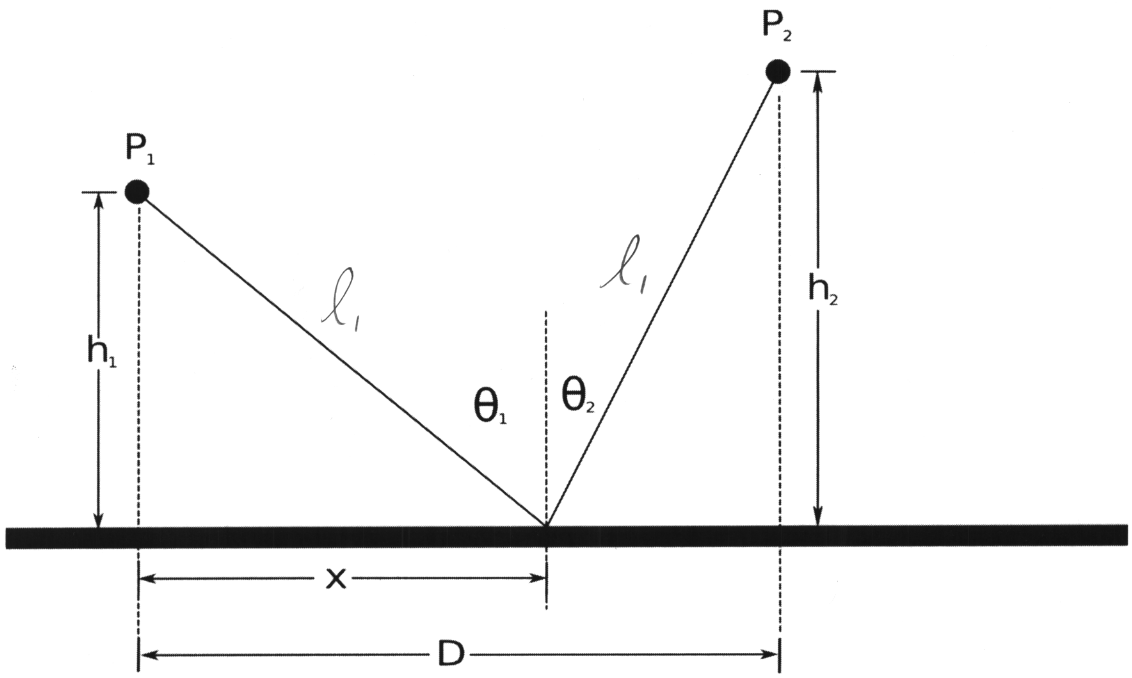
Optics – Set 1 **Page 1**

Name: ______________________ **Problems Solved __/ 8**

In the figure below, light leaves point P_1, reflects off of a mirrored surface and arrives at point P_2. Derive a relationship between the angle of incidence, θ_1, and the angle of reflection, θ_2, by varying x to minimize the light travel time the two points.



* Find expression for total trip time

$$t_T = t_1 + t_2, \quad t_1 = \frac{l_1}{v}, \quad t_2 = \frac{l_2}{v}$$

$$\Rightarrow t_T = \frac{1}{v}(l_1 + l_2)$$

$$l_1 = (h_1^2 + x^2)^{1/2}, \quad l_2 = (h_2^2 + (D-x)^2)^{1/2}$$

$$\Rightarrow \boxed{t_T = \frac{1}{v}\left((h_1^2 + x^2)^{1/2} + (h_2^2 + (D-x)^2)^{1/2}\right)}$$

* Find minimum time by taking $\frac{dt}{dx}$

minimum

$$\frac{dt_T}{dx} = \frac{1}{v}\left[\cancel{\tfrac{1}{2}}(h_1^2 + x^2)^{-1/2} \cdot \cancel{2}x + \cancel{\tfrac{1}{2}}(h_2^2 + (D-x)^2)^{-1/2} \cdot \cancel{2}(D-x)(-1)\right] = 0$$

continued ↓

Optics Set 1, P1 - continued

$$\Rightarrow \frac{x}{(h_1^2 + x^2)^{1/2}} - \frac{(D-x)}{(h_2^2 + (D-x)^2)^{1/2}} = 0$$

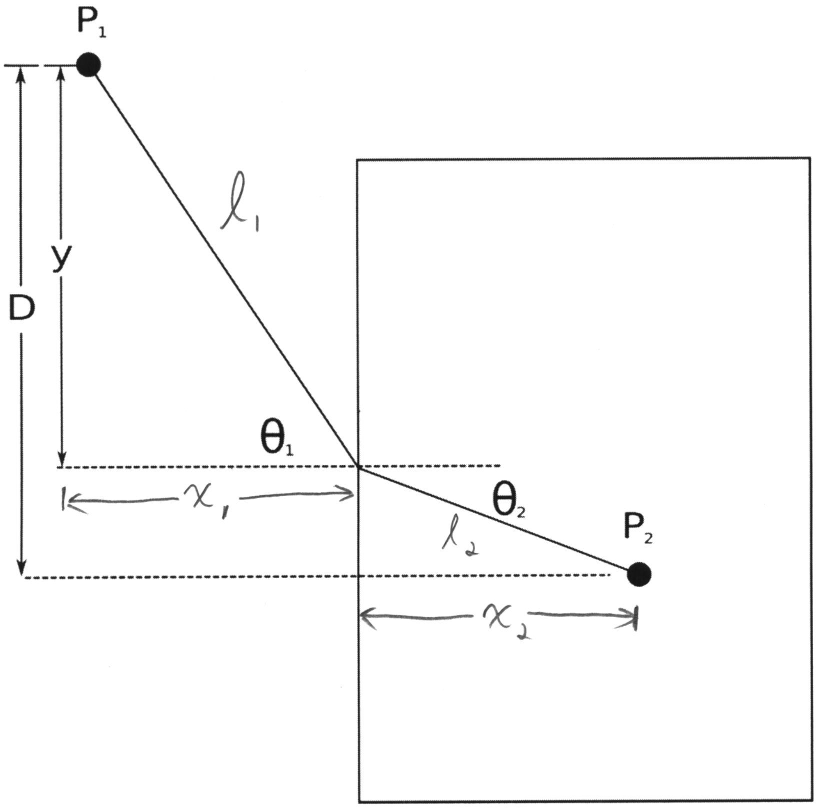
$$\Rightarrow \frac{x}{l_1} - \frac{(D-x)}{l_2} = 0 \Rightarrow \text{SIN}\,\theta_1 = \text{SIN}\,\theta_2$$

$$\Rightarrow \boxed{\theta_1 = \theta_2}$$

Angle of incidence equals
Angle of reflection

In the figure below, light leaves point P_1 traveling at a velocity v_1, enters a glass block where its velocity is v_2 and arrives at point P_2.

a) Derive a relationship between the angle of incidence, θ_1, and the angle of refraction, θ_2, by adjusting y to minimize the light travel time the two points.
b) Let the index of refraction, n, be defined as: $n = \frac{c}{v}$, where c is the speed of light in vacuum. Rewrite the relationship from part a in terms of the indices of refraction of the two mediums.



$$t_1 = \frac{l_1}{v_1}$$

$$\Rightarrow \boxed{t_1 = \frac{(y^2+x_1^2)^{1/2}}{v_1}}$$

$$t_2 = \frac{l_2}{v_2}$$

$$\Rightarrow \boxed{t_2 = \frac{((D-y)^2+x_2^2)^{1/2}}{v_2}}$$

$$t_T = \frac{(y^2+x_1^2)^{1/2}}{v_1} + \frac{((D-y)^2+x_2^2)^{1/2}}{v_2}$$

$$\frac{dt_T}{dy} = \frac{1}{v_1}\left[\cancel{\tfrac{1}{2}}(y^2+x_1^2)^{-1/2}\cdot\cancel{2}y\right] + \frac{1}{v_2}\left[\cancel{\tfrac{1}{2}}((D-y)^2+x_2^2)^{-1/2}\cdot\cancel{2}(D-y)(-1)\right] = 0$$

$$\Rightarrow \frac{1}{v_1}\cdot\frac{y}{(y^2+x_1^2)^{1/2}} - \frac{1}{v_2}\cdot\frac{(D-y)}{((D-y)^2+x_2^2)^{1/2}} = 0$$

continued ↓

Optics Set 1, P2 - continued

$$\Rightarrow \boxed{\frac{1}{v_1} \sin\theta_1 = \frac{1}{v_2} \sin\theta_2}$$

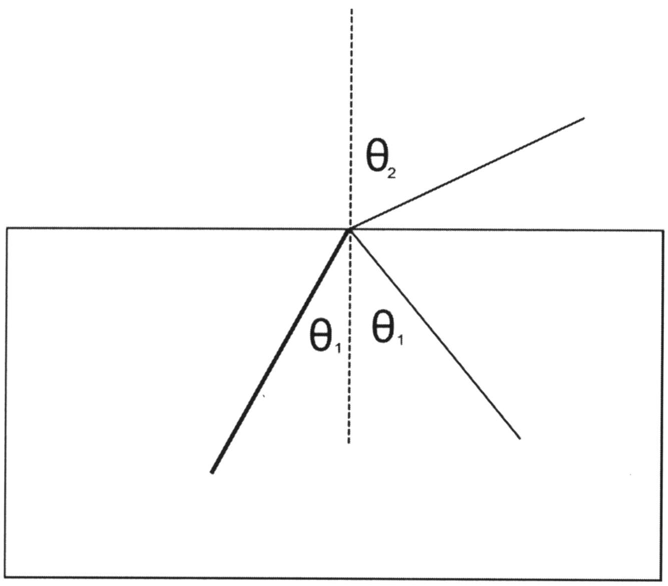
b) $n_1 = \frac{c}{v_1}$, $n_2 = \frac{c}{v_2}$

$$\Rightarrow \cancel{c} n_1 \sin\theta_1 = \cancel{c} n_2 \sin\theta_2$$

$$\Rightarrow \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$ Snell's law

When light travels from a medium with a large index of refraction to one with a lower index of refraction, the refracted ray bends away from the normal line as in the picture. At a particular angle, the critical angle θ_c, the refracted ray moves parallel to the boundary ($\theta 2=90^o$). At angles $\theta_1 > \theta_c$, there is no refracted ray and the incident ray is entirely reflected. This is known as total internal reflection.

Find an expression for θ_c.



$n_1 \sin\theta_1 = n_2 \sin\theta_2$, $\theta_1 = \theta_c$ when $\theta_2 = 90^\circ$

$\Rightarrow n_1 \sin\theta_c = n_2 \sin(90^\circ)$, $\sin(90^\circ) = 1$

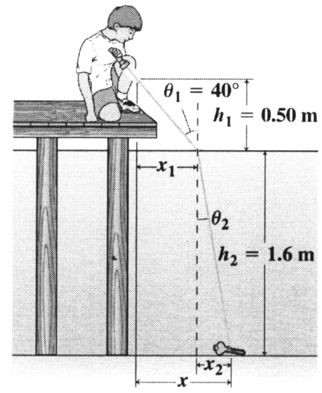
$\Rightarrow n_1 \sin\theta_c = n_2$

$\Rightarrow \boxed{\sin\theta_c = \frac{n_2}{n_1}}$

so, if $\theta_1 > \theta_c$, all reflections will be internal

if $\theta_1 < \theta_c$, some light will escape

You have dropped your car keys at night off the end of a dock into water 1.6m deep. A flashlight held directly above the dock edge and 0.50m above the water illuminates the keys when it's pointed at 40° to the vertical, as shown in the figure. What is the horizontal distance *x* from the edge of the dock to the keys?



Given

$h_2 = 1.6m$

$h_1 = 0.5m$

$\theta_1 = 40°$

$n_2 = 1.33$

$n_1 = 1.0$

Want

x

$$x = x_1 + x_2, \quad x_1 = h_1 \tan\theta_1, \quad x_2 = h_2 \tan\theta_2$$

$$\Rightarrow x = h_1 \tan\theta_1 + h_2 \tan\theta_2$$

get θ_2 from snell's law:

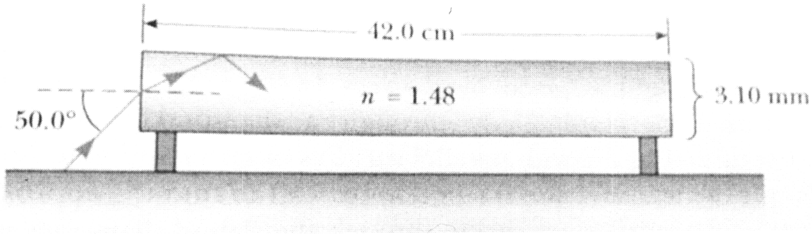
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}\left[\frac{\sin\theta_1}{n_2}\right]$$

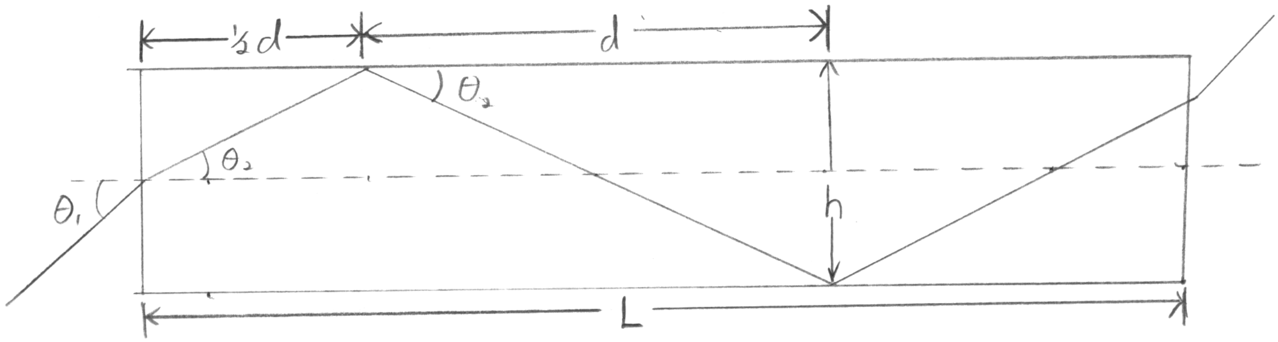
$$\boxed{\theta_2 = 29°}$$

$$\Rightarrow x = (0.5m)\tan(40°) + (1.6m)\tan(29°)$$

$$\Rightarrow \boxed{x = 1.3m}$$

A laser beam strikes the end of a slab of material as shown in the figure. The index of refraction of the slap is 1.48. Determine the number of internal reflections of the beam before it emerges from the opposite end of the slab.





d is distance between bounces, and there is one bounce at $\frac{d}{2}$.

So, the integer part of $\frac{L - \frac{1}{2}d}{d}$ will give one less than the actual number of bounces and the ceiling of $\frac{L - \frac{1}{2}d}{d}$ will give the number of bounces.

$d = \frac{h}{\tan\theta_2}$, and snell says $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$

$\theta_2 = \sin^{-1}\left(\frac{1}{1.48}\cdot\sin(50°)\right) = \boxed{31°}$

$d = \frac{3.1\times10^{-3}}{\tan(31°)} = 5.1\times10^{-3}\ \text{m}$

continued ↓

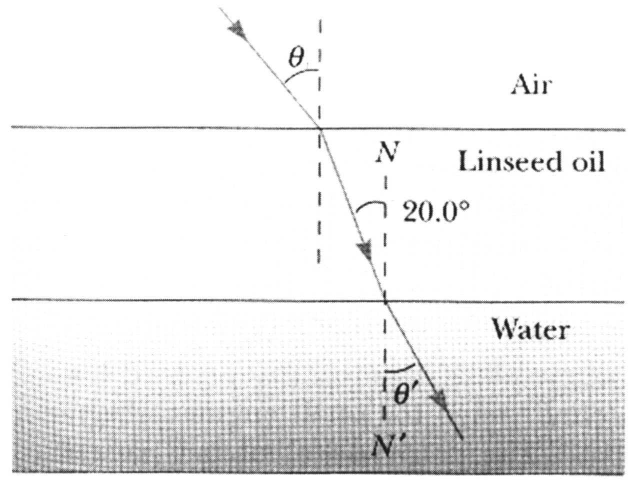
Then: $n = \text{ceil}\left(\frac{L - \frac{d}{2}}{d}\right)$

$$\Rightarrow n = \text{ceil}\left[\frac{42\times10^{-2}\,m - (\tfrac{1}{2})(5.1\times10^{-3})\,m}{5.1\times10^{-3}\,m}\right]$$

$$\Rightarrow n = \text{ceil}\left[81.85\right]$$

$$\Rightarrow \boxed{n = 82}$$

The light beam shown makes an angle of 20.0^{O} with the normal line in the linseed oil. Determine the angles θ and θ'. The index of refraction of water is 1.33, the index of refraction of air is 1.00, and the index of refraction of linseed oil is 1.48.



Given	Want
$\theta_2 = 20.0^0$	θ_1, θ_3
$n_1 = 1.0$	
$n_2 = 1.48$	
$n_3 = 1.33$	

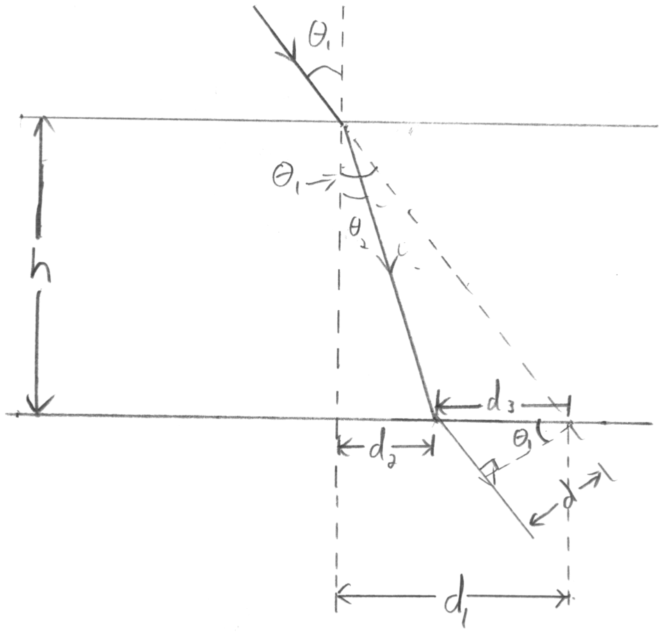
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2$$

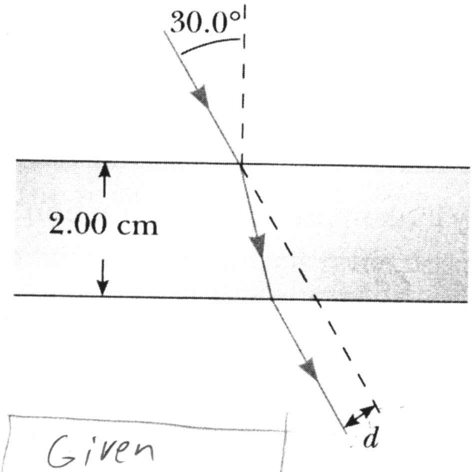
$$\Rightarrow \theta_1 = \sin^{-1}\left[\frac{n_2}{n_1}\sin\theta_2\right] \Rightarrow \boxed{\theta_1 = 30^0}$$

$$n_2 \sin\theta_2 = n_3 \sin\theta_3 \Rightarrow \sin\theta_3 = \frac{n_2}{n_3}\sin\theta_2$$

$$\Rightarrow \theta_3 = \sin^{-1}\left[\frac{n_2}{n_3}\sin\theta_0\right] \Rightarrow \boxed{\theta_3 = 22^0}$$

When the light in the figure passes through the glass block, it is shifted laterally by the distance d. Taking n=1.50, find the value of d.





Given
$\theta_1 = 30°$
$h = 2.0\text{cm}$
$n_1 = 1.00$
$n_2 = 1.50$

$d = d_3 \sin\theta_1, \quad d_3 = d_1 - d_2$

$d_1 = h\tan\theta_1, \quad d_2 = h\tan\theta_2,$

$\Rightarrow \boxed{d = h[\tan\theta_1 - \tan\theta_2]\sin\theta_1}$

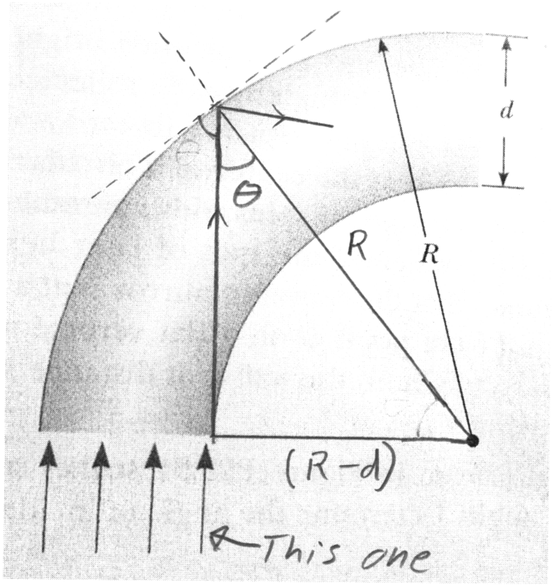
$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right]$

$\Rightarrow \boxed{\theta_2 = 19.5°}$

$\Rightarrow d = (2.00\text{cm})[\tan(30°) - \tan(19.5°)]\sin(30°)$

$\Rightarrow \boxed{d = 0.22\text{cm}}$

An optical fiber has an index of refraction n and diameter d. It is surrounded by air. Light is sent into the fiber along its axis as shown in the figure.



a) Find the smallest outside radius R permitted for a bend in the fiber if no light is to escape.
b) Does the result for part a predict reasonable behavior as D approaches zero?

a) The ray closest to the center of curvature will strike the outer wall with the smallest angle of incidence.

If R is too small, $\theta < \theta_c$ and light will escape.

What's R when $\theta = \theta_c$?

$$\sin\theta_c = \frac{n_2}{n_1}, \quad \sin\theta = \frac{R-d}{R}$$

$$\Rightarrow \frac{R-d}{R} = \frac{n_2}{n_1} \Rightarrow n_1(R-d) = n_2 R$$

$$\Rightarrow n_1 R - n_1 d = n_2 R$$

$$\Rightarrow R(n_1 - n_2) = n_1 d$$

$$\Rightarrow \boxed{R_{min} = \frac{n_1}{(n_1 - n_2)} d}$$

b) Yes, smaller fibers allow tighter curves.