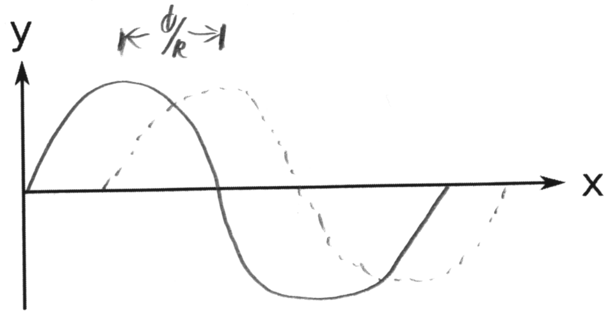
Waves – Set 3

Name: ______________________

Problems Solved __/ 6

a) Sketch a sine curve on the coordinate axis below using a solid line
b) Sketch a second sine curve at a slightly different phase (let $\varphi<\pi$) using a dashed line.
c) Label the phase shift between the two waves.



Using the picture and what you know about *super position*, fill in the table below with the relationships for constructive and destructive interference.

Relationship	Destructive	Constructive
$\varphi(n)$	$\phi=(2n+1)\pi$	$\phi=2n\pi$
$\Delta L(\varphi,k)$	$\Delta L=\frac{\phi}{k}$	$\Delta L=\frac{\phi}{k}$
$\Delta L(n,k)$	$\Delta L=\frac{(2n+1)\pi}{k}$	$\Delta L=\frac{2n\pi}{\lambda}$
$\Delta L(n,\lambda)$	$\Delta L=(n+\frac{1}{2})\lambda$	$\Delta L=n\lambda$
$\Delta t(\varphi,w)$	$\Delta t=\frac{\phi}{\omega}$	$\Delta t=\frac{\phi}{\omega}$
$\Delta t(n,\omega)$	$\Delta t=\frac{(2n+1)\pi}{\omega}$	$\Delta t=\frac{2n\pi}{\omega}$
$\Delta t(n,T)$	$\Delta t=(n+\frac{1}{2})T$	$\Delta t=nT$

$k\Delta L=\phi \Rightarrow \Delta L=\frac{\phi}{k}$, $\omega\Delta t=\phi \Rightarrow \Delta t=\frac{\phi}{\omega}$

$\Rightarrow \Delta L=\frac{(2n+1)\pi}{k}$ node $\Rightarrow \Delta t=\frac{(2n+1)\pi}{\omega}$ node

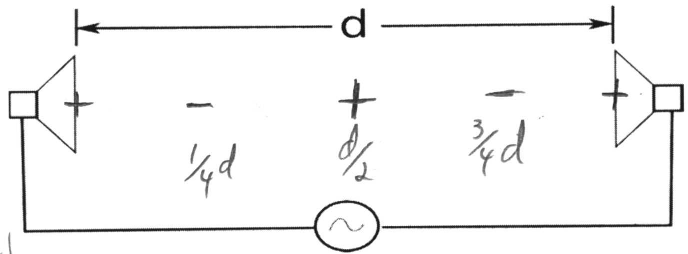
$\Rightarrow \Delta L=\frac{2n\pi}{k}$ anode $\Rightarrow \Delta t=\frac{2n\pi}{\omega}$ anode

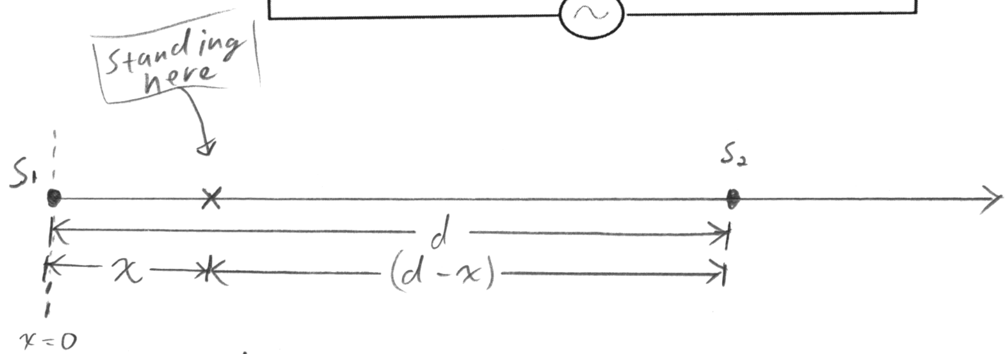
$\Rightarrow \Delta L=\frac{(2n+1)\cancel{\pi}}{2\cancel{\pi}}\lambda \Rightarrow \Delta L=(n+\frac{1}{2})\lambda$ node $\Rightarrow \Delta t=(n+\frac{1}{2})T$ node

$\Rightarrow \Delta L=\frac{2n\cancel{\pi}}{2\cancel{\pi}}\lambda \Rightarrow \Delta L=n\lambda$ node $\Rightarrow \Delta t=nT$

Two identical speakers are separated by a distance *d* are driven by the same oscillator with a frequency *f*.

a) Use the *path length difference* to find an expression for the locations of the nodes between the speakers.
b) Use the *path length difference* to find an expression for the locations of the antinodes between the speakers.
c) ~~b)~~ Let $\lambda = d$ and mark the locations of nodes with a - (minus) and the locations of antinodes with a +.





To find nodes and anti-nodes, we have to find Δl.

at any arbitrary position x: $\Delta l = |(d-x) - x|$

$$\boxed{\Delta l = |d - 2x|}$$

* To find nodes, look for points of destructive interference

$\Delta l = (n + \frac{1}{2})\lambda \Rightarrow |d - 2x| = (n + \frac{1}{2})\lambda$

Positive solution

$d - 2x = (n + \frac{1}{2})\lambda$

$\Rightarrow x = \frac{1}{2}(d - (n + \frac{1}{2})\lambda)$

$$\Rightarrow \boxed{x = \frac{d}{2} - (n + \frac{1}{2})\frac{\lambda}{2}}$$

negative solution

$2x - d = (n + \frac{1}{2})\lambda$

$$\Rightarrow \boxed{x = \frac{d}{2} + (n + \frac{1}{2})\frac{\lambda}{2}}$$

continued ↓

waves Set 3, p2 continued

so nodes are at:

$$\boxed{x = \frac{d}{2} \pm (n+\tfrac{1}{2})\frac{\lambda}{2}}$$

* To find anti-nodes, look for points of constructive interference

$\Delta \ell = 2n\pi \Rightarrow |d-2x| = n\lambda$

Positive Solution

$d - 2x = n\lambda$

$\Rightarrow x = \frac{d}{2} - n\frac{\lambda}{2}$

negative Solution

$2x - d = n\lambda$

$\Rightarrow x = \frac{d}{2} + n\frac{\lambda}{2}$

so: anti-nodes at:

$$\boxed{x = \frac{d}{2} \pm n\frac{\lambda}{2}}$$

c) let $\lambda = d$: Then -

at $n=0$, nodes at $\boxed{x = \frac{d}{2} \pm \frac{d}{4} = \frac{1}{4}d, \frac{3}{4}d}$

anodes at $\boxed{x = \frac{d}{2}}$

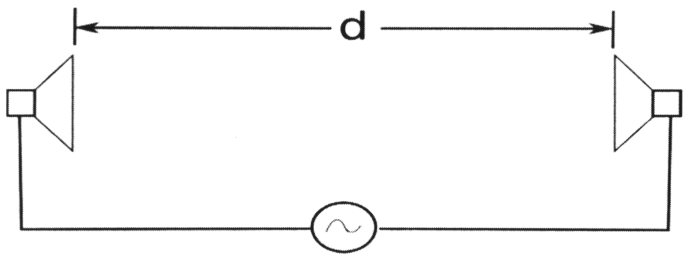
out of bounds

at $n=1$, nodes at $x = \frac{d}{2} \pm \frac{3}{4}d = -\frac{3}{8}d, +\frac{5}{4}d$

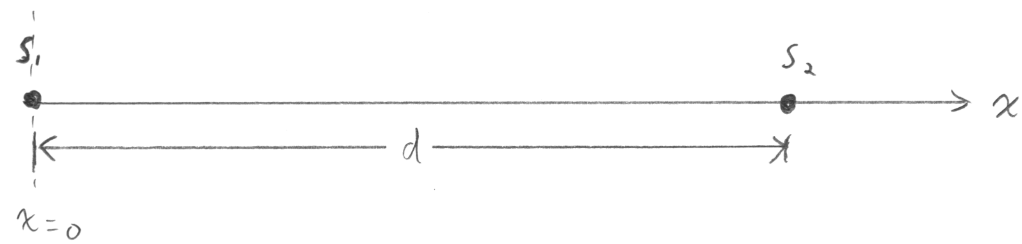
anodes at $x = \frac{d}{2} \pm \frac{d}{2} = \boxed{0, d}$

Two identical speakers are separated by a distance *d* are driven by the same oscillator with a frequency *f*.

a) Use *superposition* to show that there is an antinode directly between the two speakers.
b) Use superposition to find the amplitude of the wave to the right of the two speakers.



a)



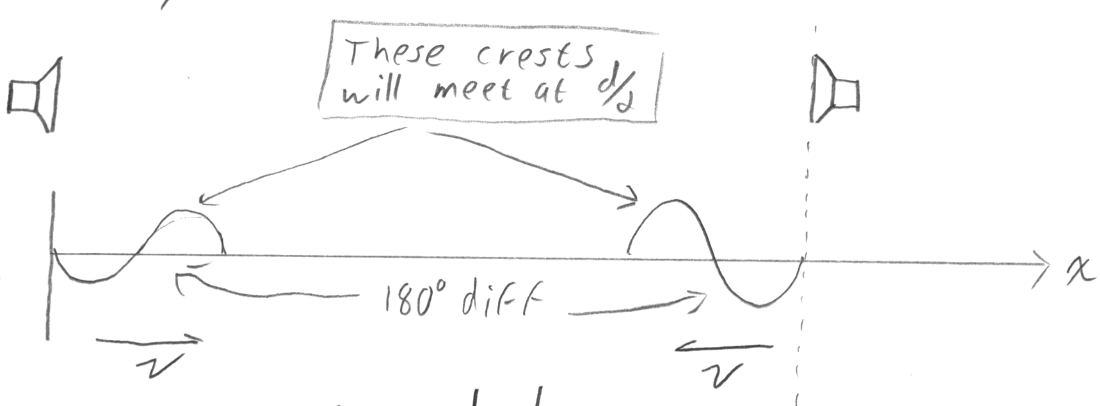
At $t=0$, let both speakers begin a SINE cycle.

Then, the left speaker emits as:

$$\boxed{y_1 = A\sin(kx - \omega t)}$$

The right speaker has a phase lag due to the spatial separation, $\boxed{\phi = -kd}$

The right speaker <u>also</u> has an addition π phase shift; let each speaker emit 1 cycle:



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But, $\sin(\theta + \pi) = -\sin(\theta)$

Traveling backwards

SO: $y_2 = A\sin(kx + \omega t - kd + \pi)$

$$\Rightarrow \boxed{y_2 = -A\sin(kx + \omega t - kd)}$$

$$y_{net} = y_1 + y_2$$

$$= A\left[\sin(kx - \omega t) - \sin(kx + \omega t - kd)\right]$$

$$\boxed{\text{Trig ID: } \sin a - \sin b = 2\cos\left[\tfrac{1}{2}(a+b)\right]\sin\left[\tfrac{1}{2}(a-b)\right]}$$

$$\Rightarrow y_{net} = 2A\cos\left[\tfrac{1}{2}(kx \cancel{-\omega t} + kx \cancel{+\omega t} - kd)\right]\sin\left[\tfrac{1}{2}(\cancel{kx} - \omega t \cancel{-kx} - \omega t + kd)\right]$$

$$\boxed{y_{net} = 2A\cos\left[kx - \tfrac{1}{2}kd\right]\sin\left[-\omega t + \tfrac{1}{2}kd\right]}$$ standing wave

* Spatial part of this wave goes as cos

anti-nodes at: $\cos\left[kx - \tfrac{1}{2}kd\right] = 1$

$$\Rightarrow kx - \tfrac{1}{2}kd = 0,$$

$$\Rightarrow \cancel{k}x = \tfrac{1}{2}\cancel{k}d \Rightarrow \boxed{x = \frac{d}{2}}$$ yay!

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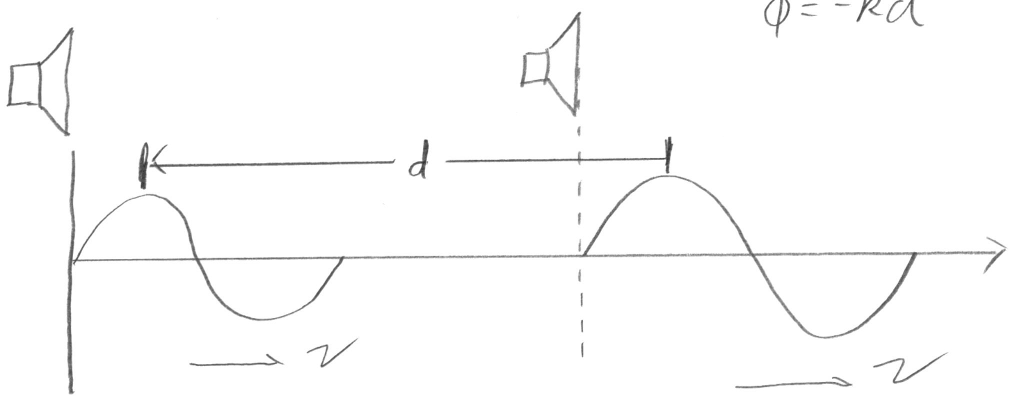
b) Beyond the second speaker, y_2 changes:

$$y_2 = +A\sin(kx - \omega t - kd)$$

$$\phi = -kd$$



So: $y_{net} = 2A\cos\left[\tfrac{1}{2}(\cancel{kx} - \cancel{\omega t} - \cancel{kx} + \cancel{\omega t} + kd)\right]\sin\left[\tfrac{1}{2}(kx - \omega t + kx - \omega t - kd)\right]$

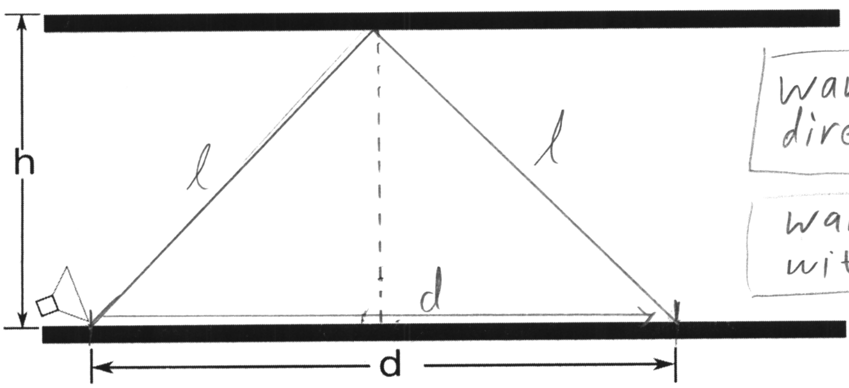
$$\Rightarrow \boxed{y_{net} = 2A\cos\left[\frac{kd}{2}\right]\sin\left[kx - \omega t - \frac{kd}{2}\right]}$$ Traveling waves

$$\boxed{A_{new} = 2A\cos\left[\frac{kd}{2}\right]}$$

In the image below, the speaker is emitting waves with a wavelength λ. Due to reflected waves from the ceiling, you find a quiet spot (node) at a distance d from the speaker.

a) Show that, due to the reflection, nodes appear when $\Delta L = n\lambda$.
b) Use the path length difference between the direct and reflected waves to find the height of the ceiling.

NOTE: When a wave reflects off of a surface, the reflected wave is 180 degrees out of phase with the incident wave and the angle of incidence is equal to the angle of reflection (that is the distance from the speaker to the ceiling is equal to the distance from the ceiling to the receiver).



wave #1 travels a distance d directly to the receiver.

wave #2 travels a distance $2l$ with one reflection.

a) we get nodes when $\boxed{\phi_D = (2n+1)\pi}$ (destructive interference)

In this case, we get a phase shift from the path length difference and an extra π radians from the reflection.

$$\boxed{\phi = k\Delta L + \pi}$$

So: $k\Delta L + \pi = (2n+1)\pi$

$\Rightarrow k\Delta L = (2n+1)\pi - \pi$

$\Rightarrow k\Delta L = 2n\pi \Rightarrow \frac{\cancel{2}\cancel{\pi}}{\lambda}\Delta L = \cancel{2}n\cancel{\pi}$

$$\Rightarrow \boxed{\Delta L = n\lambda}$$

continued ↓

b) so: $\Delta L = 2l - d$, $l = \left[h^2 + \frac{d^2}{4}\right]^{1/2}$

$\Rightarrow \Delta L = 2\left[h^2 + \frac{d^2}{4}\right]^{1/2} - d$

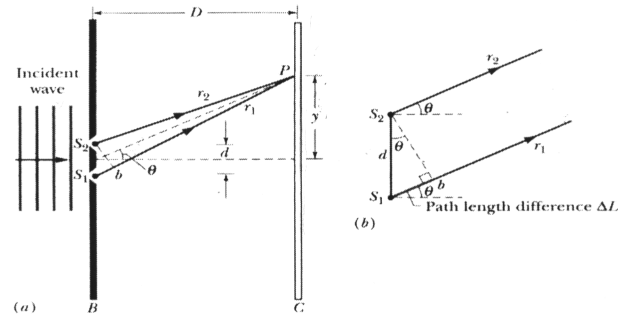
and $2\left[h^2 + \frac{d^2}{4}\right]^{1/2} - d = \lambda$ [n = 1, first node]

$\Rightarrow \left[h^2 + \frac{d^2}{4}\right]^{1/2} = \lambda + d$

$\Rightarrow h^2 + \frac{d^2}{4} = (\lambda + d)^2$

$$\Rightarrow \boxed{h = \left[(\lambda + d)^2 - \frac{d^2}{4}\right]^{1/2}}$$

The figure shows light shining on a barrier that has two slits cut into it separated by a distance d. A screen is set up a distance D away. Consider the light hitting a point P located a distance y above a horizontal line positioned midway between the two slits. Light through the bottom slit takes the path r_1 to get to the point P while light through the top slit takes the path r_2. Because $r_1 \neq r_2$, an interference pattern will appear on the screen. When $y=0$, the light intensity will be maximum.



Assume that $D >> d$ so that r_1 is parallel to r_2.
Assume that $d >> \lambda$, and $D >> y$ so that $\sin\theta \approx \theta$ and $\tan\theta \approx \theta$

a) Find an expression for the distance y_{min} to the first minimum in terms of d, D, and λ.
b) Find an expression for the distance y_{max} to the second maximum (the first being at $y=0$).

a) According to the diagram, and the $D >> d$ approximation, the path length difference goes as:

$$\Delta L = d\sin\theta \Rightarrow \phi = k\Delta L \text{ so: } \boxed{\phi = kd\sin\theta}$$

We're looking for destructive interference.

$$\phi_D = (2n+1)\pi, \Rightarrow kd\sin\theta = (2n+1)\pi$$

$$\Rightarrow \frac{2\cancel{\pi}}{\lambda} d\sin\theta = (2n+1)\cancel{\pi}$$

$$\Rightarrow \sin\theta = (2n+1)\frac{\lambda}{2d}$$

$$\Rightarrow \boxed{\theta \approx (2n+1)\frac{\lambda}{2d}}$$

But we want y: $\tan\theta = \frac{y}{D} \Rightarrow \theta \approx \frac{y}{D}$

$$\Rightarrow \frac{y}{D} = (2n+1)\frac{\lambda}{2d} \Rightarrow \boxed{y = (2n+1)\frac{D}{d}\frac{\lambda}{2}}$$

continued ↓

waves 3, p5 continued

so at $n=0$:

$$\boxed{y_{min} = \frac{D}{d} \cdot \frac{\lambda}{2}}$$

b) We want constructive interference so:

$kd\sin\theta = 2n\pi$

$\Rightarrow \frac{2\cancel{\pi}}{\lambda} d\sin\theta = \cancel{2}n\cancel{\pi}$

$\Rightarrow \sin\theta = \frac{\lambda}{d}n \Rightarrow \boxed{\theta \approx \frac{\lambda}{d}n}$

so: $\frac{y}{D} = \frac{\lambda}{d}n \Rightarrow y = \frac{D}{d}n\lambda$

at $n=1$: $\boxed{y_{max} = \frac{D}{d}\lambda}$