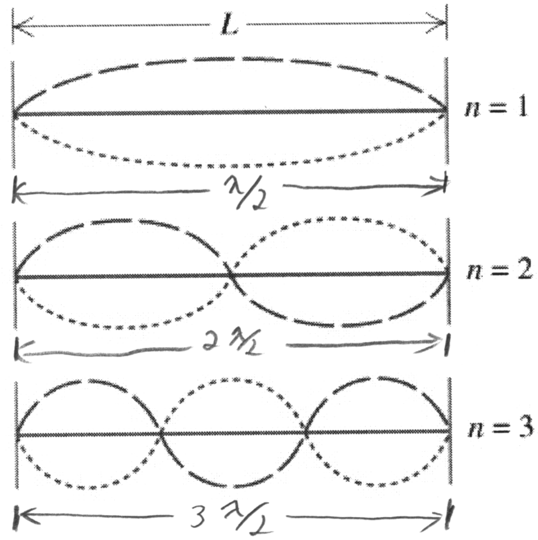
Waves – Set 4

Name: ________________________

Problems Solved __/ 6

Consider a stretched string fixed at both ends that is vibrating. The vibrations are constrained to be standing waves with nodes at the fixed ends. The first harmonic ($n=1$) is a standing wave with one anti-node. The second harmonic ($n=2$) is a standing wave with two anti-nodes, and so on.



a) What is the wavelength of the n^{th} harmonic on a string of length L?

b) What is the distance between nodes for the n^{th} harmonic on a string of length L?

a) $n=1, \quad L = \lambda/2$

$n=2, \quad L = 2\frac{\lambda}{2}$

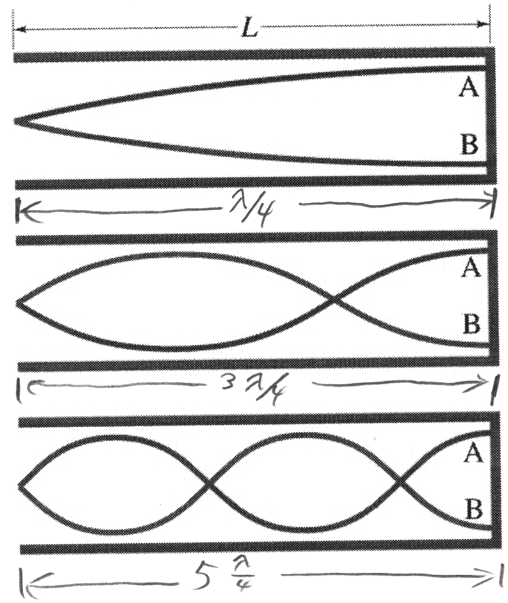
$n=3, \quad L = 3\lambda/2$

In general, $L = n\frac{\lambda}{2} \Rightarrow \boxed{\lambda = \frac{2L}{n}}$

b) Distance between nodes:

$\Delta x = \lambda/2$, always a half wavelength from node to node

$\Delta x = \frac{1}{2}\frac{2L}{n} \Rightarrow \boxed{\Delta x = \frac{L}{n}}$



Consider standing sound waves in a pipe that is closed at one end. The sound waves are constrained such that there must be an anti-node at the closed end and a node at the open end. Just to be annoying, in acoustics, the first harmonic ($n=1$) is a standing wave with one anti-node, the *third* harmonic ($n=3$) is a standing wave with two anti-nodes, the *fifth* harmonic (n=5) is a standing wave with three anti-nodes, and so on. Only the odd values of n are valid harmonics.

a) What is the wavelength of the n^{th} harmonic in a closed pipe of length L?
b) What is the distance between nodes for the n^{th} harmonic in a closed pipe of length L?

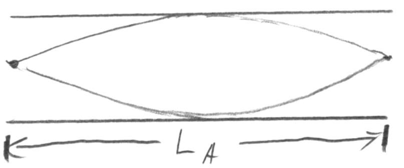
a) $n=1, \quad L = \lambda/4$

$n=3, \quad L = 3\lambda/4$ $\Rightarrow$ in general: $\boxed{L = n\lambda/4, \quad n = \text{odd} \Rightarrow \lambda = \frac{4L}{n}}$

b) $\Delta x = \frac{\lambda}{2}$ still holds.

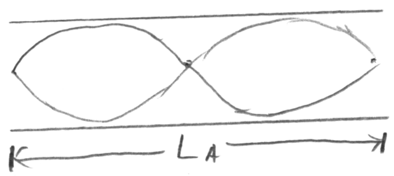
$\Rightarrow \Delta x = \frac{1}{2}\frac{4L}{n} \Rightarrow \boxed{\Delta x = \frac{2L}{n}}$

Organ pipe A, with both ends open, has a fundamental frequency of 300 Hz. The third harmonic of organ pipe B, with one end open, has the same frequency as the second harmonic of pipe A. What are the lengths of pipe A and pipe B? The speed of sound is 343 m/s.



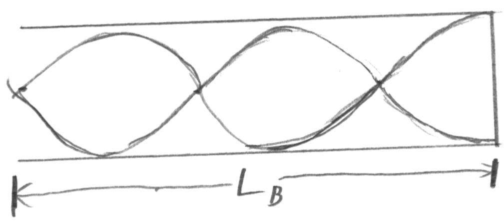
$n=1$

Given	Want
$f_{1A} = 300\ Hz$	L_A
$v = 343\ m/s$	L_B
$f_{3B} = f_{2A}$	



$n=2$

$$\lambda_{nA} = \frac{2L_A}{n}, \quad \lambda_{nB} = \frac{4L_B}{n}$$



$n=3$

Pipe A, $n=1$: $\lambda_{1A} = \frac{2L_A}{1} \Rightarrow \frac{v}{f_{1A}} = 2L_A \Rightarrow \boxed{L_A = \frac{v}{2f_{1A}}}$

Pipe A, $n=2$: $\lambda_{2A} = \frac{2L_A}{2} \Rightarrow \frac{v}{f_{2A}} = L_A \Rightarrow \frac{\cancel{v}}{f_{2A}} = \frac{\cancel{v}}{2f_{1A}} \Rightarrow \boxed{f_{2A} = 2f_{1A}}$

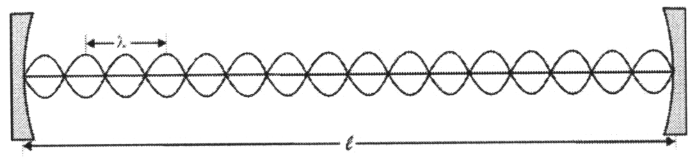
Pipe B, $n=3$: $\lambda_{3B} = \frac{4L_B}{3} \Rightarrow \frac{v}{f_{3B}} = \frac{4L_B}{3} \Rightarrow \boxed{f_{3B} = \frac{3}{4}\frac{v}{L_B}}$

Then: $f_{3B} = f_{2A} \Rightarrow \frac{3}{4}\frac{v}{L_B} = 2f_{1A}$

$$\Rightarrow \boxed{L_B = \frac{3}{8}\frac{v}{f_{1A}}}$$

$$\boxed{L_A = \frac{v}{2f_{1A}}}$$

Electromagnetic standing waves are set up in a laser cavity with two parallel, highly reflecting mirrors separated by 1.50 cm. There are nodes at both ends, just as there are for a string fixed at both ends.



a) Calculate the longest possible wavelength and lowest possible frequency of electromagnetic standing waves between the mirrors.

b) For this longest-wavelength condition, at what location(s) in the cavity does the electric field have maximum magnitude? Draw a sketch. (Note: Wavelengths on the order of cm are called microwaves, and the microwave version of a laser is called a maser.)

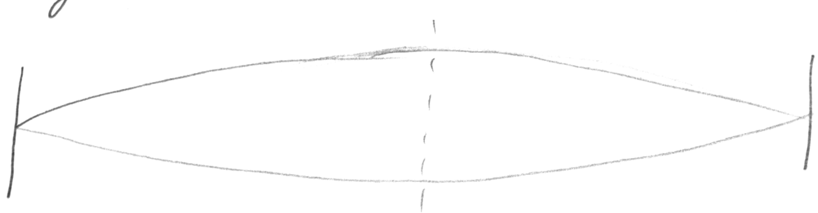
c) What harmonic (value of n) is necessary for the cavity to produce orange laser light of $\lambda_n = 600$ nm? (Note: Different types of mirrors must be used for microwave and visible light waves, but we're ignoring that detail here.)

a) This system behaves like the string: $\lambda_n = \frac{2L}{n}$

Longest wavelength at $n = 1$

$\Rightarrow \boxed{\lambda = 2L} \Rightarrow \boxed{\lambda = 3\,cm}$

b) Single anti-node at $\ell/2$

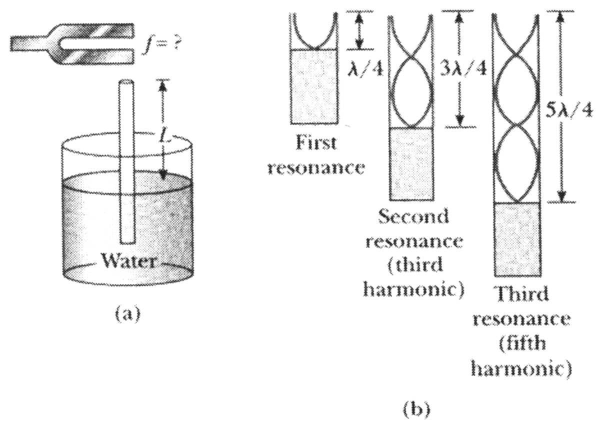


c) $\lambda_n = \frac{2L}{n} \Rightarrow \boxed{n = \frac{2L}{\lambda_n}} \Rightarrow n = \frac{3.0 \times 10^{-2}\,m}{600 \times 10^{-9}\,m} = \underline{50{,}000}$

A long vertical tube open at both ends is partially submerged in a beaker of water. A vibrating tuning fork of unknown frequency is placed near the top. The length L of the air column in the tube is adjusted by moving the tube vertically.

The smallest value of L for which a peak occurs in the sound intensity is 9.00 cm.

a) What is the frequency of the tuning fork?
b) What will L be for the next two resonances?



a) $\lambda_n = \frac{4L}{n}$, Lowest Frequency at $n=1$

$$\Rightarrow \frac{v}{f_1} = 4L \Rightarrow \boxed{f_1 = \frac{v}{4L}} \Rightarrow f_1 = \frac{343\,m/s}{4 \cdot 9 \times 10^{-2}} = \boxed{7\,kHz}$$

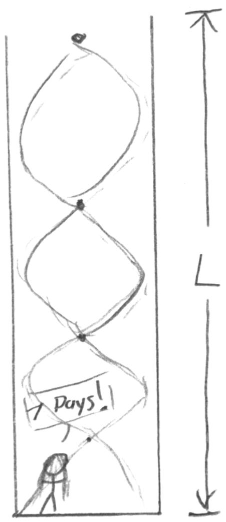
b) $$L_n = \frac{n\lambda}{4} \Rightarrow L_n = \frac{n}{4}\frac{v}{f} \Rightarrow L_n = \frac{n\cancel{v}}{\cancel{4}}\frac{\cancel{4}L_1}{\cancel{v}} \Rightarrow \boxed{L_n = nL_1}$$

$n=3$: $L_3 = 3L_1 \Rightarrow \boxed{L_3 = 27\,cm}$

$n=5$: $L_5 = 5L_1 \Rightarrow \boxed{L_5 = 45\,cm}$

Susie has fallen down a well and we need to get her out before she starts crawling out of TVs like that girl from *The Ring*. We need to quickly measure the well depth so we can buy enough rope at the hardware store, but all we have is a tone generator.

You hear two successive resonances at 51.5 Hz and 60.0 Hz. How deep is the well?



We have two succesive resonant Frequencies But we don't know what harmonic, n, we're at.

In general, $\lambda_n = \frac{4L}{n}$, $n = \text{odd}$

$$\Rightarrow \frac{v}{f_n} = \frac{4L}{n} \Rightarrow f_n = \frac{nv}{4L}$$

Then: $f_n = \frac{nv}{4L}$ and $\underbrace{f_{n+2}}_{\text{next odd } n} = \frac{(n+2)v}{4L}$

and if we take: $f_{n+2} - f_n = \frac{(n+2)v}{4L} - \frac{nv}{4L}$

$$\Rightarrow \Delta f = (n+2-n)\frac{v}{4L}$$

$$\Rightarrow \boxed{L = \frac{v}{2\Delta f}}$$