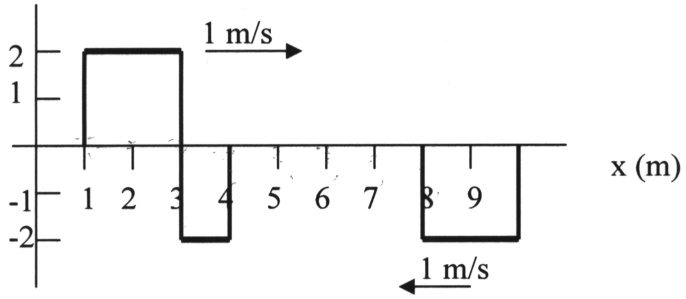
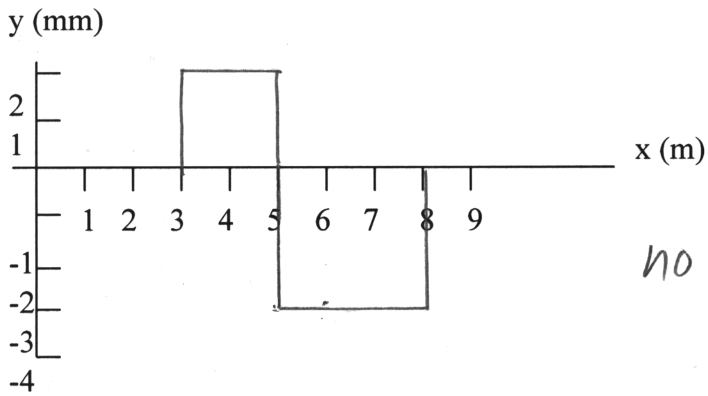
Name: ______________________ **Problems Solved** __/ 8

Below is a snapshot graph (x vs. y) of two wave pulses at t = 0s, moving with the velocities indicated in the graph.

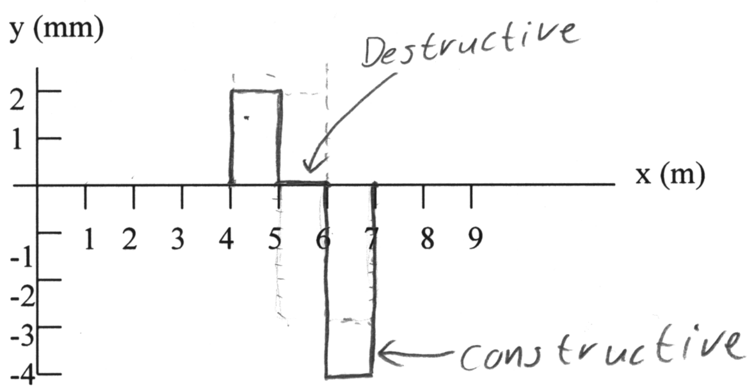


a) Draw a snapshot graph at t = 2s. At what point(s) along the x axis is there completely constructive/destructive interference?

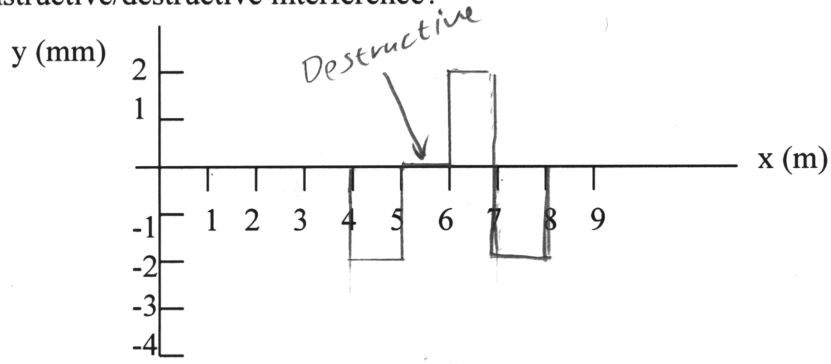


no interference yet

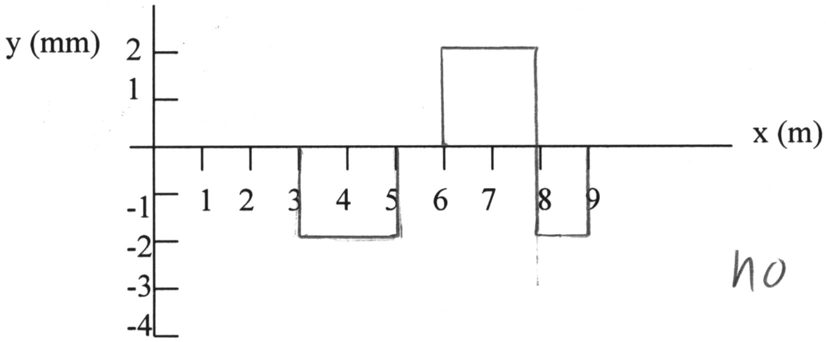
b) Draw a snapshot graph at t = 3s. At what point(s) along the x axis is there completely constructive/destructive interference?



c) Draw a snapshot graph at t = 4s. At what point(s) along the x axis is there completely constructive/destructive interference?



d) Draw a snapshot graph at t = 5s. At what point(s) along the x axis is there completely constructive/destructive interference?



no interference.

Two sinusoidal waves with identical wavelengths λ and amplitudes A travel in the same direction at a speed of v. The second wave originates from the same point as the first, but at a later time. The amplitude, A_{net}, of the resultant wave is the same as that of each of the two initial waves ($A_{net}=A$). Determine the minimum time lag between the two waves.

Given

$\lambda_1 = \lambda_2$

$A_1 = A_2$

$\vec{v}_1 = \vec{v}_2$

$\Delta x = 0$

$A_{net} = A$

want

Δt

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx - \omega t + \phi)$$

phase due to time lag $\boxed{\phi = -\omega \Delta t}$

$$y_{net} = y_1 + y_2$$

$$= A(\sin(kx-\omega t) + \sin(kx - \omega t + \phi))$$

$$\boxed{*\text{ trig ID: } \sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)}$$

$$\Rightarrow y_{net} = 2A\cos\left[\tfrac{1}{2}(\cancel{kx} - \cancel{\omega t} - \cancel{kx} + \cancel{\omega t} - \phi)\right]\sin\left[\tfrac{1}{2}(kx - \omega t + kx - \omega t + \phi)\right]$$

$$\Rightarrow y_{net} = \underbrace{2A\cos\left(-\frac{\phi}{2}\right)}_{\text{Amplitude}}\sin\left[kx - \omega t + \frac{\phi}{2}\right] \quad \text{Traveling wave}$$

$$A_{new} = 2A\cos\left(+\frac{\omega \Delta t}{\phi}\right), \quad \text{want } A_{new} = A$$

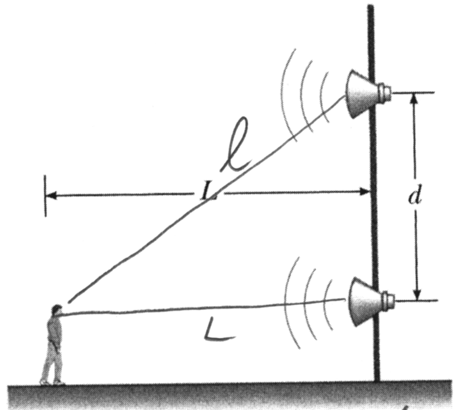
$$\cancel{A} = 2\cancel{A}\cos\left(+\frac{\omega\Delta t}{2}\right) \Rightarrow \cos\left(+\frac{\omega \Delta t}{\phi}\right) = \frac{1}{2}$$

$$\Rightarrow \frac{\omega \Delta t}{2} = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \boxed{\Delta t = \frac{2}{\omega}\cos^{-1}\left(\frac{1}{2}\right)}$$

get ω into λ: $\omega = \frac{2\pi}{T} = 2\pi f$, $v = f\lambda$

$$\Rightarrow \omega = 2\pi \frac{v}{\lambda}$$

$$\Rightarrow \boxed{\Delta t = \frac{\lambda}{\pi v}\cos^{-1}\left(\frac{1}{2}\right)}$$



$\ell = (d^2 + L^2)^{1/2}$

Two speakers are driven by the same oscillator at a frequency f. They are located a distance d from each other on a vertical pole. A man walks straight toward the lower speaker in a direction perpendicular to the poles as shown in the figure. The speed of sound is v.

Find an expression for the location of the minima between the man and the pole?

* Find phase difference

$$\phi = k\Delta\ell, \quad \Delta\ell = \ell - L = (d^2+L^2)^{1/2} - L$$

$$\Rightarrow \phi = k\left[(d^2+L^2)^{1/2}\right] - L$$

* Pick destructive interference

$$\phi_0 = (2n+1)\pi$$

Then: $(2n+1)\pi = k\left[(d^2+L^2)^{1/2} - L\right]$

Pythagorean mess

$$\Rightarrow (2n+1)\cancel{\pi} = \frac{2\cancel{\pi}}{\lambda}\left[(d^2+L^2)^{1/2} - L\right]$$

$$\Rightarrow (n+\tfrac{1}{2})\lambda = (d^2+L^2)^{1/2} - L$$

$$\Rightarrow \left[(n+\tfrac{1}{2})\lambda - L\right]^2 = (d^2+L^2)$$

$$\Rightarrow \left[(n+\tfrac{1}{2})\lambda\right]^2 - 2(n+\tfrac{1}{2})\lambda L + \cancel{L^2} = d^2 + \cancel{L^2}$$

$$\Rightarrow \boxed{L = \frac{d^2 - (n+\frac{1}{2})^2\lambda^2}{2(n+\frac{1}{2})\lambda}}$$

Two waves simultaneously present in a long string are given by the wave functions

$$y_1(x,t) = A\sin(kx - \omega t + \varphi) \text{ and } y_2(x,t) = A\sin(kx + \omega t)$$

We showed earlier that when φ=0, a standing wave is formed.

a) Show that the addition of the arbitrary phase constant changes only the position of the nodes.
b) Show that the distance between nodes is still $\frac{\lambda}{2}$.

$$y_{net} = A(\sin(kx - \omega t + \phi) + \sin(kx + \omega t))$$

$$= 2A\cos\left[\tfrac{1}{2}(\cancel{kx} - \omega t + \phi - \cancel{kx} - \omega t)\right]\sin\left[\tfrac{1}{2}(kx - \cancel{\omega t} + \phi + kx + \cancel{\omega t})\right]$$

$$\boxed{y_{net} = 2A\cos\left[-\omega t + \frac{\phi}{2}\right]\sin\left[kx + \frac{\phi}{2}\right]}$$ standing wave

nodes at $\sin\left[kx + \frac{\phi}{2}\right]$, nodes shifted by $\phi/2$

b) nodes when: $\sin\left[kx + \frac{\phi}{2}\right] = 0$

$$\Rightarrow kx + \frac{\phi}{2} = 0, \pi, 2\pi, \text{ etc...}$$

$$\Rightarrow kx + \frac{\phi}{2} = n\pi$$

$$\Rightarrow kx = n\pi - \frac{\phi}{2}$$

$$\Rightarrow \frac{2\pi}{\lambda}x = n\pi - \frac{\phi}{2}$$

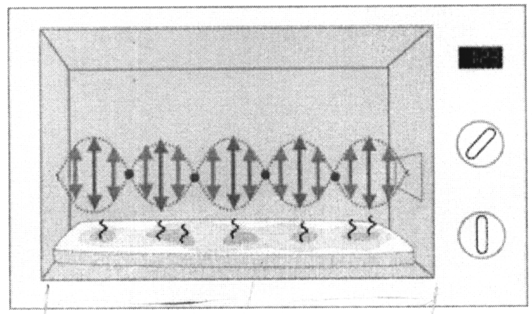
$$\Rightarrow x_n = \left(n\pi - \frac{\phi}{2}\right)\frac{\lambda}{2\pi}$$

$$\Delta x = x_{n+1} - x_n = \left[\left(\cancel{n\pi} - \cancel{\frac{\phi}{2}}\right) - \left((\cancel{n}+1)\pi - \cancel{\frac{\phi}{2}}\right)\right]\frac{\lambda}{2\pi}$$

$$\Delta x = -\frac{\cancel{\pi}\lambda}{2\cancel{\pi}} = -\frac{\lambda}{2}$$ ← oops, subtracted backwards! :)

$$\boxed{\Delta x = \frac{\lambda}{2}}$$

In microwave ovens, food is heated by standing waves of high-frequency electric fields ($f = 2.45$ GHz). Most ovens use a rotating tray because standing waves give rise to cold spots if the food is stationary. What is the shortest distance between cold spots for stationary food?



nodes occur every $\frac{\lambda}{2}$ meters

independent of the size of the cavity.

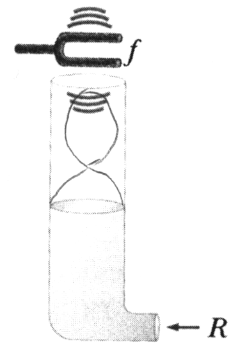
Of course, the cavity size must be chosen to give standing waves when $f = 2.45$ GHz

$d = \frac{\lambda}{2}$, $c = \lambda f$ where c is the speed of light

$\Rightarrow \boxed{d = \frac{c}{2f}} \Rightarrow d = \frac{2.99 \times 10^8 \text{ m/s}}{2 \cdot 2.45^9 \text{ cycl/s}} = .06 \text{ m}$

$\boxed{d = 6 \text{ cm}}$

In the figure, water is pumped into a tall vertical cylinder at a volume flow rate of $\frac{dV}{dt}$. The radius of the cylinder is r and at the open top of the cylinder a tuning fork is vibrating with a frequency f. As the water rises, how much time elapses between successive resonances?



* What is the distance, ΔL, between successive resonances?

In general, $L_n = n\frac{\lambda}{4}$ for a fixed wavelength λ

Then: $\Delta L = L_{n+2} - L_n = (n+2)\frac{\lambda}{4} - n\frac{\lambda}{4}$

resonance at odd n only

$$\Rightarrow \Delta L = \frac{\lambda}{4}(\cancel{n}+2-\cancel{n}) \Rightarrow \boxed{\Delta L = \frac{\lambda}{2}} \quad ①$$

* what is Δt for a change ΔL given $\frac{dV}{dt}$?

$V = \pi r^2 L$ ← Volume

$$\Rightarrow \frac{dV}{dt} = \underbrace{\pi r^2}_{\text{constant}} \frac{dL}{dt} \Rightarrow \int_{L_1}^{L_2} dL = \frac{1}{\pi r^2}\frac{dV}{dt}\int_{t_0}^{t_1} dt$$

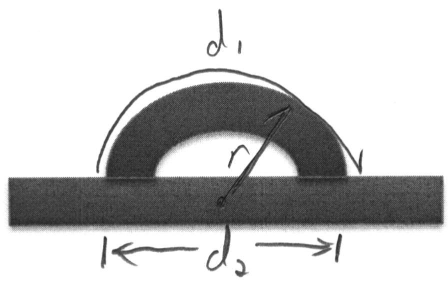
$$\Rightarrow \boxed{\Delta L = \frac{1}{\pi r^2}\frac{dV}{dt}\Delta t} \quad ②$$

So: $\frac{\lambda}{2} = \frac{1}{\pi r^2}\frac{dV}{dt}\Delta t \Rightarrow \boxed{\Delta t = \frac{\pi r^2 \lambda}{2\, dV/dt}}$

Your friend has designed a new kind of muffler for a car. Noisy exhaust gases with a 40.0cm wavelength leave the engine and travel rightward to the tailpipe through a tube that consists of a straight portion and a half-circle. Part of the sound wave travels through the half-circle and rejoins the rest of the wave, which goes directly through the straight portion. This rejoining results in interference. What is the smallest radius *r* that results in an intensity <u>minimum</u> at the tailpipe?

Given

$\lambda = 40\,cm$



Phase: $\phi = k\Delta l$

Destroy: $\phi_D = (2n+1)\pi$

$\longrightarrow (2n+1)\pi = k\Delta l$

$\Rightarrow \boxed{\Delta l = (n+\tfrac{1}{2})\lambda}$

$\Delta l = d_1 - d_2, \quad d_1 = \pi r, \quad d_2 = 2r$

$\Delta l = \pi r - 2r$

$\Rightarrow (\pi - 2)r = (n+\tfrac{1}{2})\lambda$

$\Rightarrow \quad r = \dfrac{(n+\frac{1}{2})\lambda}{(\pi - 2)}$ min r at $n=0$

$r = \dfrac{\lambda}{2(\pi-2)} = \boxed{\dfrac{\lambda}{2\pi - 1}}$