Waves – Set 1

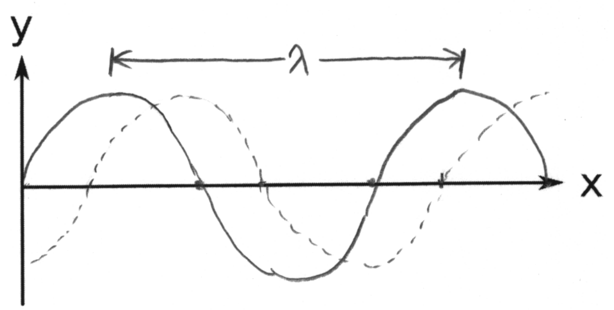
Name: ______________________

Problems Solved __/ 9

Given the equation for a sinusoidal wave on a string:

$$y(x,t) = A\sin(kx - \omega t)$$

a) Let t=0 and plot $y(x,t=0)$ on the graph below using a solid line.
b) Imagine that a small amount of time has passed such that $\omega t < 2\pi$, and plot $y(x,t=t1)$ using a dashed line.
c) The wavelength, λ, is the distance between repetitions of the wave's shape. Find an expression for the wavelength, λ, in terms of the angular wave number, k.
d) The period, T, is the number of seconds between wave repetitions. Find an expression for the period, T, in terms of the angular frequency, ω.



c) k has units of radians/m. It converts from spatial coordinates to angular coordinates.

$$\underset{\text{one spatial cycle}}{k\lambda} = \underset{\text{one angular cycle}}{2\pi} \Rightarrow \boxed{\lambda = \frac{2\pi}{k}}$$

d) ω has units of radians/sec, It converts from time coordinates to angular coordinates

$$\underset{\text{one Time cycle}}{\omega T} = \underset{\text{one angular cycle}}{2\pi} \Rightarrow \boxed{T = \frac{2\pi}{\omega}}$$

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e) The frequency, f, is the number of times the wave repeats itself in one second. Find an expression for the frequency of a wave in terms of its angular frequency, ω.

f) The *propagation velocity*, $\frac{dx}{dt}$, is the speed at which a point on the wave (a wave crest for example) moves. Find an expression for the velocity in terms of the angular wave number, k, and the angular frequency, ω.

g) Find an expression for the *propagation velocity* in terms of the frequency, f, and the wavelength, λ.

h) The *transverse velocity*, $\frac{dy}{dt}$, is the velocity of a string element moves in a direction orthogonal to the direction of propagation. Find an expression for the *transverse velocity*, ,

e) $F = \frac{1}{T} = \frac{\omega}{2\pi} \Rightarrow \boxed{F = \frac{\omega}{2\pi}}$

cycles/s   sec/cycle

F) λ is the distance that the wave moves in one period T.

$$\Rightarrow v = \frac{\lambda}{T} = \frac{\cancel{2\pi}}{k} \cdot \frac{\omega}{\cancel{2\pi}} = \frac{\omega}{k}$$

$$\boxed{v = \frac{\omega}{k}}$$

g) $v = \frac{\lambda}{T} = \lambda F \Rightarrow \boxed{v = \lambda F}$

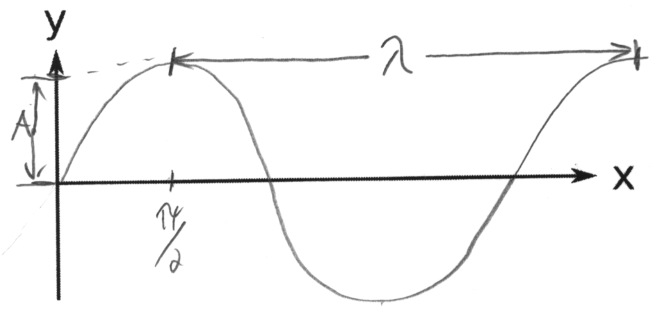
h) $y(x,t) = A\sin(kx - \omega t)$

$$\boxed{\frac{dy}{dt} = -\omega A\cos(kx - \omega t)}$$

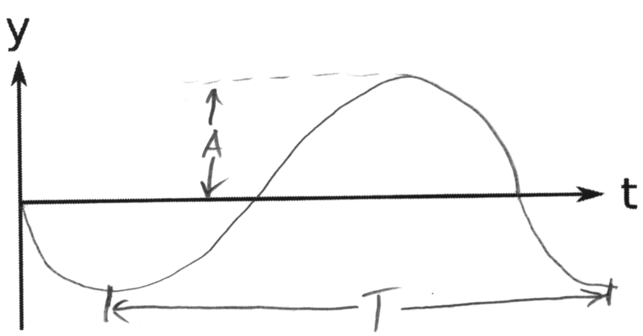
A sinusoidal wave traveling on a string is given by the following function:

$$y(x,t) = Asin(kx - \omega t)$$

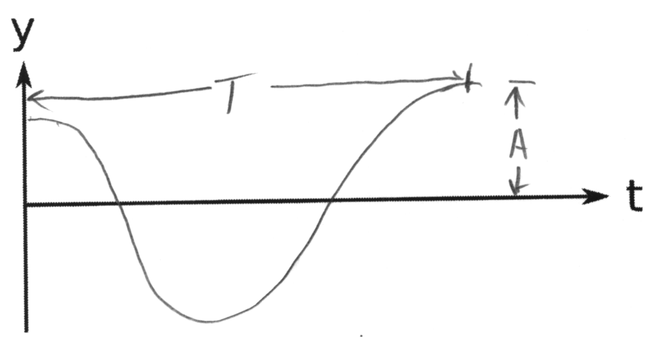
Let t=0 and plot $y(x,t=0)$ on the graph below.



Let x=0 and plot $y(x=0,t)$ on the plot below. (this is called a *history* graph)

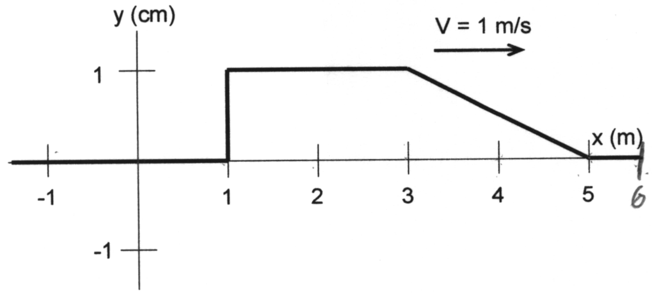


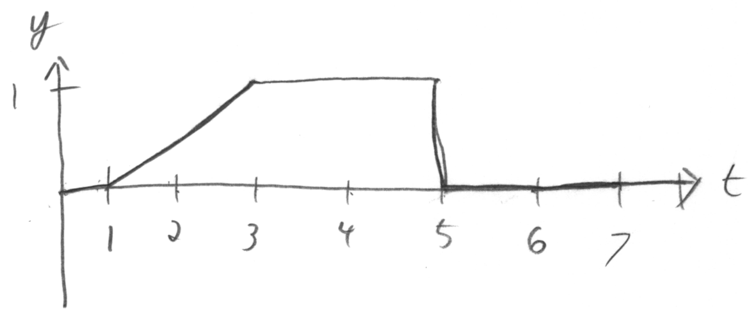
Let $kx = \frac{\pi}{2}$ and plot $y(t)$ on the plot below.



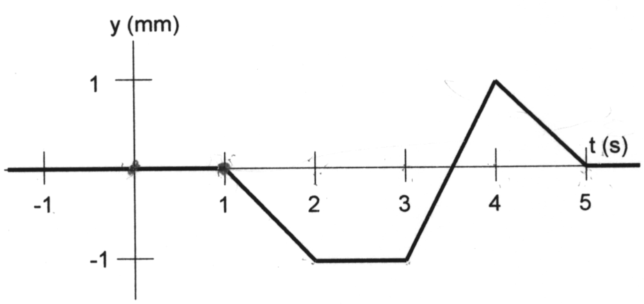
On the plots above, label the amplitude, A, wavelength, λ, and the period T.

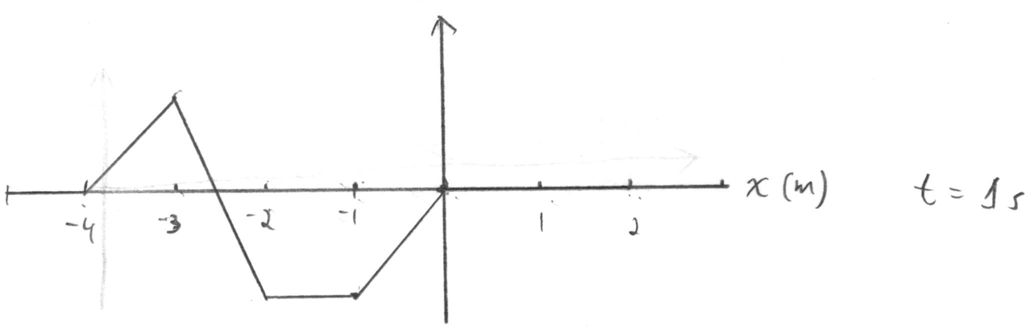
1. Draw the history graph of this wave at x = 6 m.





2. Draw the snapshot graph of this wave at t = 1 s. This graph shows the wave motion at x = 0 m, and the wave moves to the right at 1 m/s.





A sinusoidal wave on a string has a period of T=20.0 ms and travels in the negative x direction with a speed of 30.0 m/s. At t=0, an element of the string at x=0 has a transverse position of 2.0 cm and is traveling downward with a speed of 2.00 m/s.

a) What is the amplitude of the wave?
b) What is the initial phase angle?
c) What is the maximum transverse speed of an element of the string?
d) Write the complete wave function for this wave.

Given

$T = 20.0\ ms$

$v = 30.0\ m/s$

$y(t=0, x=0) = 2.0\ cm$

$v_y(t=0, x=0) = 2.0\ m/s$

Backwards traveling wave

need a phase

Starting with

$$y(x,t) = A\,SIN(kx + \omega t + \phi)$$
$$v_y(x,t) = +\omega A\,COS(kx + \omega t + \phi)$$

To write the complete wave function, we need A, ϕ, and ω
Let's apply our initial conditions.

① $y_0 = A\,SIN(\phi)$ ← $t=0, x=0,\ y_0 = y(t=0, x=0)$

② $v_{y_0} = \omega A\,COS(\phi)$ ← $v_{y_0} = v_y(t=0, x=0)$

Then we'll solve this system for A and ϕ.

Let's find ϕ first by dividing $\frac{①}{②}$:

$$\Rightarrow \frac{y_0}{v_{y_0}} = \frac{A\,SIN(\phi)}{\omega A\,COS(\phi)} \Rightarrow \frac{\omega y_0}{v_{y_0}} = \tan(\phi)$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{\omega y_0}{v_{y_0}}\right)$$

But, what's ω?

continued ↓

Waves Set 1, P4 continued.

$\omega = \frac{2\pi}{T}$, and we're given T.

so: $\boxed{\phi = \tan^{-1}\left(\frac{2\pi}{T}\cdot\frac{y_0}{v_{y0}}\right)}$ $\Rightarrow \phi = \tan^{-1}\left(\frac{2\pi}{20\times10^{-3}s}\cdot\frac{2\times10^{-2}m}{2\ m/s}\right)$

$\Rightarrow \boxed{\phi = 1.26 \text{ radians} = 0.4\pi \text{ radians}}$

* Now Find A

$y_0 = A\sin(\phi)$ and from ② $\frac{v_{y0}}{\omega} = A\cos(\phi)$

$\Rightarrow y_0^2 = A^2\sin^2(\phi)$ and $\left(\frac{v_{y0}}{\omega}\right)^2 = A^2\cos^2(\phi)$

$\Rightarrow y_0^2 + \left(\frac{v_{oy}}{\omega}\right)^2 = A^2\sin^2(\phi) + A^2\cos^2(\phi)$

$\Rightarrow y_0^2 + \left(\frac{v_{oy}}{\omega}\right)^2 = A^2\overbrace{\left(\sin^2(\phi) + \cos^2(\phi)\right)}^{1}$

$\Rightarrow \boxed{A = \left[y_0^2 + \left(\frac{v_{oy}}{\omega}\right)^2\right]^{1/2}}$ $\Rightarrow \boxed{A = \left[y_0^2 + \left(\frac{T\,v_{oy}}{2\pi}\right)^2\right]^{1/2}}$

$\Rightarrow A = \left[(2\times10^{-2}m)^2 + \left(\frac{20\times10^{-3}\cdot 2.0}{2\pi}\right)^2\right]^{1/2}$

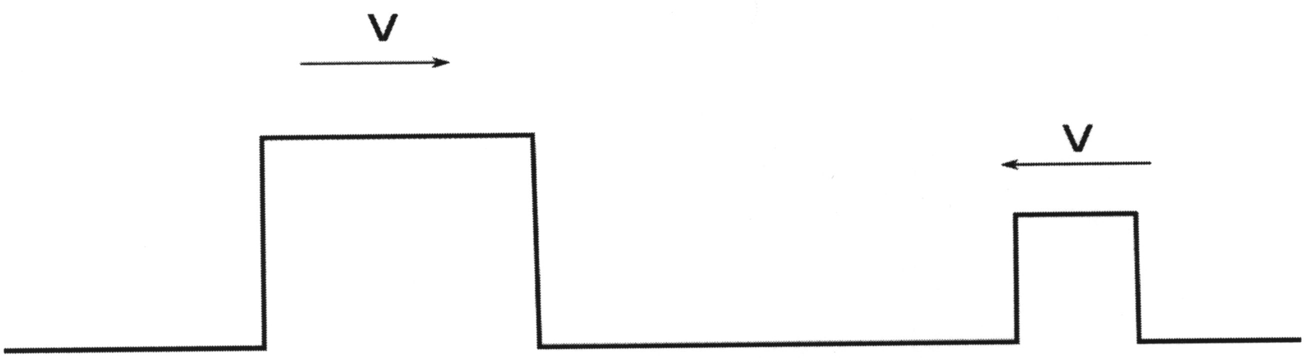
$\boxed{A = 2.1\ cm}$

Finally, $k = \frac{2\pi}{\lambda}$, $v = \lambda f = \frac{\lambda}{T} \Rightarrow \underline{\lambda = vT} \Rightarrow \boxed{k = \frac{2\pi}{vT}}$

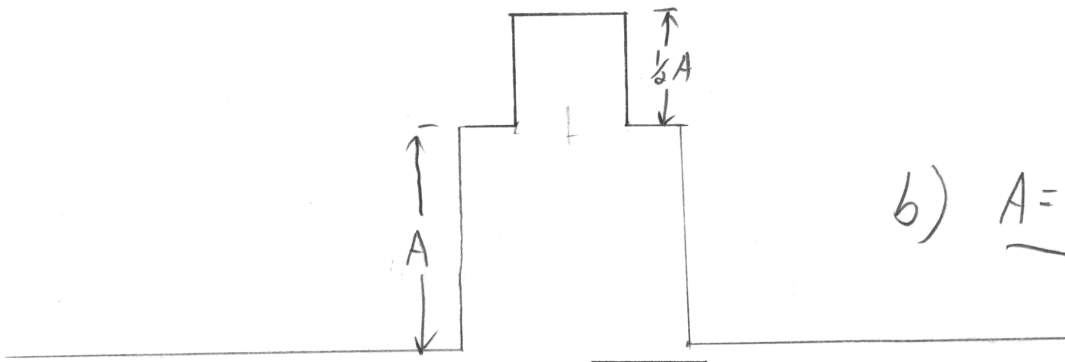
$\Rightarrow \boxed{y(x,t) = 2.1\cdot\sin\left(\frac{2\pi}{vT}x + \frac{2\pi}{T}t + 0.4\pi\right)}$

Two square pulses are traveling towards each other with a velocity v. Wave 1 is initially on the left, Wave 2 is initially on the right. Wave 2 is half the amplitude and half the width of wave 1

a) Sketch the resultant waveform when the center of wave 1 is aligned with the center of wave 2.
b) What is the resulting amplitude of the waves when their centers are aligned.
c) Sketch the resultant waveform when the center of wave 1 is aligned with the leading edge of wave 2.
d) Sketch the resultant waveform when the center of wave 2 is aligned with the trailing edge of wave 1.

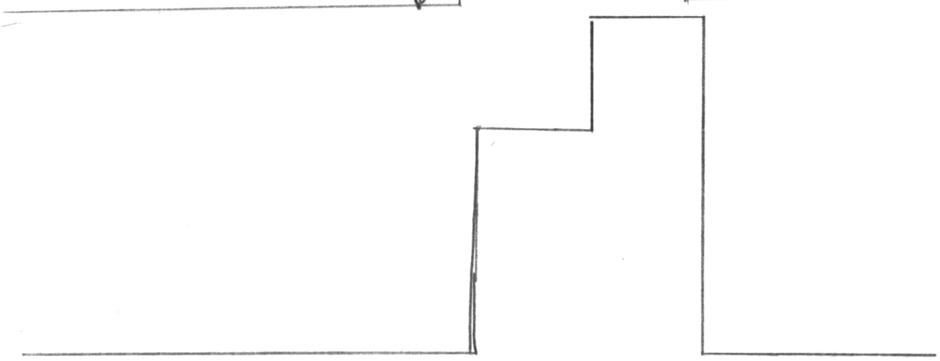


a)

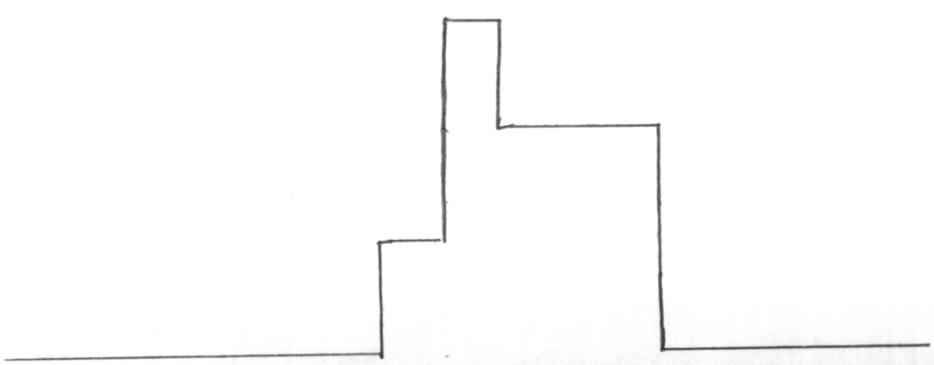


b) $A = \frac{3}{2}A$

c)



d)



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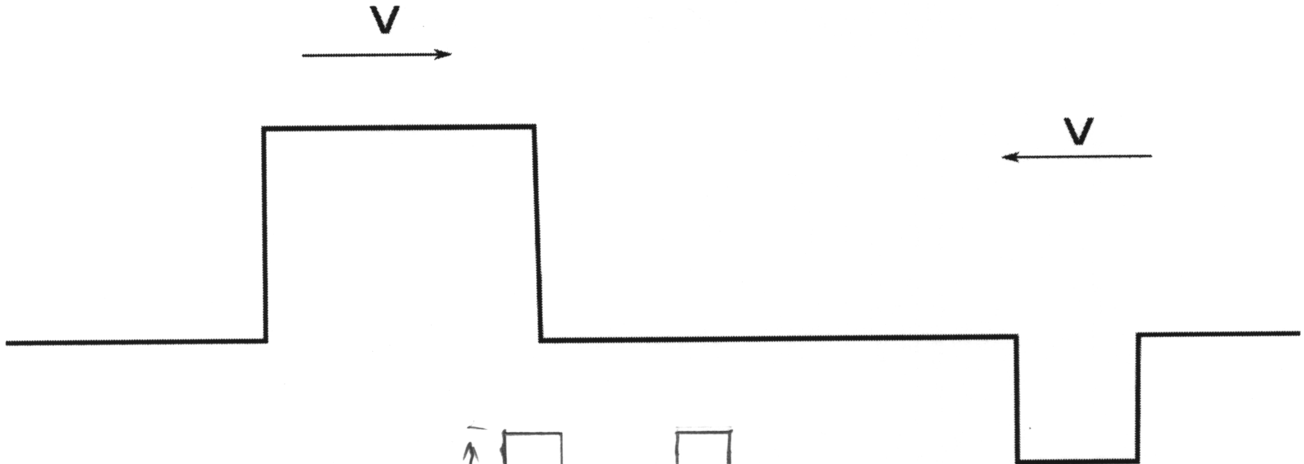
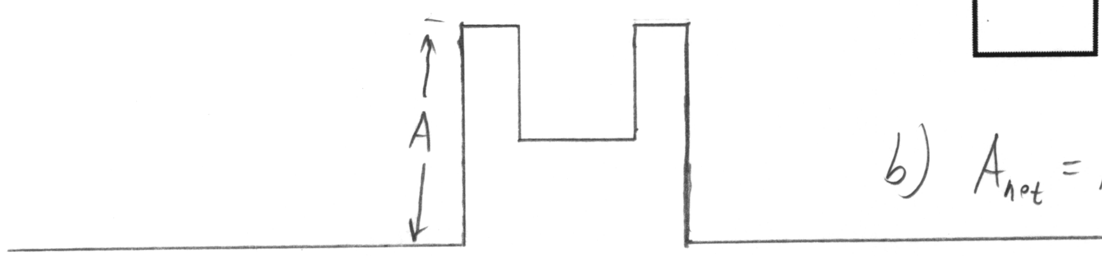
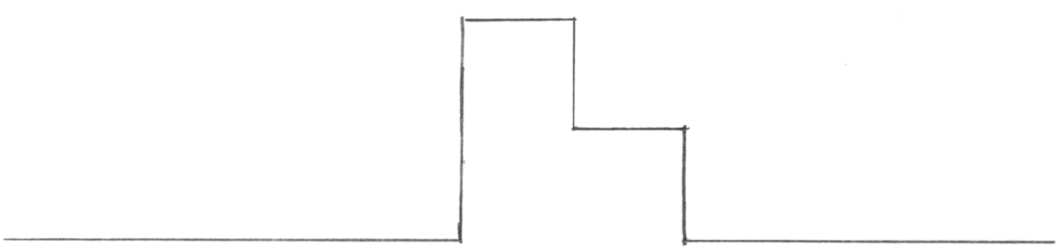


a)

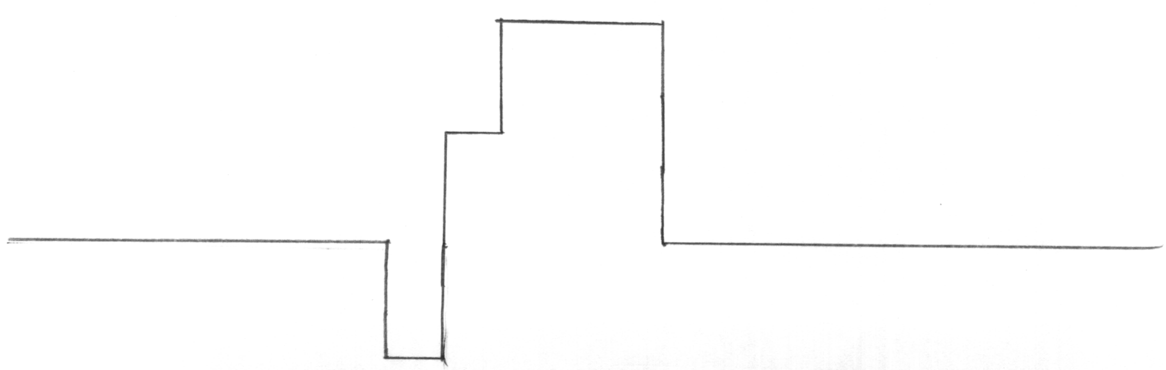


b) $A_{net} = A$

c)



d)



Two waves are traveling along a string:

$$y_1 = A\sin(kx - \omega t + \varphi)$$
$$y_2 = A\sin(kx - \omega t)$$

a) Using superposition, find the amplitude of the new wave. The following trig ID may be helpful:

$$\sin(a) + \sin(b) = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

b) Is the resulting wave a traveling wave or a standing wave? Explain.

$$y_1 + y_2 = A\left(\sin(kx - \omega t + \phi) + \sin(kx - \omega t)\right)$$

$$= 2A\cos\left[\frac{1}{2}(\cancel{kx} - \cancel{\omega t} + \phi - \cancel{kx} + \cancel{\omega t})\right]\sin\left[\frac{1}{2}(kx - \omega t + \phi + kx - \omega t)\right]$$

$$\boxed{y_{net} = \underbrace{2A\cos\left(\frac{\phi}{2}\right)}_{\text{with a new amplitude!}}\ \underbrace{\sin(kx - \omega t + \phi)}_{\text{Still a traveling wave}}}$$

And! if $\phi = 0$, $A_{new} = 2A_{old}$

So Amplitude doubles when they're in phase

And, when $\phi = \pi$, $\cos\left(\frac{\pi}{2}\right) = 0$

Then $\underline{A_{new} = 0}$, 180° out of phase cancels!

Two waves are traveling along a string:

$$y_1 = A\sin(kx - \omega t)$$
$$y_2 = A\sin(kx + \omega t)$$

a) Using superposition, find the amplitude of the new wave. The following trig ID may be helpful:

$$\sin(a) + \sin(b) = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

b) What does the plus sign in the second equation imply about the direction of propagation of the second wave?

c) Is the resulting wave a traveling wave or a standing wave? Explain.

Backwards wave

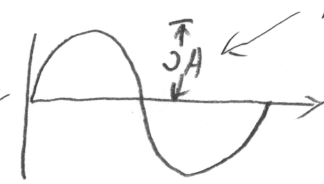
$$y_{net} = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$= 2A\cos\left[\tfrac{1}{2}(\cancel{kx} - \omega t - \cancel{kx} - \omega t)\right]\sin\left[\tfrac{1}{2}(kx - \cancel{\omega t} + kx + \cancel{\omega t})\right]$$

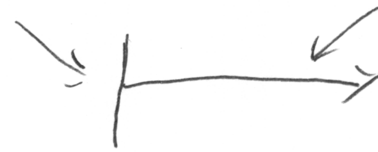
$$y_{net} = 2A\underbrace{\cos(-\omega t)}_{\text{with a time varying Amplitude!}}\underbrace{\sin(kx)}_{\text{SIN curve in } x}$$

Standing wave!

when $\cos(-\omega t) = 1$,



when $\cos(-\omega t) = 0$



when $\cos(-\omega t) = -1$